Inferring Strategic Voting
Kawai and Watanabe (2010)

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Introduction

Main Question:
“Can we identify the existence and estimate the prevalence of strategic voting empirically?”

Components:
- identification of misaligned voting in Japanese general elections
- counterfactual experiments
Definitions:

- **sincere voting**: voting according to your preferences
- **strategic voting**: voting conditioned on pivotality
- **misaligned voting**: voting for a candidate other than the most-preferred; subset of strategic voting
- **pivotality**: the state of having the decisive vote between a pair of candidates
Setup

General Information:

- plurality-rule with $K$ candidates (restriction: $K \geq 3$)
- $M$ municipalities/electoral district
- finite number of voters $N_m$ in each district

Model:

$$u_{nk} = u(x_n, z_k) + \xi_{km} + \epsilon_{nk}$$

- $x_n$: voter characteristics
- $z_k$: candidate characteristics
- $\xi_{km}$: municipality-level shocks for candidate $k$
- $\epsilon_{nk}$ individual-level shocks for candidate $k$
Voting Strategies for Individual $n$

By Type:

- **sincere**: vote for candidate $k$ IFF $u_{nk} \geq u_{nl} \ \forall l$
- **strategic**: vote for candidate $k$ IFF $\bar{u}_{nk}(T_n) \geq \bar{u}_{nl}(T_n) \ \forall l$ (see below)

Two Types of Pivotality:

1. $k$ and $l$ tied without $n$’s vote
2. $k$ one vote behind $l$ without $n$’s vote

In both cases, the difference in utility from not voting for $k$ is $\frac{1}{2}(u_{nk} - u_{nl})$. Let $T_n = \{T_{n,kl}\}_{kl}$ denote her beliefs.

**Expected Utility of Voting:**

$$\bar{u}_{nk}(T_n) = \frac{1}{2} \sum_{l \in \{1, \ldots, K\}} T_{n, kl} (u_{nk} - u_{nl})$$
Assumptions

- For at least one pair \( \{k, l\} \), \( T_{n,kl} > 0 \). Then normalize \( T_{n,kl} \) so that \( \sum_k \sum_{l>k} T_{n,kl} = 1 \).
- Let the type of voter \( n \) in municipality \( m \) be drawn from the Bernoulli distribution with \( p = \alpha_m \).

\[
\alpha_{nm} = \begin{cases} 
0 & \text{if voter } n \text{ is sincere} \\
1 & \text{if voter } n \text{ is strategic}
\end{cases}
\]

(\( \alpha_m \) is drawn iid from a common distribution \( F_\alpha \).)

- Beliefs are common across all voters in the same electoral district.
Dividing the Voting Population:

- $V^{SIN}_{k,m}$: fraction of votes cast by sincere voters for candidate $k$ in municipality $m$
- $V^{STR}(T)_{k,m}$: fraction of votes cast by strategic voters for candidate $k$ in municipality $m$

\[
V^{SIN}_{k,m} = \frac{\sum_{n=1}^{N_m} (1 - \alpha_{nm}) \cdot 1\{u_{nk} \geq u_{nl}, \forall l\}}{\sum_{n=1}^{N_m} (1 - \alpha_{nm})}
\]

\[
V^{STR}(T)_{k,m} = \frac{\sum_{n=1}^{N_m} \alpha_{nm} \cdot 1\{\bar{u}_{nk}(T) \geq \bar{u}_{nl}(T), \forall l\}}{\sum_{n=1}^{N_m} \alpha_{nm}}
\]

\[
V_{k,m}(T) = \frac{\sum_{n=1}^{N-M} (1 - \alpha_{nm})}{N_m} V^{SIN}_{k,m} + \frac{\sum_{n=1}^{N_M} \alpha_{nm}}{N_m} V^{STR}(T)_{k,m}
\]
Consistency Requirements:

A set of solution outcomes $W$ is defined as the set 
$\{T, \{\{V_{k,m}\}_{k=1}^{K}\}_{m=1}^{M}\}$, such that the following two conditions are satisfied:

1. $V_k > V_l \implies T_{kj} \geq T_{ij} \ \forall \ k, l, j \in \{1, ..., K\}$

2. $V_{km} = \frac{\sum_{n=1}^{N_m} (1 - \alpha_{nm})}{N_m} V_{k,m}^{SIN} + \frac{\sum_{n=1}^{N_m} \alpha_{nm}}{N_m} V_{k,m}^{STR}(T)$

Comments:

- $W \neq \emptyset$
- generally, $W$ is not a singleton
Data Description

- **Source:** Japanese House of Representatives (2005 election)
- **Data Selection:**
  of 480 elections, kept the 300 plurality-rule elections
  of 300 plurality-rule elections, kept the 175 that satisfied:
  - 3 or 4 candidates
  - minimum of 2 municipalities
  - no recent mergers
- municipality-level demographic information
  (taken from *Social and Demographic Statistics of Japan*)
Specification (1 of 2)

\[ u_{nmk} = u(x_n, z_{km}; \theta^{PREF}) + \xi_{km} + \epsilon_{nk} \]

- \( x_n \): voter characteristics (education, income, elderly indicator)
- \( z_{km} = \{z_{k}^{POS}, z_{km}^{QLTY}\} \): candidate characteristics
  - \( z_{k}^{POS} \): ideological characteristics
  - \( z_{km}^{QLTY} \): non-ideological characteristics
- \( \theta^{PREF} \): vector of preference parameters
- \( \xi_{km} \sim N(0, \theta_\xi) \equiv F_\xi \)
- \( \epsilon_{nk} \) follows Type-I extreme value distribution
Specification (2 of 2)

\[ u(x_n, z_{km}; \theta^{PREF}) = - (\theta^{ID} x_n - \theta^{POS} z_{k}^{POS})^2 + \theta^{QLTY} z_{km}^{QLTY} \]

Assumptions:

- unidimensional ideological space
- voter ideology is a function of demographics, \( \theta^{ID} x_n \)
- candidate ideology given by \( \theta^{POS} z_{k}^{POS} \)

and RECALL: \( \bar{u}_{nmk}(T) = \sum_{l \in \{1, \ldots, K\}} T_{kl}(u_{nmk} - u_{nml}) \)
Identification

Assumptions:

• voting games are played in $D$ districts, played independently of one another
• $D \to \infty$
• $M^d < \infty$, where $M^d$ is the number municipalities in district $d$

Types of Identification:

1. partial identification of preference parameters
2. partial identification of fraction of strategic voters
Identification: Preference Parameters

RESTRICTION: Voters do not vote for their least-preferred candidate.

PROBLEM: The restriction does not help us determine the voter’s selected candidate, which is a function of the unobservable $T^d$.

Why are the parameters only partially identified? Consider a hypothetical scenario:

1. CASE 1: $T^d_{BC} \approx 1; T^d_{AC} \approx T^d_{AC} \approx 0$
2. CASE 2: $T^d_{AB} \approx T^d_{BC} \approx 0.5; T^d_{AC} \approx 0$
Case 1
\[ T_{AB} = T_{AC} = 0, \quad T_{BC} = 1 \]

"young" municipalities

"old" municipalities

Case 2
\[ T_{AB} = T_{BC} = 0.5, \quad T_{AC} = 0 \]

"young" municipalities

"old" municipalities
Identification: Fraction of Strategic Voters

- Given preference parameters, the model can predict what the vote share would be in each municipality if the voters voted according to their preferences.

- If the proportion of strategic voters is large, though, the vote share can systematically diverge from the predicted outcome.

- Strategic voters make voting decisions conditional on the event that their votes are pivotal. If the beliefs regarding the probability of being pivotal differ across electoral districts, the behavior of strategic voters will also differ across districts. This corresponds to different outcomes being played in different districts.

- To the extent that preference parameters are only partially identified, we can vary $\theta^{REF}$ in the identified set: This allows us to trace out the identified set of the extent of strategic voting.
Estimation (1 of 2)

Parameters Estimated:
- $\theta^{PREF}$: preference parameters
- $(\theta_{\alpha 1}, \theta_{\alpha 2})$: distribution of strategic voters
- variance of $\xi$
- variance of $\theta_\epsilon$

TOOL: inequality-based estimator
Construction of Moment Inequalities

1. For some district, regress the vote share for candidate $k$ in each municipality on its demographic data to obtain the regression coefficient. (You will have $K$ total coefficients for each district.)
2. Fix the parameter $\theta$, the beliefs $T^d$, the fraction of strategic voters, and the candidate-municipality shocks; note that the latter two are vectors of length $M^d$. Given these realizations, compute the predicted vote share outcome for each municipality.
3. For each candidate, regress the simulated vote share for each municipality on its demographic information to obtain regression coefficients.
4. Vary beliefs to obtain minima and maxima for the regression coefficients.
5. Integrate out the fraction of strategic voters and candidate-municipality shocks by simulating out values of $\alpha_d$ and $\epsilon_d$.
6. Repeat steps (1) - (5) for each district to obtain a criterion function. Apply Pakes, Porter, Ho, and Ishii (2007).
## Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Confidence Interval</th>
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<tbody>
<tr>
<td>$\theta_{\alpha_1}$</td>
<td>[5.210, 6.005]</td>
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<tr>
<td>$\theta_{\alpha_2}$</td>
<td>[1.473, 1.706]</td>
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<tr>
<td>$\theta_\xi$</td>
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<td>$\theta_{\text{hometown}1}$</td>
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<td>$\theta_{\text{hometown}2}$</td>
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<td>$\theta_{\text{hometown}3}$</td>
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<td>$\theta_{\text{const}}$</td>
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<td>$\theta_{\text{income}}$</td>
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<tr>
<td>$\theta_{\text{no_experience}}$</td>
<td>[0.080, 0.083]</td>
</tr>
</tbody>
</table>
Focus: Voting Behavior

• The mean proportion of strategic voters \( \frac{\theta_{\alpha_1}}{\theta_{\alpha_1} + \theta_{\alpha_2}} \) is between 75.3% and 80.3%.

• Determining what fraction casts misaligned votes proves more challenging.

• The authors determine the upper and lower bounds as 5.5% and 2.4%, respectively. Thus, the proportion of strategic voters that cast votes insincerely lies between 3.0% and 7.3%.
Experiment 1 of 2: Proportional Representation

- Votes would not be “wasted” under PR, so less incentive exists to vote strategically.
- The authors compute the counterfactual vote share by assuming that all voters vote for the party closest to their own. Furthermore, they assume that each of the four parties fields a candidate in the voter’s district.
- Two effects:
  1. sincere-voting effect (change in behavior of strategic voters)
  2. choice-expansion effect
- To arrive at the distribution of seats, the authors multiplied each party’s vote share by the total number of districts.
The authors try to reconstruct the outcome under universally sincere voting behavior.

The change in vote share is small, but the change in the number of seats is considerable.

A large part of the decrease in vote shares in the previous experiment for the LDP and the DPJ are due to the choice-expansion effect.

Vote share for the JCP remains almost unchanged in the previous experiment, so the choice-expansion and strategic-voting effects were similar in magnitude but worked in opposite directions.
• The authors briefly consider including abstention in future models.

• A supermajority of voters fall into the category of strategic voters, meaning they are willing to cast insincere ballots conditional on their perceived pivotality.

• While the extent of misaligned voting may appear negligible, in close elections, such behavior could have a substantial impact.