Speculative Overpricing in Asset Markets with Information Flows$^1$

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Abstract

The paper derives and experimentally tests a theoretical model of speculation in multi-period asset markets with public information flows. The speculation arises from the traders’ heterogeneous posteriors as they make different inferences from sequences of public information. This leads to overpricing in the sense that price exceeds the most optimistic belief about the real value of the asset. We find evidence of speculative overpricing with both incomplete and complete markets, where the information flow is a gradually revealed sequence of imperfect public signals about the state of the world. We also find evidence of asymmetric price reaction to good news and bad news, another feature of equilibrium price dynamics under our model. Markets with a relaxed short-sale constraint exhibit less overpricing.
1 Introduction

This paper studies equilibrium pricing dynamics in a simple dynamic asset market where traders have heterogeneous beliefs and face short-selling constraints. We analyze a model that follows in a long line of theoretical research initiated by Harrison and Kreps (HK, 1978). That line of research has had a tremendous impact in the theoretical finance literature, so it is quite remarkable that there have been no attempts to try to directly observe one of the central implications of the theory, what we refer to as speculative overpricing. By speculative overpricing we refer to both a phenomenon where current price of an asset exceeds the maximum amount any trader is willing to pay if they have to hold the asset to maturity (overpricing), and the reason traders are willing to “overpay” in equilibrium: because they believe (correctly) that in equilibrium there is a chance that at some future date another trader will value it more highly than they do. The key insight of the seminal HK paper is that speculative overpricing of a multi-period asset can arise in equilibrium if there is a combination of short-selling constraints and divergent beliefs about the fundamentals determining the underlying value of the asset. We report the results of a laboratory study of trading that implements the main features of such asset markets. The transactions data from these markets are then used to test the speculative overpricing hypothesis as well as several other testable implications of the model.

The model is by design a simple one: simple enough to study easily in the laboratory using the standard multiple-unit open-book continuous double-auction market. The specification of the source of belief heterogeneity is motivated by well-documented heterogeneity in how individuals update prior beliefs after receiving a signal that is correlated with the state of the world. Specifically, some individuals over-react to signals in the sense of updating their prior beliefs more sharply than would a Bayesian, while other individuals under-react in the sense of updating their prior more conservatively than would a Bayesian. If one considers the traders in a market as drawn from a pool of decision makers consisting of a range of over-reacters and under-reacters, then the beliefs of these traders will differ even after observing the same sequence of public signals.

Thus, as public information flows into the market, different traders interpret the same information in different ways. One can imagine a number of ways this could happen. For example, much of the public information that is broadcast about financial assets is soft – i.e., subjective in nature and open to different interpretations by traders. Stock analysts rate stocks using different methods (nearly always unpublished and subjective), and to the extent that these rating methods are unknown, different traders may give such ratings higher or lower weight as they update their beliefs about the returns to the assets being rated. This can create differences in beliefs that will be sustained over time if there is
no commonly held prior belief among the traders about the joint distribution of rating announcements and the state of the world. Such heterogeneity of beliefs can also arise and persist even with "hard" information (i.e., when the distribution of signals conditional on the state is common knowledge), if some traders follow different behavioral updating rules that are non-Bayesian. In fact, this is our starting point, as laboratory choice studies by economists and psychologists have consistently found a range of violation of Bayes' rule, or judgment fallacies. In fact, at least one paper (El-Gamal and Grether 1999) classifies subjects into categories analogous to over-reacters and under-reacters. Thus, to the extent that identical pieces of news are subject to different interpretations by different traders, heterogeneous posteriors after receiving the same bit of information can be sustained. These heterogeneous posteriors will have similar properties to posteriors generated from non-common prior models, if traders disagree about the interpretation of signals and believe their interpretation is the correct one.

In addition to the model we propose for belief heterogeneity, our model is simple in a number of ways that make it more amenable to setting up a laboratory market. We assume a finite horizon, two states of the world, \(A\) and \(B\), a binary signal space, a symmetric information structure, and a single asset, a simple Arrow-Debreu security that yields a payoff of 1 in state \(A\) and a 0 in state \(B\). As with most of the literature following HK, traders are assumed to be risk neutral. In each time period, a new public information signal arrives to the market that is observed by all traders. The equilibrium price in each period is either equal to the asset valuation of the trader type with the most optimistic belief about state \(A\) being realized or higher than the valuation of any trader, in which case we say there is a speculative premium. This speculative premium, the difference between the price and the most optimistic valuation, vanishes to zero only when a sufficient amount of information has hit the market – enough so that the current most optimistic type will remain the most optimistic for all sequences of future news. The asset is held by the trader(s) with the most optimistic beliefs, and trade subsequently occurs when a sequence of public signals leads to a new trader or traders having the most optimistic beliefs. The speculative premium is positive as long as it is still possible for switching of the set of optimistic traders at some future date. Thus, one can think of the speculative premium as representing a fair-odds bet by the currently most optimistic trader that at some future time period he will profitably sell to a more optimistic trader at some later date.

The following simple three-period example illustrates how a speculative premium arises in equilibrium with heterogeneous beliefs. Suppose the prior belief shared by all traders that the state is \(A\) is .5, and suppose that there will be one public signal observed at the start of period 2 and another public signal observed at the start of period 3.
Suppose further that conditional on state $A$ the probability the signal is $a$ is .8 and the probability it is $b$ is .2; conditional on state $B$ the probability of signal $a$ is .2 and the probability it is $b$ is .8. If all traders are Bayesian, then in the last period all traders will share common beliefs that the posterior probability of $A$ is .94 following two $a$ signals and .06 following two $b$ signals and .50 following one $a$ signal and one $b$ signal. Hence $p_3(a, a) = .94$, $p_3(a, b) = p_3(b, a) = .50$ and $p_3(b, b) = .06$. Working backward, in period 2, we have $p_2(a) = (.8 * .8 + .2 * .2)p_3(a, a) + (.8 * .2 + .2 * .8)p_3(a, b) = .80$ and $p_2(b) = (.2 * .8 + .8 * .2)p_3(b, a) + (.2 * .2 + .8 * .8)p_3(b, b) = .20$. Finally we have $p_1 = (.5 * .8 + .5 * .2)p_2(a) + (.5 * .2 + .5 * .8)p_2(b) = .50$. So, as one would expect with fully Bayesian traders there is no speculative premium.

Now suppose belief heterogeneity is very simple, with the following two possible types. Type I traders are Bayesian, so their posteriors in period 3 are the same as above, and their posteriors in period 2 after the public signals $a$ or $b$ are .8 and .2 respectively. Type II traders under-react to the information, and to keep it simple suppose they act as if the signals are uninformative and therefore don’t update at all. Then the type II traders will always believe the probability of state $A$ is .5, following any signal sequence. Thus, in period 3, type II traders will be the most optimistic after two $b$ signals and value the asset at .50, so $p_3(b, b) = .50$. For the other sequences of signals, period 3 pricing will coincide with Bayesian prices: $p_3(a, a) = .94$, $p_3(a, b) = p_3(b, a) = .5$. Working backward, in period 2, we have $p_2(a) = (.8 * .8 + .2 * .2)p_3(a, a) + (.8 * .2 + .2 * .8)p_3(a, b) = .80$ as before, but now we have and $p_2(b) = (.5 * .5 + .5 * .5)p_3(b, a) + (.5 * .5 + .5 * .5)p_3(b, b) = .50$. This in turn implies that $p_1 = (.5 * .8 + .5 * .2)p_2(a) + (.5 * .2 + .5 * .8)p_2(b) = .65$. Since the highest valuation of the asset by either trader type in period 1 is .50, this implies a speculative premium of 30%. Traders are willing to pay more than .5 because the expected market price in period 2 is equal to .65 > .50.

This example also illustrates another implication of our model of belief heterogeneity concerning the trajectory of prices: asymmetric reaction to good and bad news. Since the heterogeneity involves either over-reacting or under-reacting to news, the most optimistic trader will be an over-reacter if there is more good than bad news (e.g. two $a$ signals) and will be an under-reacter if the sequence of signals has more bad than good news (e.g. two $b$ signals). Because price responses are dampened when the marginal traders are under-reacters and exaggerated when the marginal traders are over-reacters, the absolute difference between the price and .5 is larger when there are more pieces of good news than bad news, compared to when there are more pieces of bad news than good news (holding the absolutely difference between pieces of good and bad news constant). For example, the absolute difference after two $a$ signals is $|p_3(a, a) - .5| = .44$ whereas the absolute difference after two $b$ signals is $|p_3(b, b) - .5| = 0$. Furthermore, equilibrium prices are
more volatile when there has been more good news than bad news, compared to when there has been more bad news than good news. In the former case, the price increases between periods 2 and 3 from \( p_2(a) = .80 \) to \( p_3(a, a) = .94 \), while in the latter case, the price remains unchanged between periods 2 and 3, with \( p_2(b) = p_3(b, b) = .50 \).

To test these pricing predictions derived from our model, we run laboratory-controlled asset markets where asset returns are contingent upon a binary state of the world, and the information flows consist of a sequence of 10 informative public signals. In these markets, all traders are informed that the prior on state \( A \) is \( .5 \), and also told the conditional distribution of public signals given the state of the world. Theoretically, if they are all Bayesians, there will be no belief heterogeneity and thus no speculative premium. On the other hand if there is belief heterogeneity of the kind in our model, that is, if some traders are non-Bayesians and this leads them to update beliefs as if the public information contains either less or more information than is implied by the true joint distribution of signals and states, then the theory predicts speculative overpricing relative to the Bayesian benchmark prices, and asymmetric responses to good vs. bad news. The experiments consist of two different information treatments, one in which the signals are highly informative, and another in which signals are less informative.

To set up our markets to test these theories of speculative overpricing, we impose short-sale constraints and endow our traders with adequate liquidity so liquidity constraints do not bind. We find persistent and significant overpricing. That is, in both information treatments we find pricing of the assets that is above the baseline of Bayesian updating to homogeneous posteriors. We also find that trading prices under-react to bad news compared to the reaction to good news, as implied by the model. We estimate a parametric model of the distribution of trader belief types, which allows us to test for heterogeneity of beliefs and also to back out estimates of the speculative premium. We find that the estimated speculative premiums are generally positive in those periods where the theory predicts it. These pieces of evidence are supportive of the basic HK principles of speculation with short-sale constraints, and suggest that the beliefs traders hold after receiving the same pieces of information can be divergent and prices do not necessarily reflect some measure of consensus among the traders. Finally, we also conduct an individual-level analysis of trader types, and find that traders’ holding patterns are qualitatively consistent with the model with some exceptions.

To dig more deeply into the overpricing phenomenon and to identify the extent to which it depends critically on the short-sale constraints, we run two additional variations on the simple one-market model. In variation I, which we call the complete markets treatment we open a second, complementary Arrow-Debreu security market that pays of 1 in state \( B \) and 0 in state \( A \). Traders are endowed with both assets and trading
occurs simultaneously in both markets. Thus, good news for the A market is bad news for the B market, and vice versa. This has several implications. The first is that the existence of a speculative premium is very easy to identify, because it is immediately implied whenever the sum of the prices in the two markets exceeds 1. Second, while short sales are still disallowed, if prices add to more than 1, there are some (limited) arbitrage opportunities, since a trader can sell one unit of each asset and make a sure profit. We still find overpricing in these complete markets with limited arbitrage, but less so. In our final treatment, which we call the short sales treatment we continue to have both assets, but now allow short sales by permitting traders to buy, at any time the market is operating, units of a risk-free "bundled" asset, consisting of one unit of the A asset and one unit of the B asset, for a price of 1. To keep things simple, we only allow trading in the A asset. However, this means that if the price of the A asset is higher than a trader’s valuation, any trader can buy a risk free asset bundle, and then sell the unit of A, generating an expected profit. Thus, it relaxes the short-sale constraint. We find lower prices in the relaxed short-sale constraint treatment that are closer to the homogeneous-belief Bayesian pricing.

Section 2 gives some background and discusses some of the related literature. The model and the theoretical results are presented in Section 3. Section 4 describes the experimental design and procedures. Results are presented in Section 5. Section 6 concludes with a summary of findings and suggestions for future work.

2 Background and discussion of related literature

2.1 Asset pricing experiments

There are three relevant classes of asset pricing experiments that provide a useful background and contrast with the experiment presented in this paper. First, there are a number of published multi-period asset experiments that were designed to test rational expectations equilibrium with no uncertainty, where the asset paid off certain dividends in each period and perfect foresight pricing was easily calculated. These date back to the initial study by Forsythe et al. (1982). There is a connection with this paper, in that the pricing was determined by a very simple recursive calculation starting from the last period, and equilibrium had the property that in each period the price was determined by exactly one trader type who values the asset the highest. There were two key findings in that experiment, that have been successfully replicated with a number of variations (Forsythe et al., 1984; Friedman et al., 1984). First, prices converged over time toward the rational expectations prices. Second, prices always converged from below; that is,
prices never exceeded the rational expectations prices. No speculative premium was ever observed. The current experiment differs from these experiments by introducing state uncertainty, sequential public information signals and Arrow-Debreu securities that pay off only in the last period.

A second class of asset pricing experiments, initiated by Plott and Sunder (1982) and reviewed in Sunder (1995), explicitly focuses on the questions of whether and under what conditions state-contingent claims markets successfully aggregate private information in static markets; i.e., rational expectations equilibrium in the sense of Radner (1979) and Grossman and Stiglitz (1980). The asset markets in these studies can be thought of as prediction markets, but without any information flows. Traders are endowed with private information at time 0, the market opens and clears at time 1, and private information is fully revealed by the equilibrium price as if it had been public information from the start. Thus, unlike our model in which traders receive a steady flow of many pieces of public information over the course of the market, there is usually a one-shot revelation of private or public information at the beginning or middle of the market. The central finding in these experiments is that whether or not prices are able to fully reveal all the private information held by different traders depends on the fine details of the information structure and market design (Plott and Sunder 1982, 1988; Camerer and Weigelt, 1991). More recent studies have dug deeper into questions about why standard predictions about price response to information (Asparouhova et al. 2009) and the distribution of asset holdings (Bossaerts et al. 2007) may fail. In contrast to the present paper, these approaches are based on the standard capital asset pricing model, and explore the role of heterogeneity in attitudes towards risk and ambiguity while our approach centers around heterogeneous beliefs.

The third class of experiments are the "bubble experiments" initiated by Smith et. al. (1988). Like the first class, these are multi-period asset markets where the assets generate a stream of dividends. The dividends in each period are i.i.d. draws from a known distribution. Thus, unlike our model, realizations of the outcomes in each period provide no information about the future value of the asset. Rather, the expected value of the asset is known at all points in time so there is no possibility for heterogeneous beliefs. Since dividends accrue each period, the fundamental asset value declines over time. Thus the equilibrium price dynamics for such markets are completely different from markets that share the properties of our model. In fact, if all traders are risk neutral, equilibrium prices simply decline linearly to zero over time. If there are T periods remaining, the value is simply equal to T times the expected per-period dividend of the asset.

Indeed the observed price dynamics in these bubble experiments are completely different from the equilibrium price dynamics in our model. The pricing more closely resem-
bles the original FPP experiment. In early periods, transaction prices are significantly below the equilibrium price as if there is a negative speculative premium. Because the equilibrium price declines over time while the price adjustment process drives the below-equilibrium prices upward, the transaction prices eventually catch up with equilibrium prices. When they finally reach the equilibrium price, which has been falling, the price adjustment stops, and level out. However, the equilibrium price continues to fall. This results in a situation where prices exceed fundamental value - a bubble. The surprising observation in these experiments is that transaction prices often remain approximately constant for a while even though the fundamental value is declining. Volume declines as well, and then the price collapses to its fundamental value at or near the time the terminal period when the asset expires. This is obviously not an equilibrium phenomenon, at least within the class of models that motivated those experiments or the class of models considered here. A second finding from those experiments that mirrors the FPP class of experiments is that the disequilibrium pricing (both the underpricing in early periods and the overpricing in middle-to-later periods) diminishes with experience, leading to convergence in the direction of the rational expectations equilibrium. Also noteworthy is that equilibrium pricing in the basic bubble experiment doesn’t depend on assumptions about short sales, liquidity, trader heterogeneity, complete markets, and so forth. In fact, there have been variations run with futures markets and other variations on the market organization which generally lead to similar conclusions. In one variation particularly relevant to the present paper (Porter and Smith, 2003), short sales are allowed, and the bubble phenomenon persists (and if anything is even more pronounced).

2.2 Theories of speculative trade in asset markets

Models in the finance literature have analyzed the impact of speculative trading due to heterogenous beliefs on asset prices when no short-selling is allowed. Biais and Bossaerts (1998) consider several types of heterogeneity in beliefs such as common knowledge about the belief formation rules only and derive the implied speculative value of the assets under each type. Scheinkman and Xiong (2003) find speculative bubbles with high volume and volatility in their model of differences in beliefs due to overconfidence. Our model is closest the one studied in Harris and Raviv (1993), where they look at heterogeneity in beliefs in a model with a continuum of public signals, but some traders have market power so prices are not determined competitively. Like Scheinkman and Xiong, they focus on the relationship between trading volume and price volatility.¹

¹There is a large empirical literature looking at the relationship between volume and volatility, much of it spawned by the Harris and Raviv paper. We observe a significant positive correlation between volume and volatility in our data. See Section 5.
Morris (1996) builds a dynamic version of the HK speculative trading model to show that small differences in prior beliefs can lead to a significant speculative premium. In the HK model, the heterogeneity in expectation of others’ beliefs that drove the speculative buying in anticipation of reselling was taken as given. Morris models this heterogeneity as initial differences in beliefs regarding the fundamental value of the asset so that as beliefs converge over time to the true probability, the speculative premium would tend towards zero as well. He also formalizes Miller’s (1977) claim that the most optimistic trader would hold all the assets assuming sufficient liquidity and that the most optimistic valuation would drive the equilibrium pricing. Ottaviani and Sorensen (2007) analyze the REE price dynamics in a binary prediction market where traders have heterogeneous priors and private information. They find that the prices actually under-react to information under the assumption that traders are liquidity-constrained or risk averse. They also find that more information released over time correct this initial under-reaction so that the price approaches the Bayesian posterior. Finally, Asparouhova et al. (2009) explore the implications of a different kind of behavioral bias in beliefs, by studying asset market equilibria with ambiguity averse traders.

Our model builds on these ideas about speculation and belief heterogeneity and maintains the same institutional assumptions of sufficient liquidity, risk neutrality, and short-sale constraints. However, we depart from the assumption of heterogeneous prior and updating about the probability of future dividends based on the history of dividends because ours is a model of an asset that only pays off at the end of the market based on the state of the world. The traders in our model receive public pieces of information over the life of the asset but draw different inferences about the state of the world from this information and arrive at heterogeneous posteriors. We find that the price and speculative premium depend on the sequence of information revealed and that prices track the belief of the most optimistic trader in the latter part of a market rather than converge towards a common Bayesian valuation.

3 The Model

Nature chooses the state of the world, \( \varpi \in \{A, B\} \), where the probability of \( A \) is \( p \in (0, 1) \). There is an asset market with \( T + 1 \) trading periods, \( t = \{0, 1, 2,...,T\} \) and \( I \) risk-neutral traders, \( i = \{1, 2,...,I\} \). There is one type of asset in this market. Each unit a trader holds at the end of period \( T \) pays off 1 if \( A \) is the state of the world and 0 if the state of the world is \( B \). There are no intermediate direct returns from holding the asset in periods \( 0, ..., T - 1 \). Traders observe a sequence of public signals, \( s = \{s_1, ..., s_T\} \), where \( s_t \) denotes the signal observed at the beginning of trading period \( t \). There are two sources
of earnings in these markets: trading profits or losses from transactions made during the market and the one-time state-dependent payoff for the final asset holdings at the end of the market. Each trader is initially endowed with \( x_i \) units of this risky asset and \( y_i \) units of a safe asset that pays 1 in both states of the world (“cash”). We assume traders are risk neutral, so if trader \( i \)'s final allocation of the risky asset is \( x_i^T \), and final allocation of cash is \( y_i^T \), then \( i \)'s utility is \( E^i = y_i^T + x_i^T I_A \) where \( I_A = 1 \) in state \( A \) and \( I_A = 0 \) in state \( B \).

Signals are binary, with \( s_t \in \{a, b\} \), and are generated by a symmetric stochastic process that is independent and identically distributed across periods, conditional on the state.\(^2\) Conditional on \( \varpi = A \), then \( s_t = a \) with probability \( q > 0.5 \) and \( s_t = b \) with probability \( 1 - q \). Conditional on \( \varpi = B \), \( s_t = b \) with probability \( q > 0.5 \) and \( s_t = a \) with probability \( 1 - q \). In the initial trading period, traders have no information about the state of the world except the prior \( p_0 \). Since the asset pays off only in state \( A \), we sometimes refer to the asset as \( \text{asset} \ A \) and sometimes refer to a signal \( s_t = a \) as \( \text{Good News} \) and a signal \( s_t = b \) as \( \text{Bad News} \).

### 3.1 Equilibrium Prices with Bayesian Traders

First, suppose that all traders are Bayesians and use a common Bayesian updating rule, based on the “true” stochastic process generating the signals. That is, \( q \) is common knowledge and all traders update using Bayes rule. Denote by \( \rho_t = \Pr(\varpi = A) \) be the common belief that the state of the world is \( A \), given the history of signals \( \{s_1, s_2, ..., s_t\} \). Note that \( \rho_0 = p \) because no piece of information has yet been revealed. Given \( \rho_t \), the common posterior at \( t + 1 \) if \( s_{t+1} = a \) is

\[
\rho_{t+1}^{s_t=a} = \frac{q \rho_t}{q \rho_t + (1-q)(1-\rho_t)} \tag{1}
\]

and the common posterior at \( t + 1 \) if \( s_t = b \) is

\[
\rho_{t+1}^{s_t=b} = \frac{q \rho_t}{q \rho_t + (1-q)(1-\rho_t)} \tag{2}
\]

Given that the asset pays off 1 in state \( A \) and 0 in state \( B \), and given that all agents are symmetric and risk neutral, this common posterior at period \( t \) is also the valuation of the asset at period \( t \). This is the (Bayesian) equilibrium price of the asset after any history.

\(^2\)Most of the theoretical results hold for more general signal structures. Assumptions such as a binary signal space, independence, symmetry and identical distributions over time are used for simplicity of exposition and to keep the theoretical model as close as possible to the experimental implementation.
3.2 Equilibrium with Heterogeneous Beliefs

This section contains a theory of pricing in the asset A market if traders have heterogeneous beliefs of a particular kind. As in the HK models, the traders agree to disagree. At every point in time, each trader thinks his own belief is absolutely correct. Traders have rational expectations about the distribution of future prices, in the sense that they agree on the mapping from sequences of signals to the equilibrium price, and disagree only about the fundamental value of the asset.

The traders could have subjective priors and start out with different homegrown prior beliefs $p_i^0$ that the state is $A$. However, since we state clearly to the traders that states $A$ and $B$ are equally likely in the instructions, this type of belief heterogeneity is unlikely.

We focus on a model where different traders have different perceptions about the informativeness of each signal. In this case, traders starts out in period 0 with the common prior, $p_0$, but each trader has a his own personal estimate, $q^i$, of the informativeness of the signal. These $q^i$’s could differ from the objective $q$ of the signal.

This subjective updating leads to heterogeneity in the degree to which different traders will update in response to identical sequences of signals. Specifically, it is possible that some traders over-react to news, and other traders under-react to news (relative to how a Bayesian with $q^i = q$ updates). Past experiments (e.g. Anderson and Sunder 1995; Goeree et al. 2007; El-Gamal and Grether 1995) have found evidence for this kind of judgment bias, including heterogeneity. Over-reaction to the signals is sometimes referred to as base-rate neglect or a base-rate fallacy, and under-reaction is sometimes referred to as conservatism (Camerer 2003).

3.2.1 Trader Types with Subjective Updating Heterogeneity

Consider possible trader types characterized by the parameter $\theta \in [0, \infty]$. A trader with type $\theta_i$ will treat a single signal as if it had the informational equivalent of $\theta$ independent signals, each of informativeness $q$. Thus, $\theta_i$ measures how much trader $i$ under-reacts ($\theta_i < 1$) or over-reacts ($\theta_i > 1$) to the public signal, relative to $q$.

Let $\rho_{it} = \Pr(\varpi = A)$ be trader $i$’s belief at the beginning of period $t$ that the state of the world is $A$ given some history of public signals $\{s_1, s_2, ..., s_t\}$. Since traders share a common prior when no information has yet been revealed, $p_{i0} = p$ for all $i \in I$. Given $\rho_{it}$ trader $i$’s updated posterior after observing $s_{t+1} = a$ if $i$ is type $\theta_i$ equals:

$$\rho_{it+1}^{s_{t+1}=a}(\theta_i) = \frac{q^\theta_i \rho_{it}}{q^\theta_i \rho_{it} + (1 - q)^\theta_i (1 - \rho_{it})} \tag{3}$$

and after observing $s_t = b$ equals:
\[ \rho_{it+1}^{s_{t+1}=b} (\theta_i) = \frac{q^\theta_i \rho_{it}}{q^\theta_i \rho_{it} + (1 - q)^\theta_i (1 - \rho_{it})} \]  

Three examples illustrate this. We refer to traders with \( 0 \leq \theta < 1 \) as Skeptical types. At one extreme is the \( \theta = 0 \) type. Traders of this type believe that the signals are just noise, as if the signal distribution was independent of the state. They do not update their prior after either signal \( a \) or signal \( b \). Such a type’s probabilistic belief that \( A \) is the state of the world remains unchanged for any sequence of signals. That is,

\[ \rho_{it+1}^{s_{t+1}=a} (0) = p \quad \forall t, s_1, ..., s_t \]

We refer to \( \theta = 1 \), as the Bayesian type. Traders of this type simply update as if they are receiving signals of strength \( q \) so the posteriors are equivalent to those of a Bayesian.

\[ \rho_{it+1}^{s_{t+1}=a} (1) = \frac{q \rho_{it}}{q \rho_{it} + (1 - q)(1 - \rho_{it})} \]

We refer to \( \theta > 1 \) as Fickle types. Traders of this type update as if the informativeness of signals is higher than \( q \). For extremely high values of \( \theta \), gullible traders treat a signal as nearly a full revelation of the state. For example, if \( p = .5 \), \( q = .7 \) and \( \theta_i = 10 \), then, after the first signal, if \( s_1 = a \), trader \( i \)'s posterior is

\[ \rho_{i1}^{s_1=a} (10) = \frac{.7^{10}}{.7^{10} + .3^{10}} = .9998 \]

Of course, this does not imply that Fickle types’ beliefs get immediately stuck near 0 or 1. In fact, exactly the opposite is the case. In the above example, if \( s_2 = b \), then \( i \)'s beliefs go back to \( \rho_{i2}^{s_1=a, s_2=b} = .5 \), and then if \( s_3 = b \) again, the belief would be \( \rho_{i3}^{s_1=a, s_2=b, s_3=b} = .0002 \). Thus, fickle types have relatively volatile beliefs while skeptical types have relatively sticky beliefs.

### 3.2.2 Equilibrium Prices

We maintain the assumptions of no short-sale (implemented in the experimental design) and sufficient liquidity so that any trader can hold all units of the risky asset, for any price less than or equal to 1. Under these assumptions, we can apply arguments similar to HK models and characterize the equilibrium price dynamics in our model.

For the remainder of the paper we will assume \( p = 0.5 \).\(^3\) In this case, the updating process depends only on the number of \( a \) signals, which we denote by \( \alpha \), and the number of \( b \) signals, which we denote by \( \beta \equiv t - \alpha \). Hence, in the baseline case of homogeneous

\(^3\)The model extends in a straightforward way to the more general case of \( p \neq -0.5 \).
Bayesian, beliefs, \((\theta_i = 1 \ \forall i)\), the equilibrium price of the asset at period \(t\), \(P^B_t\), following any history in which the number of \(a\) signals is \(\alpha\) equals:

\[
P^B_t = \rho_t = \frac{q^\alpha(1 - q)^{t-\alpha}}{q^\alpha(1 - q)^{t-\alpha} + q^{t-\alpha}(1 - q)^\alpha}
\]

Given the way we have defined our different trader types, and with the additional assumption that \(p = 0.5\), a trader’s posterior beliefs and equilibrium prices will depend only on trader types, and the difference between the number of good news signals and bad news signals, \(\delta = \alpha - \beta\). Specifically, the current belief of trader type \(\theta_i\) can be expressed as:

\[
\rho^\alpha_{it}(\theta_i) = \frac{q^{\theta_i\alpha}(1 - q)^{\theta_i(t-\alpha)}}{q^{\theta_i\alpha}(1 - q)^{\theta_i(t-\alpha)} + q^{\theta_i(t-\alpha)}(1 - q)^{\theta_i\alpha}}
= \frac{1}{1 + (\frac{1-q}{q})^{\theta_i\delta}}
\]

Define \(\bar{\rho}_t(\alpha) = \max_{\theta_i \in I} \{\rho^\alpha_{it}(\theta_i)\}\) to be the most optimistic belief among the traders at period \(t\) about \(A\) being the state of the world and define \(\theta^*_t = \arg \max_{\theta_i \in I} \{\rho^\alpha_{it}(\theta_i)\}\). That is, \(\bar{\rho}_t(\alpha) = \rho^\alpha_{it}(\theta^*_t)\). The equilibrium price of the asset at period \(t\) given the number of \(a\) signals, \(P_t(\alpha)\), must be equal to the highest expect return of holding it to the next period \(t + 1\) in equilibrium. If the price is strictly lower than the highest expected return, then the trader(s) with the highest expected return would demand infinite units of the asset and the market would not clear. On the other hand, if the price is strictly higher than the highest expected return, then the demand for the asset would be zero and that price cannot be the equilibrium price.

Let \(\varphi_t(\alpha)\) denote the most optimistic belief about the probability of an \(s_{t+1} = a\) after \(\alpha\) \(a\) signals up to period \(t\). Then:

\[
\varphi_t(\alpha) = \bar{\rho}_t(\alpha)(\frac{q^{\theta^*}}{q^{\theta^*} + (1 - q)^{\theta^*}}) + (1 - \bar{\rho}_t(\alpha))(\frac{(1 - q)^{\theta^*}}{q^{\theta^*} + (1 - q)^{\theta^*}})
\]

(5)

Note that this is not equivalent to the most optimistic belief about \(A\) being the state of the world because \(\varpi = A\) does not necessarily mean \(s_{t+1} = a\). Traders can only update their beliefs and asset valuations based on the sequence of signals revealed so pricing depends upon the signals revealed and expectations about future signals. The \(\theta\) type with the most optimistic belief about the state of the world being \(A\) also has the most optimistic belief about the next signal being \(a\). Now we can specify the equilibrium price

\[
P_t(\alpha) = \varphi_t(\alpha)P_{t+1}(\alpha + 1) + (1 - \varphi_t(\alpha))P_{t+1}(\alpha)
\]

(6)
The first term on the RHS is equal to most optimistic belief about an \( a \) signal being revealed next period multiplied by the price next period if \( s_{t+1} = a \). The second term is equal to the corresponding belief about a \( b \) signal being revealed next period multiplied by the price next period if \( s_{t+1} = b \).

At the last period, period \( T \), the price is equal to the maximum of the types’ respective beliefs that the state is \( A \).

\[
P_T(\alpha) = \bar{\rho}_T(\alpha)
\]

(7)

The equilibrium pricing scheme is uniquely pinned down by these two equations because we can now solve backwards for the equilibrium price at every period. Note that our model and this specification of the equilibrium price dynamics departs from the original HK and Morris models in two specific ways. First, while they are looking at a finite truncation of an infinite market, we analyze a market with \( T < \infty \) periods. Because of our finite horizon, we can rule out immediately other possible pricing trajectories involving bubbles or Ponzi schemes that Harrison and Kreps and Morris consider. Second, while the uncertainty in their analysis is whether the asset will pay off a dividend after each period, the asset that we analyze only pays off at the end of the market after \( T \) periods. In their analysis, the price dynamics and speculative premiums are driven by heterogenous beliefs about dividend payoffs in future periods based on the past dividend stream. In our analysis, the price dynamics and speculative premiums are driven by heterogeneous updating of beliefs about the state of the world that determines final asset payoff.

### 3.3 Speculative Premium

The traders are characterized by a realized distribution of \( \theta \) ranging from \( \theta_{\min} \) to \( \theta_{\max} \) where \( 0 \leq \theta_{\min} < \theta_{\max} \). We compare the price in each period to the traders’ valuations and derive several results. Recall that \( \alpha \) is the number of \( a \) signal being realized, \( \beta = t - \alpha \) is the number of \( b \) signals, and there are \( t = 0, 1, 2, \ldots, T \) periods. Following Morris (1996), we first give two definitions of optimistic traders.

**Definition 1** Trader \( k \) is a current optimist at \( t \) if \( \rho_{kt}(\alpha) = \bar{\rho}_t(\alpha) \).

**Definition 2** Trader \( k \) is a permanent optimist at \( t \) if \( \rho_{kt'}(\alpha') = \bar{\rho}_{t'}(\alpha') \) for all \( t' = t + 1, \ldots, T \) and for all \( \alpha' = \alpha, \alpha + 1, \ldots, \alpha + t' - t \).

In words, a permanent optimist at \( t \) has the (weakly) most optimistic belief at \( t \) that \( A \) is the state of the world, and this remains true for every possible continuation sequence of signals. We can now define the speculative premium, \( \pi_t(\alpha) = P_t(\alpha) - \bar{\rho}_t(\alpha) \). The speculative premium is the amount by which the price exceeds the maximum hold-to-maturity valuation of all traders. The speculative premium can be calculated recursively.
by:

\[ \pi_t(\alpha) \equiv \varphi_t(\alpha)[\pi_{t+1}(\alpha + 1) + \bar{\rho}_{t+1}(\alpha + 1)] + (1 - \varphi_t(\alpha))[\pi_{t+1}(\alpha) + \bar{\rho}_{t+1}(\alpha)] - \bar{\rho}_t(\alpha) \]

It is straightforward to prove that \( \pi_t(\alpha) \geq 0 \) for all \( t = 0, \ldots, T \) and for all \( \alpha = 0, 1, \ldots, \).

The following result shows that the speculative premium is strictly positive if and only if there is no permanent optimist.

**Proposition 1**

(i) If \( |\delta| < T - t \), then no trader is a permanent optimist and \( P_t(\alpha) > \rho_{it}(\alpha) \) \( \forall i \), and \( \pi_t(\alpha) > 0 \)

(ii) If \( |\delta| \geq T - t \), then there is a permanent optimist, and \( \pi_t(\alpha) = 0 \)

**Proof.** See Appendix A.

In our experimental setup, there are 10 signals released in each market so \( T = 10 \). In this case, the condition for positive speculative premium stated in Part (i) of Proposition 1 simplifies to \( \alpha < 5 \) and \( \beta < 5 \). With less than 5 pieces of both Good and Bad News, there is always a possibility of enough additional pieces of either Good or Bad News before the end of the market so that the current optimist at period \( \alpha + \beta = t \) is no longer the optimist. However, if \( \alpha \) is greater than or equal to 5, this is not possible. The \( \theta_{max} \) trader(s) is the permanent optimist because there will always be at least as many pieces of Good News as there are Bad News regardless of future pieces of information. Similarly, if \( \beta \) is equal to or greater than 5, then the \( \theta_{min} \) trader(s) is the permanent optimist. The permanent optimist(s) will continue to hold the assets until the end of the market so there is no speculative premium once a permanent optimist exists.

### 3.4 Asymmetric Response to Good versus Bad News

We also compare by how much the price at time \( t \) differs from the flat prior \( p = 0.5 \) when \( \alpha \) pieces of Good News have been revealed versus when \( t - \alpha \) pieces of Good News have been revealed. An implication of our model is that equilibrium prices react more to pieces of Good News than pieces of Bad News.

**Proposition 2** \( 1 - P_t(\alpha) < P_t(t - \alpha) \) \( \forall \alpha > \frac{t}{2} \)

**Proof.** See Appendix A.

### 3.5 Horizon Effect

Next we explore another pattern in the speculative premiums: the horizon effect. As the number of periods until the end of the market decreases, the speculative premium is
non-increasing. The first part of this horizon effect follows directly from Proposition 1: if a sufficiently large number of Good or Bad News signals have been revealed (|δ| ≥ T − t), then the speculative premium, π_t(α), will equal zero for all subsequent periods. This is true because with enough pieces of Good News or Bad News, relative to the number of periods remaining, there is no possibility that the most optimistic trader will change, no matter how many pieces of Good News or Bad News follow.

The second part of the horizon effect is that in periods where |δ| < T − t, the speculative premium is non-decreasing in the horizon for fixed δ. With fewer trading periods left in the market, the probability of δ switching between positive and negative also decreases; therefore, the speculative premium cannot increase.

**Proposition 3**  
π_t(α) ≤ π_{t−2}(α − 1) ∀T ≥ t > 1 and α < t

**Proof.** See Appendix A.

Note that since t = α + β the value of δ is the same at histories (α, t) and (α − 1, t − 2). Thus the proposition is simply that the speculative premium is (weakly) higher in earlier periods, holding δ = α − β constant.

### 4 Experimental Design and Procedures

We conducted six baseline sessions with a total of 68 individual traders. Table 1 describes the experimental setup for all baseline sessions. The traders were registered Caltech students who were recruited by e-mail solicitation. Sessions were conducted at the Social Science Experimental Laboratory at Caltech. Instructions were read out loud and screen displays were explained using a Powerpoint slide show in front of the laboratory at the beginning of each session. All interactions in a session took place through the computer interface. The trading interface used the open source software package Multistage Games⁴.

In each market of a session, a coin is flipped before the market opens to determine the state of the world: either State A (heads) or State B (tails). The result of the coin-flip is not announced until the market closes. We then organize and allow trading in a single asset market, where each subject can take trading positions as buyers and/or sellers.⁵ To ensure adequate liquidity, all traders had a large initial cash endowment. Traders are endowed with three units of the asset. No short selling is allowed. There is also a bankruptcy constraint which does not allow any trader to engage in any transaction if her cash holdings go below zero. Each trader receives payoffs at the end of the market based on final asset holdings and cash holdings. All prices are in integers values. In state

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⁴[http://multistage.ssel.caltech.edu/](http://multistage.ssel.caltech.edu/)

⁵For more procedural details, see the sample instructions in Appendix B.
A, each unit of the Asset pays off 100 experimental dollars at the end of the market; in State B, each unit of the Asset pays off 0.\textsuperscript{6}

There are eleven trading periods in each market, each period lasting for 50 seconds. Trading is opened for the first trading period, and follows an open continuous double auction procedure. Subjects can type in bids to buy and/or offers to sell as many units of the asset as they want subject to the liquidity and short sale constraints. When a bid or offer is entered, it immediately shows up on the public bid and offer book, which is displayed in the center of each subject’s screen. Only improving bids and offers could be made, and only the most recent current bid and offer are active. Subjects can accept a bid or offer by highlighting it with the mouse and clicking the “accept” button, subject to the bankruptcy and short sales constraints\textsuperscript{7}. Subjects can also cancel an active bid or offer they had previously posted. At the 50 second mark, all unfilled bids and offers are cleared from the book, and the second 50-second trading period begins.

At the start of the second trading period, the binary public signal (Good News or Bad News) is drawn according to the distribution conditional on the original coin flip and publicly announced to all subjects. Holdings are carried over across periods. Trading occurred in the second period following the same rules and procedures as in the first period. After 50 seconds, the book is again cleared and a new public signal is drawn and announced. This continued for 11 trading periods (until 50 seconds after the 10th public signal had been announced). After the last trading period, the market closes, the state of the world is revealed, and each trader’s cash on hand is credited based on final holdings of the asset. We then proceed to open another market, with procedures identical to the first market. The experimenter again tosses a flip coin to determine the state, trading screens are reset, asset endowments are reset at three units for each trader, and cash holdings are carried over from the first market. This continues until a total of six markets are conducted, after which each subject is paid in private the sum of their earnings in all six markets, plus a show-up fee of $10. Each session lasted between 1.5 to 2 hours, including instructions and payment.

The public signal is generated by rolling a die each time as described in the instructions (see appendix). In three of the sessions, the signal distribution corresponds to an informativeness of \( q = \frac{5}{9} \), and in the other three sessions, the signal informativeness was \( q = \frac{6}{9} \). These conditional signal distributions were explained carefully and accurately to

\textsuperscript{6}In four of the sessions, the state 2 payoffs equaled 20 instead of 0. In the analysis of data, all transactions and prices and rescaled on a 0 to 100 scale. Experimental dollars were converted to U.S. dollars using an exchange rate of either .01 or .02, depending on the session.

\textsuperscript{7}If a bid (offer) was for multiple units, a seller (buyer) can sell to (buy from) the bidder (offerer) multiple units by clicking “accept” repeatedly. After each acceptance, the book reduces the number of units available for sell (buy) to (from) that bidder (seller) by one unit.
the subjects.

<table>
<thead>
<tr>
<th>Session</th>
<th>Signal</th>
<th>Payoffs</th>
<th># Mkts</th>
<th># Subs</th>
<th>Period (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.56</td>
<td>100,20</td>
<td>6</td>
<td>12</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>0.56</td>
<td>100,20</td>
<td>6</td>
<td>12</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>0.56</td>
<td>100,0</td>
<td>6</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>0.67</td>
<td>100,20</td>
<td>6</td>
<td>12</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>0.67</td>
<td>100,20</td>
<td>6</td>
<td>12</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>0.67</td>
<td>100,0</td>
<td>6</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 1: Summary of Baseline Experimental Sessions

Two additional sessions with a total of 20 traders were conducted using a complete market design. That is, there were two assets, one for each state of the world. Traders were endowed with three units of each asset, and both asset markets were open simultaneously. In other respects they were conducted the same as the markets described above. They are discussed in more detail in section 5.6.1. We also conducted three sessions with 36 more traders where short-selling was allowed. These are described in Section 5.6.2.

5 Results

The results are organized as follows. First, we analyze whether our markets produce prices that are in excess of prices that would arise if traders had homogeneous beliefs. We refer to this as the overpricing hypothesis, and it is the primary hypothesis we examine in this paper. This section also includes a summary of the pricing data from all the baseline sessions.

We then go on to test another important implication of the model: asymmetric reaction to Good News versus Bad News. Is the absolute difference between the asset price and 50 after $\alpha$ pieces of good news and $\beta < \alpha$ pieces of bad news are revealed higher than the price after $\alpha$ pieces of bad news and $\beta$ pieces of good news are revealed?

In the next section, we offer some finer tests of the implications of the theoretical model based on heterogeneous beliefs. We use a minimum squared deviation procedure to estimate $\theta_{\min}$ and $\theta_{\max}$ of traders in each session. We also obtain estimates for a model of homogeneous beliefs by constraining $\theta_{\min} = \theta_{\max}$. This allows us to conduct a nested test to see whether our data rejects the null hypothesis of homogeneous beliefs. This is done separately for each session. We then use these session-by-session estimates to calculate the implied speculative premium in every trading period, i.e., the difference between the price and the valuation of the most optimistic type. We then analyze the estimated speculative premiums for two horizon effects.

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We then examine the relationship between trading volume and price volatility as well as the dynamics of individual asset ownership from period to period as information is gradually revealed. We use individual trading data to divide subjects into types based on how their individual holdings data varies with $\delta$. Theoretically, the ownership distribution shift over time, depending on whether $\delta$ is positive or negative, in accordance with our model. The most optimistic trader types should be holding the assets change over time with the arrival of new information. That is, traders with low $\theta$ should be net buyers when $\delta < 0$ and net sellers when $\delta > 0$; traders with high $\theta$ should be net buyers when $\delta > 0$ and net sellers when $\delta > 0$; traders with intermediate $\theta$ should be net sellers regardless of $\alpha$ and $\beta$. The model makes no predictions about asset ownership when $\delta = 0$.

Finally, we look at the impact of complete markets and relaxing the short-sale constraint on prices in the four additional sessions.

5.1 Asset Prices and the Overpricing Hypothesis

In this section, we address the principal hypothesis we are testing with this experiment. Is there speculative overpricing? We calculate the median price of all transactions in each trading period and use this as our price observation for a trading period. For the analysis in this section of the paper, we aggregate prices by the amount of information revealed. To do this, we code the history of public signals that has been revealed up to period $t$ by counting the number, $\alpha$, of Good News signals and the number, $\beta$, of Bad News signals. The observations for our analysis are aggregated at the period level. However, for ease of presentation in this section of the analysis, we construct an aggregate price for all periods in all markets of a treatment that share the same $\delta$. That is, we use the median of the median transaction prices over all trading periods with the same value of $\delta$. The $\delta = 0$ trading periods are further broken down into two categories, depending on whether it was the initial trading period of a market ($\alpha = \beta = 0$) or a later trading period ($\alpha = \beta > 0$).

5.1.1 One-Asset Markets

We conducted six one-asset market sessions. Three of the sessions use a signal precision $q = \frac{5}{9}$ and the other three a signal precision of $q = \frac{6}{9}$.

**Signal Strength**: $q = \frac{5}{9}$ Table 2 presents the aggregate prices for the $q = \frac{5}{9}$ sessions, for each value of $\delta$ as well as the predicted prices for the homogeneous Bayesian updating model. $N$ is not the number of periods but rather the number of transactions.

Figure 1 plots the prices in the $q = \frac{5}{9}$ sessions by the difference in Good versus Bad News signals along with the predicted prices under the null model where all traders
<table>
<thead>
<tr>
<th>δ</th>
<th>Median Price (N)</th>
<th>Bayesian ($\theta = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>37.5 (4)</td>
<td>20.8</td>
</tr>
<tr>
<td>-5</td>
<td>48.1 (4)</td>
<td>24.7</td>
</tr>
<tr>
<td>-4</td>
<td>51.2 (24)</td>
<td>29.1</td>
</tr>
<tr>
<td>-3</td>
<td>50 (52)</td>
<td>33.9</td>
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<tr>
<td>-2</td>
<td>52.5 (113)</td>
<td>39</td>
</tr>
<tr>
<td>-1</td>
<td>55.6 (182)</td>
<td>44.4</td>
</tr>
<tr>
<td>0 (initial period)</td>
<td>53.8 (136)</td>
<td>50</td>
</tr>
<tr>
<td>0</td>
<td>58.8 (148)</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>60 (138)</td>
<td>55.6</td>
</tr>
<tr>
<td>2</td>
<td>61.2 (58)</td>
<td>61</td>
</tr>
<tr>
<td>3</td>
<td>75.2 (24)</td>
<td>66.1</td>
</tr>
<tr>
<td>4</td>
<td>87.5 (35)</td>
<td>70.9</td>
</tr>
<tr>
<td>5</td>
<td>93.1 (7)</td>
<td>75.3</td>
</tr>
</tbody>
</table>

Table 2: Median Prices by Information Revealed ($q = \frac{5}{9}$)

Bayesian update to the common posterior after receiving each signal (i.e., no heterogeneity). The actual transaction prices remain above the predicted ones regardless of the difference between Good and Bad News signals. The actual transaction prices and predicted prices under the null model are significantly different from each other according to the Wilcoxon signed-rank test ($p < 0.01$). Although the traders are receiving informative signals about the state of the world, they may still be using non-Bayesian updating heuristics. To the extent there is heterogeneity of these heuristics, asset prices can deviate systematically from those predicted under the assumption of perfect Bayesian updating and lead to overpricing according to the multiple-$\theta$ model of speculation.
The homogenous beliefs model predicts the price to be 50 in all periods where there are equal pieces of Good and Bad news, $\delta = 0$. A Wilcoxon signed-rank test reveals that the median prices in these periods are significantly higher than 50 ($p < 0.01$). Next we turn to the price in the initial period when there has been no announcements. Under the null model, the price in the initial period of each market when no information has been revealed should be 50 to reflect the flat prior and this prediction does not hinge upon any assumptions about the belief updating process. In fact, in this treatment condition, the median price is above 50 in all 18 initial periods (Wilcoxon signed-rank: $p < 0.01$) The greater than 50 transaction prices in the initial periods may offer the clearest evidence of speculative trading. Since no information has been revealed, if the prices are above 50, at least some traders must be trading based on speculation about price changes in future periods.

**Signal Strength:** $q = \frac{6}{9}$ Table 3 and Figure 2 display the results for the $q = \frac{6}{9}$ sessions. We find the predicted Bayesian prices track the trajectory of actual prices more closely than the case with $q = \frac{5}{9}$. However, the actual price is still greater than the predicted price for nearly all $\delta$ and these differences are significant according to the Wilcoxon signed-rank test ($p < 0.01$).
<table>
<thead>
<tr>
<th>$\delta$</th>
<th>Median Price (N)</th>
<th>Bayesian ($\theta = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>0.9 (6)</td>
<td>0.8</td>
</tr>
<tr>
<td>-6</td>
<td>2.2 (8)</td>
<td>1.5</td>
</tr>
<tr>
<td>-5</td>
<td>4.4 (2)</td>
<td>3</td>
</tr>
<tr>
<td>-4</td>
<td>10.2 (12)</td>
<td>5.9</td>
</tr>
<tr>
<td>-3</td>
<td>13.1 (30)</td>
<td>11.1</td>
</tr>
<tr>
<td>-2</td>
<td>21.9 (57)</td>
<td>20</td>
</tr>
<tr>
<td>-1</td>
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<td>33.3</td>
</tr>
<tr>
<td>0 (initial period)</td>
<td>58.1 (110)</td>
<td>50</td>
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<td>0</td>
<td>62.5 (98)</td>
<td>50</td>
</tr>
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<td>75 (146)</td>
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<td>97.5 (39)</td>
<td>94.1</td>
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<td>6</td>
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<td>7</td>
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<td>99.2</td>
</tr>
<tr>
<td>8</td>
<td>100 (1)</td>
<td>99.6</td>
</tr>
</tbody>
</table>

Table 3: Median Prices by Information Revealed ($q = \frac{6}{9}$)

Figure 2: Median Prices versus Bayesian Predictions ($q = \frac{6}{9}$)
We again look at the median prices in all periods where \( \delta = 0 \) and the prices are predicted to be 50 under the null model with homogeneous Bayesian updating just as in the \( q = \frac{5}{9} \) treatment. We find that the median prices are also significantly higher than 50 in this treatment (Wilcoxon signed-rank: \( p < 0.01 \)). The median prices in the 18 initial periods are not as uniformly high as they were in the \( q = \frac{5}{9} \) treatment. Nevertheless, they are still significantly higher than 50 (Wilcoxon signed-rank: \( p < 0.05 \)) with only 4 periods having a median price lower than 50 and three more periods right at 50.

Result 1 (Overpricing): Prices in the one-asset market are systematically higher in all treatments than equilibrium prices based on the null model of correct homogeneous trader beliefs about \( q \).\(^8\)

5.2 Asymmetric Pricing in Good News versus Bad News Regimes

The pricing asymmetry hypothesis is that prices react more strongly in Good News regimes (\( \delta \) positive) than in Bad News regimes (\( \delta \) negative). The reason is that the marginal trader is a high-\( \theta \) type in Good News regimes and a low-\( \theta \) type in Bad News regimes. The asymmetric price response to information in Good News regimes compared to Bad News regimes is already evident in Table 2 and Figure 1. The median price never goes below 30 for information flows in Bad News regimes (\( \delta < 0 \)), while the price reach above 90 in Good News regimes (\( \delta \geq 0 \)).

To test Proposition 2 more carefully, we run the following regression by treatment and one pooled across the treatments to test if the price is indeed less sensitive to Bad News than to Good News. The dependent variable is the deviation of the median price from 50. This is calculated by subtracting the price from 50 if \( \delta < 0 \) and subtracting 50 from the price if \( \delta \geq 0 \).\(^9\) The independent variables are interaction terms, one between the absolute difference between \( a \) signals and \( b \) signals, \(|\delta|\), and a dummy for this difference being negative, and another between the difference and a dummy for a non-negative difference.

\[
|50 - P| = \pi + \gamma_1|\delta| * I(\delta \geq 0) + \gamma_2|\delta| * I(\delta < 0) + \epsilon
\]

We hypothesize that \( 0 \leq \gamma_2 < \gamma_1 \) because, from Proposition 2, the price should be further from 50 if \( \delta \geq 0 \) than if \( \delta < 0 \). Table 4 reports the regression results.

\(^8\)The transaction prices are also inconsistent with a model of homogeneous but incorrect beliefs about \( q \) (i.e., homogeneous \( \theta \), but \( \theta \neq 1 \)). For the model with homogeneous \( \theta < 1 \), prices are predicted to be less than the null model when \( \delta > 0 \); and for the model with homogeneous \( \theta > 1 \), prices are predicted to be less than the null model when \( \delta < 0 \). Both predictions are rejected in our data.

\(^9\)We run the same regressions with the absolute deviation from 50 as the dependent variable and the qualitative results remain the same.
Table 4: Price Reaction to Good News vs. Bad News

<table>
<thead>
<tr>
<th></th>
<th>5/9**</th>
<th>6/9*</th>
<th>Pooled**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>7.35 (0.74)</td>
<td>8.40 (1.02)</td>
<td>9.40 (0.56)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-2.53 (1.29)</td>
<td>5.45 (1.18)</td>
<td>1.93 (0.61)</td>
</tr>
<tr>
<td>Constant</td>
<td>3.10 (1.08)</td>
<td>11.66 (2.19)</td>
<td>5.85 (1.03)</td>
</tr>
</tbody>
</table>

We find that $\gamma_2$ is significantly less than $\gamma_1$ in both the separate and pooled regressions as hypothesized ($p < 0.05$). When there are already at least as many $a$ signals as $b$ signals ($\delta \geq 0$), the estimated price reaction to an additional $a$ signal ($\hat{\gamma}_1$) ranges from 7.35 to 9.40, and is significantly greater than zero in all cases ($p < 0.01$). In contrast, if there are already fewer $a$ signals than $b$ signals ($\delta < 0$), the estimated price reaction to an additional $b$ signal ($\hat{\gamma}_2$) is not significantly different from zero in the $q = \frac{5}{9}$ treatment and is significantly greater than 0 for the $q = \frac{6}{9}$ treatment and for the pooled sample. We also note that the constant is significantly greater than 0 in all the regressions which indicates speculative overpricing when $\delta = 0$.

Finally we test one additional implication of the asymmetric response hypothesis. We specifically compare the absolute price change when going from $\delta = 1$ to $\delta = 2$ versus going from $\delta = -1$ to $\delta = -2$.\(^\text{10}\) Pooled across both treatments, the absolute price change when $\delta$ changes from 1 to 2 ($N = 27$), 8.38 on average, is significantly higher than the absolute price change when $\delta$ changes from -1 to -2 ($N = 31$), 5.54 on average (Mann-Whitney: $p < 0.05$).

**Result 2 (Asymmetry):** Market prices react asymmetrically to information in Good and Bad news regimes, and this asymmetry is consistent with the equilibrium price dynamics predicted by the heterogeneous $\theta$ updating model.

### 5.3 Testing the Heterogeneous-$\theta$ Model: Type Estimation

For a more formal approach to the data we use our transaction price data to estimate the maximum and minimum $\theta$ type for each of our sessions using the following procedure. From the theoretical model, given any pair of values, ($\theta_{\min}, \theta_{\max}$) one can calculate the theoretical price trajectory for every sequence of information flows in our data, using the recursive formulae in Section 3. Depending on the exact sequence of public signals, either the maximum $\theta$ type or the minimum $\theta$ type will be the most optimistic traders and this is what determines the asset price trajectories in our model. Recall that $\theta = 0$

\(^{10}\)There are too few observations to compare $\delta = 2$ to $\delta = 3$ and $\delta = -2$ to $\delta = -3$ (or higher levels of $\delta$).
corresponds to a trader who acts as if signals contain no useful information about the state, and \( \theta = 1 \) corresponds to a trader who Bayesian updates with \( q \) equal to either \( \frac{5}{9} \) or \( \frac{6}{9} \), depending on the treatment. We compute the equilibrium price trajectories for all pairs of \( \theta_{\text{min}} = 0, 0.1, 0.2, \ldots \) and \( \theta_{\text{max}} = 0, 0.1, 0.2, \ldots \) such that \( \theta_{\text{min}} \leq \theta_{\text{max}} \). This produces a matrix of prices that depends on \( \alpha \) and \( \beta \). Note that our estimation procedure also allows for the constrained model of homogenous beliefs where \( \theta_{\text{min}} = \theta_{\text{max}} \). This implies a nested test for heterogeneous beliefs. With homogeneous beliefs, there is no speculative premium.

For each possible \((\theta_{\text{min}}, \theta_{\text{max}})\) pair, we sum up the squared deviations of the median price in each trading period of each market from the theoretical price for that pair. Formally, let \( P_{gmt} \) be the median transacted price in trading period \( t \) of market \( m \) of session \( g \). Let \( \alpha_{gmt} \) and \( \beta_{gmt} \) denote, respectively, the number of \( a \) signals and \( b \) signals up to and including period \( t \) in market \( m \) of session \( g \). Let \( P_t^* (\alpha_{gmt}, \beta_{gmt}|\theta_{\text{min}}, \theta_{\text{max}}) \) denote the equilibrium prices from our theoretical model. Then we define the model error as the sum of squared deviations of the pricing data in session \( g \) from the theoretical model, evaluated at parameters \((\theta_{\text{min}}, \theta_{\text{max}})\):

\[
e_g(\theta_{\text{min}}, \theta_{\text{max}}) = \sum_{m,t} \left[ P_{gmt} - P_t^* (\alpha_{gmt}, \beta_{gmt}|\theta_{\text{min}}, \theta_{\text{max}}) \right]^2.
\]

The estimated parameters of the model for session \( g \) are given by

\[
(\hat{\theta}^g_{\text{min}}, \hat{\theta}^g_{\text{max}}) = \arg \min_{0 \leq \theta_{\text{min}} \leq \theta_{\text{max}}} \{ e_g(\theta_{\text{min}}, \theta_{\text{max}}) \}.
\]

Both the price and predicted price are normalized to 0 to 1. We also pooled the sessions for each of the treatments together to estimate a treatment-level \( \hat{\theta}_{\text{min}} \) and \( \hat{\theta}_{\text{max}} \). The results are displayed in Table 5.11

Column 3 of Table 5 shows the estimated \((\hat{\theta}^g_{\text{min}}, \hat{\theta}^g_{\text{max}})\) pairs. Column 4 displays the best fitting homogeneous \( \theta \) model. Column 6 contains the \( F \) test statistic for the null hypothesis \( \hat{\theta}^g_{\text{min}} = \hat{\theta}^g_{\text{max}} \), where

\[
F = \frac{e_g(\theta_{\text{min}}=\theta_{\text{max}}) - e_g(\theta_{\text{min}}<\theta_{\text{max}})}{(n-1)-(n-2)}
\]

and \( n \) is the number of trading periods. The fit is uniformly worse for the single \( \theta \) estimations compared to the \( \theta \) pair ones, suggesting that our model with heterogeneous posterior beliefs among the traders better captures the price dynamics. The homogenous belief model is rejected at the 5% level for all sessions except one. In the one exception,

11In 12 out of 396 trading periods there was no transaction. These are treated as missing data.
<table>
<thead>
<tr>
<th>Session</th>
<th>$q$</th>
<th>$\hat{\theta}<em>{\min}, \hat{\theta}</em>{\max}$</th>
<th>$\hat{\theta}<em>{\min} = \hat{\theta}</em>{\max}$</th>
<th>F-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5/9</td>
<td>0.2, 1.7</td>
<td>0.6</td>
<td>236.1*</td>
</tr>
<tr>
<td>2</td>
<td>5/9</td>
<td>0, 1</td>
<td>0.3</td>
<td>123.1*</td>
</tr>
<tr>
<td>3</td>
<td>5/9</td>
<td>0, 1.8</td>
<td>0.3</td>
<td>227.3*</td>
</tr>
<tr>
<td>pooled</td>
<td>5/9</td>
<td>0, 1.5</td>
<td>0.5</td>
<td>484.4*</td>
</tr>
<tr>
<td>4</td>
<td>6/9</td>
<td>0.5, 1</td>
<td>0.9</td>
<td>46.6*</td>
</tr>
<tr>
<td>5</td>
<td>6/9</td>
<td>1.1, 1.3</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>6</td>
<td>6/9</td>
<td>0, 1.5</td>
<td>0.8</td>
<td>131*</td>
</tr>
<tr>
<td>pooled</td>
<td>6/9</td>
<td>0.6, 1.3</td>
<td>0.9</td>
<td>78.6*</td>
</tr>
</tbody>
</table>

Table 5: Type Estimation by Session

Note: * = homogeneous model (no speculative premium) rejected ($p < 0.05$)

the price dynamics suggest that most of the traders’ perceptions are close to the objective signal strength.

Some additional observations can be gleaned from the estimation results for both the unconstrained and constrained models. For the homogeneous belief model, $\hat{\theta}_{\max} = \hat{\theta}_{\min}$ is less than one for 5 of the 6 sessions. If we assumed that all traders updated their beliefs in the same way, then the price trajectory would suggest on-average under-reaction to the information flow. On the other hand, under the heterogeneous belief model, $\hat{\theta}_{i,\max}$ is estimated to be at least one for all 6 sessions. Furthermore, $\hat{\theta}_{i,\min}$ and $\hat{\theta}_{i,\max}$ span a wide range for most of the sessions. Given the better of fit of this model for nearly all the sessions, the likely heterogeneity across traders ranging from those who react little to information to those who overreact would have been masked by a homogeneous belief model.\textsuperscript{12}

Result 3 (Heterogeneity): We estimate significant heterogeneity in updating rules across subjects in all sessions. The range varies across treatments, with $\hat{\theta}_{\max}$ greater than or equal to 1.

\textsuperscript{12}We would expect such variation within and across sessions if there is some underlying distribution of $\theta^i$ in the population and we are drawing small (10 or 12) independent samples of traders from this distribution.
Session 1, Market 1, $q=5/9$ ($\theta_{\text{min}}=0.2$ and $\theta_{\text{max}}=1.7$)

Session 2, Market 5, $q=5/9$ ($\theta_{\text{min}}=0$ and $\theta_{\text{max}}=1$)

Session 4, Market 5, $q=6/9$ ($\theta_{\text{min}}=0.5$ and $\theta_{\text{max}}=1.0$)

Session 5, Market 3, $q=6/9$ ($\theta_{\text{min}}=1.1$ and $\theta_{\text{max}}=1.3$)

Session 5, Market 5, $q=6/9$ ($\theta_{\text{min}}=1.1$ and $\theta_{\text{max}}=1.3$)
To illustrate how the observed sequences of transacted prices compare to the prices in the estimated model with heterogeneity, Figure 3 displays pricing graphs of five asset markets from four different sessions. These graphs present all the bids, offers, and transactions in each market in addition to the price trajectory of our model estimates for that session. Transacted prices appear as large dots in the graph, unaccepted bids to buy appear as small diamonds and unaccepted offers to sell appear as small triangles. The estimated prices from our model appear as solid lines.

5.4 Estimating the Speculative Premium

We use the session-specific $\hat{\theta}_{\min}^g$ and $\hat{\theta}_{\max}^g$ to calculate the speculative premium for each period of each session. Recall the speculative premium is the difference between the price and the maximum valuation of the asset among all traders which is determined by either the $\theta_{\min}$ or $\theta_{\max}$ trader depending on the information revealed: $P_t(\alpha) - \overline{p}_t(\alpha)$. Table 6 presents the median speculative premium as a function of $\delta$.

5.4.1 Testing of the Overpricing Hypothesis Using the Estimated Speculative Premium

Overpricing should be reflected in a positive speculative premium in periods where $\alpha < 5$ and $\beta < 5$ since there are no permanent optimists. The median speculative premium for these periods is 5.00 in the $q = \frac{5}{9}$ treatment, 3.38 in the $q = \frac{6}{9}$ treatment, and 5.00 overall, all significantly positive (Wilcoxon signed-rank test: $p < 0.05$). The speculative premiums are also significantly positive across all trading periods (Wilcoxon signed-rank test: $p < 0.05$) in the $q = \frac{5}{9}$ treatment (median: 4.80), in the $q = \frac{6}{9}$ treatment (median: 1.89) and pooled across both treatments (median: 3.75).

Result 4 (Positive Speculative Premium): The speculative premium is significantly positive in periods with no permanent optimist.

5.4.2 Horizon Effects

There are two parts to the horizon effect. The first part follows from Proposition 1, which implies that the speculative premium should be zero when enough Good News signals (or Bad News signals) have accumulated, relative to the number of periods remaining in the market ($T - t \leq |\delta|$), but strictly positive if there are a sufficient number of periods remaining ($T - t > |\delta|$). The second part concerns the relationship between the speculative premium when $T - t > |\delta|$ (i.e., for our markets, if $\alpha < 5$ and $\beta < 5$) and the number
Table 6: Speculative Premium by Information Revealed

<table>
<thead>
<tr>
<th>δ</th>
<th>$q = \frac{5}{9}$ (N)</th>
<th>$q = \frac{6}{9}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>-7.18 (6)</td>
<td>-7.18 (6)</td>
</tr>
<tr>
<td>-6</td>
<td>-6.85 (4)</td>
<td>-8.92 (8)</td>
</tr>
<tr>
<td>-5</td>
<td>3.68 (4)</td>
<td>-10.65 (2)</td>
</tr>
<tr>
<td>-4</td>
<td>5.00 (24)</td>
<td>-0.70 (12)</td>
</tr>
<tr>
<td>-3</td>
<td>3.34 (52)</td>
<td>1.09 (30)</td>
</tr>
<tr>
<td>-2</td>
<td>4.73 (113)</td>
<td>1.81 (57)</td>
</tr>
<tr>
<td>-1</td>
<td>5.63 (182)</td>
<td>10.45 (80)</td>
</tr>
<tr>
<td>0</td>
<td>3.75 (136)</td>
<td>8.13 (110)</td>
</tr>
<tr>
<td>0</td>
<td>8.75 (148)</td>
<td>12.50 (98)</td>
</tr>
<tr>
<td>1</td>
<td>2.59 (138)</td>
<td>3.61 (146)</td>
</tr>
<tr>
<td>2</td>
<td>0.27 (58)</td>
<td>1.11 (88)</td>
</tr>
<tr>
<td>3</td>
<td>-0.16 (24)</td>
<td>0.028 (53)</td>
</tr>
<tr>
<td>4</td>
<td>7.81 (35)</td>
<td>0.97 (39)</td>
</tr>
<tr>
<td>5</td>
<td>6.17 (7)</td>
<td>1.78 (5)</td>
</tr>
<tr>
<td>6</td>
<td>-0.34 (8)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.15 (2)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.39 (1)</td>
<td></td>
</tr>
</tbody>
</table>

of periods left in the market: for any fixed value of $\delta$, the speculative premium is weakly increasing in $T - t$.

We start with a test of the first part of the horizon effect. Specifically, the speculative premium should be higher when $T - t > |\delta|$ than in periods where the horizon is too short ($T - t \leq |\delta|$). Indeed, this is what we observe: the speculative premiums are higher on average in periods where both $\alpha$ and $\beta$ are less than $T/2 = 5$ (Table 7) and this difference is significant in both treatments. Second, a somewhat stronger prediction is that the speculative premium should be positive if and only if both $\alpha < 5$ and $\beta < 5$. Across all periods where either $\alpha$ or $\beta$ is greater than $T/2 = 5$ ($T - t \leq |\delta|$), the speculative premiums are only significantly different from zero in the $q = \frac{5}{9}$ treatment. The null hypothesis that the speculative premiums are zero cannot be rejected for the $q = \frac{6}{9}$ treatment or pooled across both information treatments. Furthermore, the speculative premium is significantly greater than 0 when both $\alpha < 5$ and $\beta < 5$. Thus, with the one exception of trading periods when $T - t \leq \delta$ in the $q = \frac{5}{9}$ treatment, we find strong support for part 1 of the horizon effect.

For a test of the more stringent second part of the horizon effect, we first construct a horizon measure which we specify as the number of trading periods that still remain in the market; thus it ranges from 10 for the initial period to 0 for the last period. For
Table 7: Median Speculative Premiums

<table>
<thead>
<tr>
<th></th>
<th>$q = \frac{5}{9}$</th>
<th>$q = \frac{6}{9}$</th>
<th>Pooled†</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha \geq 5 \text{ or } \beta \geq 5$</td>
<td>3.19*</td>
<td>0.99</td>
<td>2.00</td>
</tr>
<tr>
<td>$\alpha &lt; 5 \text{ and } \beta &lt; 5$</td>
<td>5.00*</td>
<td>3.38*</td>
<td>5.00*</td>
</tr>
</tbody>
</table>

Each treatment, we regress the estimated speculative premium on the horizon variable, controlling for the difference in the pieces of Good versus Bad News, $\delta$. These regressions were restricted to the periods where $\alpha < 5$ and $\beta < 5$ because our theory only predicts this second part of the horizon effect for these periods. The regression coefficients are reported in Table 8.

Table 8: The Horizon Effect in Speculative Premiums for Periods with $\alpha < 5$ and $\beta < 5$

<table>
<thead>
<tr>
<th></th>
<th>$q = \frac{5}{9}$</th>
<th>$q = \frac{6}{9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T - t$</td>
<td>-0.27 (0.15)</td>
<td>-0.62 (0.56)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.68 (0.60)</td>
<td>-1.88 (1.05)</td>
</tr>
<tr>
<td>Constant</td>
<td>7.09** (1.30)</td>
<td>10.96* (5.19)</td>
</tr>
</tbody>
</table>

We find no evidence of the second part of the horizon effect from this analysis. Constant terms are significantly positive and coefficients on $\delta$ are significantly negative, consistent with equilibrium pricing. However, the coefficients on the horizon variable are not significantly different from zero.

Result 5 (Horizon Effect): We find support for the first horizon effect, but do not find evidence for the second horizon effect.

5.5 Asset Allocation, Individual Behavior, and Trading Volume

5.5.1 Volatility and Volume

One of the key implications in many models of speculation with heterogeneous beliefs, and a central focus of the papers by Sheinkman and Xiong (2003) and Harris and Raviv (1993), is that a higher volume of trade is associated with periods where price movements are greater. This correlation between volume and volatility arises because both the volume of trade is high and absolute price movements are large when information that changes the set of traders who are willing to hold the asset in positive quantities hits the market.
Rows 1 and 2 of Table 9 report the Spearman rank correlation between volume of trade and volatility, $\rho$, by treatment ($q = \frac{5}{9}$ and $q = \frac{6}{9}$, respectively). Each observation corresponds to one trading period of a market. For each observation, we measure volume, $v_{mt}$, by the total number of units of the asset that are transacted in trading period $t$ of market $m$. Volatility, $\Delta_{mt} = |P_{mt} - P_{mt-1}|$, is measured by the absolute difference between the median transacted price in period $t$ and the median transacted price in period $t - 1$. In both cases the correlation is highly significant. It is also significant pooling across treatments. Figure 4 shows the scatter plot of volume and volatility for the two treatments.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>$\rho$</th>
<th>p – value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{5}{9}$</td>
<td>0.17</td>
<td>0.026</td>
</tr>
<tr>
<td>$\frac{6}{9}$</td>
<td>0.33</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Pooled</td>
<td>0.14</td>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>

Table 9: Spearman rank correlation between volatility and volume.

Figure 4: Volume and Volatility by Treatment.

Thus, $\Delta_{mt}$ is only defined if there is at least one transaction in both period $t$ and period $t - 1$. Other periods are treated as missing data.
Result 6 (Volatility and Volume): Volatility and volume are positively correlated.

5.5.2 Trader Types

In addition to properties of equilibrium asset prices, our model also suggests some hypotheses about the dynamics of asset ownership among the traders as a function of the information revealed. Under the assumptions of risk neutrality and sufficient liquidity, the trader(s) who are the current optimists should hold all the assets in each period. In theory, all available assets could be held by just one trader, the unique most optimistic trader, if this trader has sufficient liquidity. Of course, the identity of this trader can change depending on information revealed over time but the overall distribution of holdings need not. If traders are risk averse or there are multiple “optimists,” then theory is less precise on the exact distribution of assets across traders.

One implication of our model for ownership dynamics is that different traders hold the assets over time depending on the pieces of information revealed up to that point. Specifically, when more signals of Good News than Bad News have been revealed ($\delta > 0$), the $\theta_{\text{max}}$ traders are the optimists and should be net buyers of the asset and other types should sell the asset. On the other hand, when more signals of Bad News than Good News have been revealed ($\delta < 0$), the $\theta_{\text{min}}$ traders are the optimists and should hold the asset. To investigate these predicted switches, we compare the distribution of asset holdings across traders in periods with $\delta > 0$ to the holdings distribution in periods with $\delta < 0$ in each session.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Always Sell</th>
<th>Fickle</th>
<th>Skeptical</th>
<th>Always Buy</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/9</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>5</td>
<td>34</td>
</tr>
<tr>
<td>6/9</td>
<td>10</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>34</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td>17</td>
<td>19</td>
<td>12</td>
<td>68</td>
</tr>
</tbody>
</table>

Table 10: Asset Allocation by Trader Type

We categorize each trader into one of four behavioral types based on the his/her net holdings (end-of-period holdings minus initial endowment). A trader is counted as having zero net holdings if a trader’s mean net holdings are less than the standard error of the net holdings. Traders who have positive holdings when $\delta > 0$ and negative or zero net holdings when $\delta < 0$, or zero holdings when $\delta > 0$ and negative holdings when $\delta < 0$ are categorized as *Fickle* or $\theta_{\text{max}}$ type. Traders with positive holdings when $\delta < 0$ and negative or zero net holdings with $\delta > 0$, or zero net holdings when when $\delta < 0$ and negative holdings when $\delta > 0$ are categorized as *Skeptical* or $\theta_{\text{min}}$ type. The *Always Buy*
types have non-negative holdings in both $\delta > 0$ and $\delta < 0$ periods and the *Always Sell* types have non-positive holdings in both $\delta > 0$ and $\delta < 0$ periods. Periods where $\delta = 0$ are not included in the analysis because the model makes no prediction about asset holdings in these periods.

The vast majority of traders (82.4%) in our markets are categorized as *Fickle*, *Skeptical*, or *Always Sell* types, which is consistent with the heterogeneity model. The *Always Sell* traders correspond to intermediate $\theta$s so they always sold to either the *Fickle* or *Skeptical* type depending on the information flow. The *Always Buy* type is difficult to reconcile with the existing model without expanding it to include possible behavioral motivations for trading. As shown in Table 10, only 12 out of 68, or 17.6%, of the traders fall under this type across all treatments.

**Result 7 (Trader Types):** Most trader types fall in one of the three categories expected by the theoretical model: fickle, skeptical, and intermediate, corresponding to high, low, and intermediate values of $\theta$ respectively.

### 5.6 Complete Markets and Relaxing the Short-Sale Constraint

#### 5.6.1 Complete Markets: Both Assets Traded

We conducted two trading sessions (one for each $q$) where both the Asset A market (pays off 100 in State A) and the Asset B market (pays off 100 in State B) were open simultaneously and traders could transact in both markets. Hence, these markets offer the opportunity for limited arbitrage, suggesting the hypothesis that speculative overpricing will be diminished in these markets. Here we compare the price trajectories in the two markets in this complete markets environment to those in the incomplete market environment where only one asset is traded.

The prices in these two markets do reach substantially lower levels, in the 20s and 30s when $\delta < 0$, which happens rarely in the one-asset sessions. This suggests that allowing both assets to be traded has allowed for some degree of incomplete arbitrage against the speculation. However, we still observe prices significantly above 50 (Wilcoxon signed-rank: $p < 0.01$) in periods where $\delta = 0$, a median price of 57 for Asset A and 59 for Asset B. Furthermore, these above-value (Wilcoxon signed-rank: $p < 0.01$) median prices also are observed in the initial periods when no information has been revealed in both markets, 56 for Asset A and 57 for Asset B.

Observing prices with complete markets provides an opportunity for an especially
simple test of the speculative premium hypothesis, by comparing the sum of the two assets’ prices to 100 in any trading period. Proposition 2 implies that the sum should be greater than the no-arbitrage price of 100 if \( \alpha < 5 \) and \( \beta < 5 \). The alternative hypothesis, based on arbitrage pricing, is that the sum of the two prices should not be significantly different from 100.

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( q = \frac{5}{9} )</th>
<th>( q = \frac{6}{9} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>100 (16)</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td></td>
<td>79 (27)</td>
</tr>
<tr>
<td>-4</td>
<td></td>
<td>99 (35)</td>
</tr>
<tr>
<td>-3</td>
<td>136 (8)</td>
<td>115 (36)</td>
</tr>
<tr>
<td>-2</td>
<td>132 (26)</td>
<td>120 (36)</td>
</tr>
<tr>
<td>-1</td>
<td>128 (34)</td>
<td>91 (48)</td>
</tr>
<tr>
<td>0</td>
<td>129 (73)</td>
<td>104 (65)</td>
</tr>
<tr>
<td>0</td>
<td>132 (46)</td>
<td>106 (38)</td>
</tr>
<tr>
<td>1</td>
<td>132 (77)</td>
<td>108 (24)</td>
</tr>
<tr>
<td>2</td>
<td>124 (72)</td>
<td>108 (15)</td>
</tr>
<tr>
<td>3</td>
<td>129 (43)</td>
<td>115 (6)</td>
</tr>
<tr>
<td>4</td>
<td>130 (11)</td>
<td>110 (6)</td>
</tr>
<tr>
<td>5</td>
<td>132 (5)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>101 (2)</td>
</tr>
</tbody>
</table>

Table 11: Sum of Median Prices in Complete Markets Sessions (N=number of transactions)

It is evident from Table 11 that in nearly all cases (19/23) the two asset prices sum to greater than 100. In the \( q = \frac{5}{9} \) market, it occurs in all 10 cases. The effect is somewhat muted in the \( q = \frac{6}{9} \) treatment, where we observe prices in excess of the no-arbitrage price in 9 of 13 cases. Of possible interest is the observation that all of the exceptions arise when \( \delta < 0 \) and \( q = \frac{6}{9} \). Also worth noting is the fact that the sum of the prices sometimes exceeds 100 by a large amount. In fact, the sum of the prices is 15% or more above the no-arbitrage prices more than half of the time (13 out of 23 cases). The sum of prices is significantly greater than 100 in each of the treatment and pooled across all treatments according to the Wilcoxon signed-rank test for price sums in all periods in which there was at least one trade in both markets. (\( p < 0.01 \) for \( q = \frac{5}{9} \) treatments and both treatments pooled; \( p < 0.05 \) for \( q = \frac{6}{9} \).)

**Result 8 (Complete Markets):** Prices in the two-asset markets are systematically higher than no arbitrage prices. That is, the sum of the prices across
the two markets is greater than 100 for nearly all values of $\delta$. This is observed for both treatments.

5.6.2 Relaxing the Short-Sale Constraint

To explore the effect of the short-sales constraint on asset prices, we conducted three additional sessions where markets were organized to allow traders to engage in short-sales. Specifically, at any time the market was open, any trader was allowed to purchase, from the “bank”, a safe asset consisting of one unit of Asset A and one unit of Asset B at a risk-free price of 100. Traders were allowed to purchase as many units of the safe asset as they wished subject to the cash-on-hand constraint.\textsuperscript{14} This allows any trader who has zero Asset A holdings to engage in a strategy that mimics short-selling Asset A, by purchasing the safe asset and then unbundling it by selling off the Asset A part.

All three sessions use the signal strength $q = \frac{5}{9}$ and there are six 11-period markets with 12 traders in each session. The procedures were otherwise the same as the one-asset market sessions: traders could hold units of both A and B assets, but only the A market was open for trading. Table 12 presents the aggregate prices for the three sessions, for each value of $\delta$ as well as the predicted prices for the homogeneous Bayesian updating model. In 5 out of the 11 values of $\delta$, the median price is actually below the Bayesian price.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>Median Price (N)</th>
<th>Bayesian ($\theta = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>14.25 (42)</td>
<td>20.8</td>
</tr>
<tr>
<td>-5</td>
<td>22.75 (60)</td>
<td>24.7</td>
</tr>
<tr>
<td>-4</td>
<td>34 (56)</td>
<td>29.1</td>
</tr>
<tr>
<td>-3</td>
<td>44.5 (121)</td>
<td>33.9</td>
</tr>
<tr>
<td>-2</td>
<td>41.75 (181)</td>
<td>39</td>
</tr>
<tr>
<td>-1</td>
<td>49 (235)</td>
<td>44.4</td>
</tr>
<tr>
<td>0 (initial period)</td>
<td>51.5 (213)</td>
<td>50</td>
</tr>
<tr>
<td>0</td>
<td>52 (302)</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>55 (310)</td>
<td>55.6</td>
</tr>
<tr>
<td>2</td>
<td>65 (191)</td>
<td>61</td>
</tr>
<tr>
<td>3</td>
<td>64 (82)</td>
<td>66.1</td>
</tr>
<tr>
<td>4</td>
<td>70 (70)</td>
<td>70.9</td>
</tr>
</tbody>
</table>

Table 12: Median Prices in Markets with Short Sales (N=number of transactions)

Figure 5 shows the disparity between the prices in the sessions with and without the option of buying and selling asset bundles. The median prices in the sessions with a\textsuperscript{14}Traders could also re-sell risk-free assets back to the bank for a price of 100.
relaxed short-sale constraint are significantly lower (Mann-Whitney: \( p < 0.01 \)) than the baseline markets. In fact, the median price is lower for every value of \( \delta \) except for \( \delta = 3 \). Allowing short sales substantially essentially eliminates speculative overpricing.\(^{15}\)

![Figure 5: Median Prices in Short Sales sessions versus Bayesian Predictions and Markets with No Short Sales](image)

Result 9 (Short Sales): Allowing short sales significantly reduces the level of overpricing. Prices fall to approximately the level of \( \theta = 1 \) homogeneous Bayesian prices.

6 Conclusion

We study pricing in asset markets with public information flows and short-sale constraints when traders have heterogeneous beliefs. We analyze a simple parsimonious model of such heterogeneity with a single parameter that indexes whether a trader overweights or underweights new information relative to Bayesian updating. Building on Harrison and

\(^{15}\)Even though the median prices in these sessions are very close to the Bayesian prices, statistically they are still slightly higher (Wilcoxon signed-rank: \( p < 0.01 \)).
Kreps (1978) and Morris (1996), this model generates equilibrium price dynamics that exhibit two key properties: overpricing and asymmetric response to good and bad news.

Overpricing in our model takes the form of a positive speculative premium, that is the difference between the equilibrium price and the maximum hold-to-maturity valuation among the traders. Traders are willing to pay more for the asset than their “true” valuation as long as there is a positive probability that some other trader may value it even higher after some future sequence of information flows. We report data from a series of experimental asset markets to test these predictions. Assets whose terminal payoffs are contingent upon the state of the world are traded. At regular intervals during the trading, ten informative but imperfect signals are revealed to our traders at regular intervals.

We find consistent overpricing of the asset compared to what would be predicted by a model of homogeneous Bayesian updating to a common posterior. This overpricing is robust to the signal strength and whether markets are complete or incomplete.

Another theoretical property of the equilibrium price dynamics is an asymmetric price response to information in Good News versus Bad News regimes in that prices respond more strongly to information when Good News has outweighed Bad News. This arises in the model because traders who update most conservatively have the most optimistic valuation in Bad News regimes, and this valuation will determine the price thus dampen the effect of information. In contrast, the high-$\theta$ types value the asset highest when there has been more Good News than Bad News, and they update the most aggressively. We confirm the predicted asymmetric response to Good versus Bad News in our price data.

We also observe overpricing in a complete market environment with the set of two Arrow-Debreu securities being traded. Consistent with the asymmetric response, we find that the prices of the two assets add up to more than the certain payoff of having one of each asset, 100. Just as in the one-market sessions, we observe prices that are significantly higher than 50 in the initial periods when no information has been revealed and also in period with equal numbers of Good and Bad News signals.

We construct an econometric model to directly test for the presence of belief heterogeneity as posited in our theoretical model. We estimate the maximum and minimum $\theta-$type in each session. We find significant heterogeneity for each of the treatments using session-pooled data. Estimating each session separately, we find significant heterogeneity of beliefs in all but one session. Using the $\theta$-model estimates, we calculate the implied speculative premium in each period of each market by subtracting the most optimistic valuation based on the type estimation from the price. We find that the estimated speculative premium is significantly positive in periods with no permanent optimist, adding further support to the overpricing hypothesis. In contrast, the estimated speculative pre-
mium is significantly lower when there is a permanent optimist, providing evidence for the first part of the horizon effect. However, an attempt to directly measure the second part of the horizon effect found no effect.

The model also generates predictions about volume of trade, and patterns of asset ownership depending on the sequence of public signals. Concerning volume of trade, we observe a significantly positive correlation between the volatility of transaction prices and volume of trade. To study the predictions about individual asset ownership, we categorize traders into several categories depending on whether their trading behavior is consistent with being a high $\theta$-type (net buyer with good news, net seller with bad news), a low-$\theta$ type (net buyer with bad news, net seller with good news), or an intermediate-$\theta$ type (net seller). Most traders’ net holdings patterns are consistent with the heterogeneous-$\theta$ model, with two caveats: most traders do not reduce their holding to exactly zero when they are net sellers; and a few traders average positive net holdings in both Good News regimes and Bad News regimes.

Finally, we ran some additional sessions where the market organization was manipulated in two different ways: (a) by having complete markets with the full set of two Arrow-Debreu securities being traded in parallel, one market for Asset A and one for Asset B; and (b) a market where the short sales were allowed. In the complete market environment, we still observe overpricing: the observed transaction prices of the two assets add up to more than the certain payoff of holding one unit of each asset, 100. Just as in the one-market sessions, we observe prices that are significantly higher than 50 in the initial periods when no information has been revealed and also in periods with equal numbers of Good and Bad News signals.

To relax the short-sales constraint, we allowed traders to buy from the “bank” units of a safe asset consisting of one unit of each asset at the risk-free price of 100 from the “bank.” A trader with zero holdings of Asset A could effectively short sell a unit of Asset A by purchasing one unit of the safe asset and then selling off the Asset A unit. In these markets, the overpricing was significantly reduced and the prices were approximately equal to the Bayesian benchmark.

We conclude that the heterogeneous beliefs model is broadly supported by our data. Methodologically, the “public information flow” asset market design used here is an innovation to laboratory markets that makes it possible to address important theoretical questions about the asset pricing dynamics. There are a variety of different directions to take this work and our findings are suggestive of some interesting theoretical and experimental extensions. On the theoretical side, one could enrich the type space by considering multidimensional time-dependent types where $\theta_{it}$ varies over time, or variation across traders of their subjective belief about $q$. Also on the theoretical side, it would
be useful to have an extension of the Morris model (and our model as well) to include risk averse traders or incorporate private information. In principle one would expect the qualitative properties of the dynamic trajectory of asset prices (speculative premium, asymmetry and horizon effects) to continue to hold in these more general models, but the holdings predictions would not be as extreme. However, until these difficult theoretical problems are resolved, one can only make conjectures.
References


Appendix A

Proof of Proposition 1

The posterior belief that \( \omega = A \) for traders of type \( \theta \), given a sequence of Good and Bad News announcements \( (\alpha, \beta = t - \alpha) \), is

\[
\rho_{it}(\theta) = \frac{q^{\alpha \theta} (1 - q)^{\beta \theta}}{q^{\alpha \theta} (1 - q)^{\beta \theta} + q^{\theta - \alpha}(1 - q)^{\beta \theta}}
\]

Since \( q > 0.5 \), \( \theta_{\text{max}} = \arg \max_{i \in \mathcal{I}} \{\rho_{it}(\theta_i)\} \) if \( \delta > 0 \), and \( \theta_{\text{min}} = \arg \max_{i \in \mathcal{I}} \{\rho_{it}(\theta_i)\} \) if \( \delta < 0 \). Traders with the highest \( \theta \) place the greatest weight on signals so their posterior for state \( A \) will be highest of all traders when \( \alpha > \beta \). On the other hand, traders with the lowest \( \theta \) underweight the signals the most so their posterior for state \( A \) will be highest of the traders when \( \beta > \alpha \). When \( \alpha - \beta = \delta = 0 \), then \( \rho_{it}(\alpha) = p = 0.5 \) \( \forall i \), i.e., all traders’ beliefs coincide, and every trader is a current optimist.

To prove (i) consider period \( t \) and any sequence such that \( |\delta| + t < T \). Thus, if all future signals are \( a \)-signals (i.e., \( s_{t+1} = ... = s_T = a \)) then the current optimist at period \( T \) is a \( \theta_{\text{max}} \) trader. Similarly, if all future signals are \( b \)-signals (i.e., \( s_{t+1} = ... = s_T = b \)) then the current optimist at period \( T \) is a \( \theta_{\text{min}} \) trader. Therefore, there is no permanent optimist at period \( t \). That is, there is no permanent optimist at \( t \) if and only if it is uncertain which trader type will be the current optimist in the final period, \( T \).

To complete the proof, we need to show that the speculative premium is positive. Suppose without loss of generality, that in period \( t \) there has been a sequence of signals \( \{s_1, ..., s_t\} \) with \( 0 \leq \delta < T - t \) so a \( \theta_{\text{max}} \) trader is a current optimist. (The argument is the same for the case of \( 0 \leq -\delta < T - t \) and \( \theta_{\text{min}} \) is a current optimist.) Let \( t' = t + 1 + \alpha - \beta \). Note that \( t' \leq T \) since \( \alpha - \beta + t < T \). Consider all sequences of signals \( s'_{t'} = \{s_{t+1}, ..., s_{t'}\} \).

For exactly one such sequence, \( s''_t = (b, ..., b) \), the sequence with all \( b \) signals, a \( \theta_{\text{max}} \) trader is not a current optimist at \( t' \). (Instead, a \( \theta_{\text{min}} \) trader will be the current optimist.) The equilibrium price following this sequence is just \( P_{t'}(\alpha) \). Because \( \theta_{\text{max}} \) is not a current optimist at \( t' \) for this sequence, it implies that \( P_{t'}(\alpha) > \rho_{it}^\alpha(\theta_{\text{max}}) \).

For all other sequences \( s'_t \neq (b, ..., b) \), a \( \theta_{\text{max}} \) trader is still a current optimist at \( t' \). Let \( z(s''_t) \) denote the \( \theta_{\text{max}} \) trader’s belief at \( t' \) that the exact sequence of signals from period \( t + 1 \) to period \( t' \) will be \( s''_t \), and denote by \( \tilde{\alpha}(s''_t) \) the total number of \( a \) signals out of all signals \( s_1, ..., s_{t'} \), given that \( \alpha \) of the first \( t \) signals were \( a \) signals. Thus, the \( \theta_{\text{max}} \) trader’s belief at \( t' \) that the state of the world is \( A \) equals \( \rho_{it}^{\alpha}(\tilde{\alpha}(s''_t))(\theta_{\text{max}}) \). Because \( \theta_{\text{max}} \) is a current optimist at \( t' \), it implies that \( P_{t'}(\tilde{\alpha}(s''_t)) \geq \rho_{it}^{\alpha}(\tilde{\alpha}(s''_t))(\theta_{\text{max}}) \) for all \( s''_t \neq (b, ..., b) \). However, by the recursive definition of prices, and because \( \theta_{\text{max}} \) is a current optimist for every period from \( t \) to \( t' - 1 \)
(since we constructed \( t' = t + 1 + \alpha - \beta \)), the current price is given by

\[
P_t(\alpha) = z(b, \ldots, b)P_{t'}(\alpha) + \sum_{s_t' \neq (b, \ldots, b)} z(s_t')P_{t'}(\tilde{\alpha}(s_t'))
\geq z(b, \ldots, b)P_t(\alpha) + \sum_{s_t' \neq (b, \ldots, b)} z(s_t')P_{t'}(\theta_{\max})
> z(b, \ldots, b)\rho_t^\alpha(\theta_{\max}) + \sum_{s_t' \neq (b, \ldots, b)} z(s_t')\rho_{t'}(\theta_{\max})
= \rho_t^\alpha(\theta_{\max}).
\]

The last line follows from the fact that, given the updating formulas \( \rho_t^\alpha(\theta_{\max}) = \sum_{s_t'} z(s_t')P_{t'}(\tilde{\alpha}(s_t'))(\theta_{\max}) \). Hence \( P_t(\alpha) > \rho_t^\alpha(\theta_{\max}) \), and there is a positive speculative premium when there is no permanent optimist.

We now move to the proof of (ii). If at some \( t \) we have \( \alpha - \beta \geq T - t \), then \( \alpha \geq \beta \) and hence \( \rho_t^\alpha(\theta_{\max}) \geq \rho_t^\alpha(\theta_i) \forall i \). Furthermore for all \( t' = t + 1, \ldots, T \) and all possible sequences of signals \( \{s_t+1, \ldots, s_t\} \), \( \alpha \geq t' - \frac{T}{2} \geq t' - \alpha = \beta \), \( \theta_{\max} \) will be the current optimist: \( \rho_{t'}(\theta_{\max}) \geq \rho_{t'}(\theta_i) \forall i \). Thus a \( \theta_{\max} \) trader is a permanent optimist at \( t \) following any sequence of signals \( \{s_1, \ldots, s_t\} \) such that \( |s| = \{s_1, \ldots, s_t\} |s = a\} = \alpha \geq \frac{T}{2} \). By a similar argument a \( \theta_{\min} \) trader is a permanent optimist at \( t \) following any sequence of signals \( \{s_1, \ldots, s_t\} \) such that \( |s| = \{s_1, \ldots, s_t\} |s = b\} \geq \frac{T}{2} \).

To complete the proof of (ii), let \( \tau \) index the number of periods left until the end of the market. We prove by induction on \( \tau \) that if \( \theta_{\max} \) is a permanent optimist at \( t \), then for all possible continuation sequences of signals up to period \( T - \tau \) (i.e., for all \( \alpha' = \alpha, \alpha + 1, \ldots, \alpha + T - t - \tau \)), \( P_{t}(\alpha') = \rho_t^{\alpha'}(\theta_{\max}) \leq 0, \ldots, T - t \). First, note that this is trivially true for \( \tau = 0 \) since this means \( t = T \) and so it is the last period. The last period price is given by \( P_T(\alpha') = \tilde{P}_T(\alpha') = \rho_T^{\alpha'}(\theta_{\max}) \). The last holds because \( \theta_{\max} \) is a permanent optimist at \( t \) and \( \alpha' \geq \alpha \geq \frac{T}{2} \). Next we show that if \( 0 < \tau < T - t \) and \( P_{T-\tau}(\alpha') = \rho_T^{\alpha'}(\theta_{\max}) \) for all \( \alpha' = \alpha, \alpha + 1, \ldots, \alpha + T - t - \tau \), then \( P_{T-(\tau+1)}(\alpha') = \rho_T^{\alpha'}(\theta_{\max}) \) for all \( \alpha' = \alpha, \alpha + 1, \ldots, \alpha + T - t - (\tau + 1) \). Fix some \( \alpha' \in \{\alpha, \alpha + 1, \ldots, \alpha + T - t - (\tau + 1)\} \).

By definition, in period \( T - (\tau + 1) \), the equilibrium price is

\[
P_{T-(\tau+1)}(\alpha') = \varphi_{T-(\tau+1)}(\alpha')P_{T-\tau}(\alpha' + 1) + (1 - \varphi_{T-(\tau+1)}(\alpha'))P_{T-\tau}(\alpha')
= \varphi_{T-(\tau+1)}(\alpha')\rho_T^{\alpha'+1}(\theta_{\max}) + (1 - \varphi_{T-(\tau+1)}(\alpha'))\rho_T^{\alpha'}(\theta_{\max})
= \rho_T^{\alpha'}(\theta_{\max})
\]
Proof of Proposition 2

As before, let $\tau$ index the number of periods left until the end of the market. We again prove the result by induction on $\tau$. First we show the proposition is true for $\tau = 0$ and $\tau = 1$. Note that since $\alpha > \frac{1}{2}$, $\rho_t^e(\alpha) = \rho_t^{\theta_{\max}}(\alpha)$ because $\delta > 0$. Similarly, $\rho_t^e(t - \alpha) = \rho_t^{\theta_{\min}}(t - \alpha) > \rho_t^{\theta_{\max}}(t - \alpha)$. Since $\rho_t^{\theta_{\max}}(\alpha) + \rho_t^{\theta_{\max}}(t - \alpha) = 1$, $\rho_t^{\theta_{\min}}(t - \alpha) > 1 - \rho_t^{\theta_{\max}}(\alpha)$.

If $\tau = 0$ or $1$, $t \geq T - 1$. Thus, $\alpha > \frac{1}{2}$ implies $\alpha \geq \frac{T}{2}$. By Proposition 1, if $\alpha \geq \frac{T}{2}$ then $P_t(\alpha) = \rho_t^e(\alpha)$ and $P_t(t - \alpha) = \rho_t^e(t - \alpha)$. Therefore

$$P_t(t - \alpha) = \rho_t^{\theta_{\min}}(t - \alpha) > (1 - \rho_t^{\theta_{\max}}(\alpha)) = (1 - P_t(\alpha)).$$

Note that given the definition of $\tau$, if $\tau$ corresponds to $T - t$, then $\tau + 1$ corresponds to $T - t - 1$. To complete the proof, letting $t = T - \tau$, we show that for any $T - 2 > \tau > 1$ and for all $t > \alpha > \frac{1}{2}$, $1 - P_t(\alpha) < P_t(t - \alpha)$ implies $1 - P_{t-1}(\alpha) < P_{t-1}(t - 1 - \alpha)$. By the equilibrium pricing equation:

$$P_{t-1}(\alpha) = \varphi_{t-1}(\alpha)P_t(\alpha + 1) + (1 - \varphi_{t-1}(\alpha))P_t(\alpha) = \varphi_{t-1}^{\theta_{\max}}(\alpha)P_t(\alpha + 1) + (1 - \varphi_{t-1}^{\theta_{\max}}(\alpha))P_t(\alpha).$$

The second line follows because $\delta > 0$. The equilibrium pricing equation after a sequence with equal number of signals $(t - 1)$ but the numbers for the pieces of Good News and Bad News reversed is

$$P_{t-1}(t - 1 - \alpha) = \varphi_{t-1}(t - 1 - \alpha)P_t(t - \alpha) + (1 - \varphi_{t-1}(t - 1 - \alpha))P_t(t - 1 - \alpha) = \varphi_{t-1}^{\theta_{\min}}(t - 1 - \alpha)P_t(t - \alpha) + (1 - \varphi_{t-1}^{\theta_{\min}}(t - 1 - \alpha))P_t(t - 1 - \alpha).$$

The second line is due to the fact that $t - 1 - \alpha < \frac{1}{2}$, so the number of $a$ signals is less than the number of $b$ signals.

Because the proposition is assumed to be true for $\tau = T - t$, we have $1 - P_t(\alpha + 1) < P_t(t - (\alpha + 1)) = P_t(t - 1 - \alpha)$ and $1 - P_t(\alpha) < P_t(t - \alpha)$. Thus
1 - \( P_{t-1}(\alpha) = \varphi_{t-1}^{\theta_{\max}}(\alpha)[1 - P_t(\alpha + 1)] \)
\[ + (1 - \varphi_{t-1}^{\theta_{\max}}(\alpha))[1 - P_t(\alpha)] \]
\[ < \varphi_{t-1}^{\theta_{\max}}(\alpha)P_T(t - 1 - \alpha) + (1 - \varphi_{t-1}^{\theta_{\max}}(\alpha))P_t(t - \alpha) \]
\[ < (1 - \varphi_{t-1}^{\theta_{\min}}(t - 1 - \alpha))P_T(t - 1 - \alpha) + \varphi_{t-1}^{\theta_{\min}}(t - 1 - \alpha)P_t(t - \alpha) \]
\[ = P_{t-1}(t - 1 - \alpha) \]

because \( P_T(t - 1 - \alpha) < P_t(t - \alpha) \) and \( \varphi_{t-1}^{\theta_{\max}}(\alpha) > 1 - \varphi_{t-1}^{\theta_{\min}}(t - 1 - \alpha) \), the latter following from \( \theta_{\max} > \theta_{\min} \).

**Proposition 3 Proof**

Again \( \tau \) indexes the number of periods left until the end of the market. We prove by induction that \( \pi_{T-\tau}(\alpha) - \pi_{T-\tau-2}(\alpha - 1) \leq 0 \) for all \( \tau = 0, \ldots, T - 2 \) and for all \( \alpha = 1, \ldots, T - \tau - 1 \).

We first show that this is true for \( \tau = 0 \). By Proposition 1, \( \pi_{\tau=0}(\alpha) = \pi_T(\alpha) = 0 \) and \( \pi_{T-\tau-2}(\alpha - 1) = \pi_{T-2}(\alpha - 1) \geq 0 \). Thus \( \pi_{T-\tau}(\alpha) - \pi_{T-\tau-2}(\alpha - 1) \geq 0 \) when \( \tau = 0 \).

Note that given the definition of \( \tau \), if \( \tau \) corresponds to \( t \), then \( \tau + 1 \) corresponds to \( t - 1 \). To complete the proof, letting \( t = T - \tau > 2 \), we need to show that if \( \pi_t(\alpha) \leq \pi_{t-2}(\alpha - 1) \) for all \( \alpha = 1, \ldots, t - 1 \), then \( \pi_{t-1}(\alpha) \leq \pi_{t-3}(\alpha - 1) \) for all \( \alpha = 1, \ldots, t - 2 \). To prove this, pick any \( \alpha \in \{1, \ldots, t - 2\} \). We have:

\[
\pi_{t-1}(\alpha) = \varphi_{t-1}^{\ast}(\alpha)[\pi_t(\alpha + 1) + \rho_t^{\ast}(\alpha + 1)]
+ (1 - \varphi_{t-1}^{\ast}(\alpha))\pi_t(\alpha) + \rho_t^{\ast}(\alpha) - \rho_{t-1}^{\ast}(\alpha)
= \pi_t(\alpha) + \rho_t^{\ast}(\alpha) - \rho_{t-1}^{\ast}(\alpha)
+ \varphi_{t-1}^{\ast}(\alpha)\pi_t(\alpha + 1) - \pi_t(\alpha) + \rho_t^{\ast}(\alpha + 1) - \rho_t^{\ast}(\alpha).
\]

while the speculative premium for a sequence with the same \( \delta \) but two more periods left before the end of the market, \( \pi_{t-3}(\alpha - 1) \), is

\[
\pi_{t-3}(\alpha - 1) = \varphi_{t-3}^{\ast}(\alpha - 1)[\pi_{t-2}(\alpha) + \rho_{t-2}^{\ast}(\alpha)] - \rho_{t-3}^{\ast}(\alpha - 1)
+ (1 - \varphi_{t-3}^{\ast}(\alpha - 1))[\pi_{t-2}(\alpha - 1) + \rho_{t-2}^{\ast}(\alpha - 1)]
= \pi_{t-2}(\alpha - 1) + \rho_{t-2}^{\ast}(\alpha - 1) - \rho_{t-3}^{\ast}(\alpha - 1)
+ \varphi_{t-3}^{\ast}(\alpha - 1)[\pi_{t-2}(\alpha) - \pi_{t-2}(\alpha - 1) + \rho_{t-2}^{\ast}(\alpha) - \rho_{t-2}^{\ast}(\alpha - 1)].
\]
We now calculate the difference between these two speculative premiums, \( \pi_{t-3}(\alpha - 1) - \pi_{t-1}(\alpha) \). A number of terms cancel out in this difference because \( \rho_{t}^{*}(\alpha) = \rho_{t-2}^{*}(\alpha - 1) \), \( \rho_{t-1}^{*}(\alpha) = \rho_{t-3}^{*}(\alpha - 1) \), \( \rho_{t}^{*}(\alpha + 1) = \rho_{t-2}^{*}(\alpha) \), and \( \varphi_{t-1}^{*}(\alpha) = \varphi_{t-3}^{*}(\alpha - 1) = \tilde{\varphi}^{*} \in (0, 1) \). Thus we are left with

\[
\pi_{t-3}(\alpha - 1) - \pi_{t-1}(\alpha) = (1 - \tilde{\varphi}^{*})(\pi_{t-2}(\alpha - 1) - \pi_{t}(\alpha)) \\
+ \tilde{\varphi}^{*}(\pi_{t-2}(\alpha) - \pi_{t}(\alpha + 1)).
\]

We know that \( \pi_{t-2}(\alpha - 1) - \pi_{t}(\alpha) \geq 0 \) and \( \pi_{t-2}(\alpha) - \pi_{t}(\alpha + 1) \geq 0 \) by the induction hypothesis. Therefore, since \( \tilde{\varphi}^{*} \in (0, 1) \), we get \( \pi_{t-3}(\alpha - 2) - \pi_{t-1}(\alpha - 1) \geq 0 \).

**Appendix B**

**Sample Instructions**

You are going to participate in a market where you trade a financial asset that pays off real money. The market is divided into eleven 50-second trading periods. The asset you will be trading has a value that depends on the flip of a coin before the market opens. If the coin comes up heads, then each unit of the asset that you hold at the end of the last trading period of the market pays off 100 Experimental dollars. If the coin comes up tails, then each unit of the asset pays off 20 Experimental dollars. We will call it a High Payoff State if the coin comes up heads and a Low Payoff State if the coin comes up tails. Your earnings in the market come from two sources. One source is these payments you receive at the end of the last trading period for your final asset holdings. The second source of earnings (or losses) comes from buying and selling units of the asset while the market is open.

Each of you begins the first trading period with three units of the asset. You will also start with an initial cash holding of 800 experimental dollars which is a loan. You are allowed to sell units of the asset only if you have some units to sell. You are allowed to buy units of the asset only if you have enough cash holdings to pay for the purchase. When you sell a unit, your cash holdings increase by the transaction price. When you buy a unit, your cash holdings decrease by the transaction price. At the end of the last trading period, any payoffs for holding units of the asset will be added to your cash holdings. Remember, those units pay off 100 experimental dollars/unit in a High Payoff State and 20 in a Low Payoff State. Also, the 800 Experimental dollars loan will be subtracted from your total earnings at the end of the experiment.

At 50-second intervals you will receive a News Announcement that gives you some information about the Payoff State. To do this, we will roll a die. In the High Payoff
state, if the die roll was a 1, 2, 3, or 4, then the News Announcement is “Good News”. Otherwise the News Announcement is “Bad News”. In the Low Payoff State, if the die roll was a 1, 2, 3, or 4, then the News Announcement is “Bad News”. Otherwise, the News Announcement is “Good News”. Therefore, Good News is twice as likely as Bad News in the High Payoff State. Bad News is twice as likely as Good News in the Low Payoff State. News Announcements are made at the end of each trading period.

We will now start the program and explain how to use the computers. Please click on the icon pw2 and when prompted, type in your first and last name and press submit. Then wait for everyone to finish connecting to the server.

The first screen tells you to wait while I flip a coin to determine the payoff state for the market, and enter it into the computer. [coin flip] The coin came up _____ so the Payoff State is ______. This brings up the next screen. Please note that you do not receive a News Announcement until the end of the first trading period. Please click OK. After everyone has done this, the trading screen appears and the first trading period begins. Please do not use your computers yet, but pay attention carefully while I explain how you trade.

[Sequence of screens to show trading. Go over the regions of the trading screens and explain how to make bids, offers, cancel, etc.]

Trading occurs when one trader makes a bid to buy or offer to sell and another trade accepts that bid or offer. If you wish to buy a unit of the asset, you may place a bid to buy that unit by specifying a price in the box next to bid and then pressing bid. All bids and offers must be positive integers up to 100. Everyone in the room will see that you have placed that bid. If anyone wishes to sell to you at your bid price, they can click to accept your bid. If that happens, a transaction takes place, and you will have bought one unit of the asset from the seller who accepted your bid. Your cash holdings decrease by the bid price, and your asset holding increase by one unit. The seller’s cash holdings increase by the price, and their asset holdings decrease by one unit. Similarly, if you wish to sell a unit of the asset, you may place an offer to sell that unit by entering a price in the box next to offer and then pressing offer. Everyone in the room will see your offer price. If anyone wishes to buy from you at that price, they can accept your offer. If that happens, a transaction is completed, and you have sold one unit of the asset to the buyer who accepted your offer. Your cash holdings increase by the offer price, and your asset holding decrease by one unit. The buyer’s cash holdings decrease by your offer price, and their asset holding increase by one unit.

You may have at most one bid or offer on the market at one time. If you have an outstanding bid and would like to make an offer instead, you must cancel it before you can make an offer. Similarly if you have an outstanding offer, you must cancel it if you want to make a bid. The market uses an improvement rule. If there is an active bid to
buy on the screen, any new bid to replace it must higher. If there is an active offer to sell on the screen, any new offer must be lower. Whenever an improving bid or offer is made, the previous bid or offer is automatically cancelled and the new one becomes the active bid or offer. If the current bid is accepted by a seller, then there are no active bids, so the next bid can be any amount from 1 to 100. Similarly, if the current offer is accepted by a buyer, then there is no active offer. Also, there is no active offer if the current offer is cancelled and no active bid if the current bid is cancelled. The current bid and offer will be canceled automatically at the end of each trading period. You may cancel your own current bid or offer at any time by clicking on it and pressing cancel. You may also replace it with an improving bid or offer simply by entering the new bid or offer.

The first trading period has ended. I will now roll a die. The die comes up a ___ and recall the Payoff State was ___, so the News Announcement is "". If the Payoff State had been ___, the News Announcement would be "". Explain the history panel then when the period ends, explain the rest of history panel.

We will now have a short practice market with three 50-second trading periods. You will not be paid for this practice market. It is only to help you learn how to use the trading screen. At the beginning of the practice market, you will begin with 800 Experimental dollars and 3 units of the asset. You will receive the first News Announcement after the first 50-sec trading period. Are there any questions?

For this experiment, we will have 6 markets, which are completely independent of each other. We will now proceed to the first market. After the 11 trading periods of the first market are over, we will open a new market that will also have 11 trading periods. The rules will be the same for every market but we will flip a coin at the beginning of each market to determine the payoff state for that market. If the coin comes up heads, it is a High Payoff State and if it comes up tails, it is a Low Payoff State.

You must remember the following important things.

- At the beginning of the first market you start with 800 Experimental dollars and 3 units of the asset. The 800 is a loan that will be subtracted from your final earnings at the end of the experiment.

- Each market will consist of 11 50-sec trading periods. You will receive a News Announcement at the end of each trading period.

- Each unit of the asset pays off 100 experimental dollars/unit in a High Payoff State and 20 in a Low Payoff State.

- Good News is twice as likely as Bad News in the High Payoff State. Bad News is
twice as likely as Good News in the Low Payoff State. News Announcements are made at the end of each trading period.

- Different markets may have different Payoff States, depending on the coin flip at the beginning of the market.

- The units of the asset you hold at the end of a market do not carry over to the next market. They expire at the end of a market. You will always start with exactly three units of the asset at the beginning of each market.

- Your cash holdings DO carry over to the next market.

If you have any questions from now on, please raise your hand and I will come to your computer station and answer your question in private.

[after end of first market]

Remember, your cash holdings will be carried over to the second market and cash holdings will be carried over from market to market until the end of the experiment. However, your units of the asset do not carry over. We will give each of you exactly 3 units of the asset to start with in market 2.

[after end of last market]

We have now completed the experiment. Please note that the 800 loan has been subtracted from your earnings. We will pay you the sum of your earnings in the markets plus your show-up fee of $10.00. We will now pay each of you in private in the next room. Please take all belongings with you when you leave to receive payment. You are under no obligation to reveal your earnings to the other players. Please remain in your seat until we call you to be paid. Please leave the dividers pulled out. Do not talk or socialize with the other participants or use cell phones or computers. Thank you for your patience.