1. The double dividend of environmental taxation

Motivation. This problem has two goals. First, to practice computing the optimal policy associated with public goods such as pollution. Second, to explore how the revenues raised by Pigouvian taxes can be used to reduce the DWL of distortionary taxes.

Consider a simple economy with two produced goods ($q_1$ and $q_2$) that are separable, in the sense that neither their demands nor their supplies are interrelated. In particular, suppose that the inverse aggregate demand in each market is given by

$$p^D(Q) = 1000 - Q,$$

and that the aggregate supply in each of the markets is given by

$$p^S(Q) = Q.$$

In addition, suppose that the production of good 1 generates a total marginal damage to consumers (measured in $\$s$) given by $2Q_1$, but that there are no externalities associated with good 2.

A. Compute the competitive equilibrium in each market.

B. Compute the Pareto optimal allocation in this economy.

Suppose that the government needs to finance $125,000 in absolutely critical government expenditures, and that for political reasons it must do so using a per-unit tax on consumers in market 2.

C. What is the resulting tax in market 2? What is the associated DWL?
D What is the optimal Pigouvian tax necessary to restore Pareto optimality in market 1? How much revenue does it raise?

E What is the total DWL in the economy if the revenue from the Pigouvian tax is returned to the consumers using lump-sum-transfers?

F What is the total DWL if instead it is used to finance the critical expenditures, and the tax in market 2 is reduced accordingly?

2 The canonical public goods problem

Motivation. The goal of this problem is to show you how the analysis of the public goods case is very similar to the one for public bads.

Consider a market with $N$ identical consumers, each of whom have a utility function given by

$$U(q, e, \eta) = a \log(q) + e + \sigma \eta,$$

where $\eta$ denotes the total amount of pollution in the economy. Pollution is generated by the production and consumption of good $q$. In particular, each unit of good $q$ leads to one unit of pollution, so that

$$\eta = \sum_c q_c.$$

The good $q$ is produced by firms with a constant marginal cost of production equal to $\mu$. Assume that $\mu > \sigma N^2$

A What are the marginal social benefit (MSB) and marginal social cost (MSC) curves in this model (as a function of the total amount of production $Q$)?

B What is the total damage (as a function of $Q$)?

C What is the optimal amount of production and pollution?

D What is the equilibrium outcome in the absence of government intervention?

E Compute the equilibrium outcome and social surplus as a function of the size of a Pigouvian tax $\tau$ on the $q$-good.

F At what value of $\tau$ is social surplus maximized?

G How does this $\tau$ compare to the total marginal damage at the no-tax equilibrium? At the optimum?