Neural Random Utility and Measured Value

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Neurobiological dataset

Medial Prefrontal Cortex (mPFC)

Functional Magnetic Resonance Imaging (fMRI) scanner

*Levy and Glimcher (2012), Bartra et al (2013): meta-studies indicating that **activity in mPFC is tightly correlated with the values subjects place on choice objects**
Neural Random Utility Model

subjective value
(observable)

\[ u_{i,t} = v_{i,t} + \eta_{i,t} \]

Note: subjective value can be measured even in the absence of the choice set
Neural Random Utility Model

\[ u_{i,t} = v_{i,t} + \eta_{i,t} \]

binary choice trial \( t \)

item \( i \)

Note: subjective value can be measured even in the absence of the choice set
Neural Random Utility Model

Neural RUM

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Neural Random Utility Model

subjective value
(observable)

$u_{i,t} = \nu_{i,t} + \eta_{i,t}$

Choice Mechanism

$v_1$

$v_2$

$v_I$

choice

binary choice trial t

item i

stochasticity in subjective value due to perception (e.g., temperature)

preference stochasticity

Note: subjective value can be measured even in the absence of the choice set

Ryan Webb (2013):
Drift Diffusion implies Random Utility
Neural Random Utility Model

\[ u_{i,t} = v_{i,t} + \eta_{i,t} \]

- \( f(X_i) + \omega_i \)
- Observable attributes
- Subjective value (observable)
- Specification error
- Preference stochasticity
- Choice stochasticity

Choice Mechanism
- \( v_1 \)
- \( v_2 \)
- \( v_I \)
Neural Random Utility Model

$$u_{i,t} = v_{i,t} + \eta_{i,t}$$

$$f(X_i) + \omega_i + v_{i,t}$$

$$u_{i,t} = f(X_i) + \varepsilon_{i,t}$$
Neural Random Utility Model

Subjective value
(observable)

$u_{i,t} = \nu_{i,t} + \eta_{i,t}$

Observable attributes

$f(X_i) + \omega_i + \nu_{i,t}$

Specification error

Preference stochasticity

Choice stochasticity

Choice

NRUM

$u_{i,t} = f(X_i) + \varepsilon_{i,t}$

Webb, Glimcher et al. (2013)
Neural Random Utility Model

\[ u_{i,t} = \nu_{i,t} + \eta_{i,t} \]

---

\[ f(X_i) + \omega_i + \nu_{i,t} \]

---

\[ u_{i,t} = Ev_{i,t} + \nu_{i,t} + \eta_{i,t} \]

---

**RUM**


Neural RUM

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\[ u_{i,t} = v_{i,t} + \eta_{i,t} \]

DM chooses \( i \) vs. \( j \) on trial \( t \) if \( u_{i,t} > u_{j,t} \) (consider only binary choices) \( \Rightarrow y_{ij,t} = 1 (u_{i,t} > u_{j,t}) \)

\[ P [y_{ij,t} = 1 \mid v_{i,t}, v_{j,t}] = P [\tilde{v}_{ij,t} > \tilde{\eta}_{ji,t} \mid v_{i,t}, v_{j,t}] \]

where \( \tilde{v}_{ij,t} \equiv v_{i,t} - v_{j,t} \), \( \tilde{\eta}_{ji,t} \equiv \eta_{j,t} - \eta_{i,t} \)

\[ \text{assume}^4 \tilde{\eta}_{ji,t} \sim \text{iid } N \left(0, \sigma^2_{\tilde{\eta}}\right) \quad \leftarrow \text{A1} \]

\[ \Rightarrow P [y_{ij,t} = 1 \mid v_{i,t}, v_{j,t}] = \Phi \left(\frac{\tilde{v}_{ij,t}}{\sigma_{\tilde{\eta}}}\right) \]

\[ \text{assume} v_t = (v_{1,t}, \ldots, v_{I,t}) \text{ is independent over trials} \quad \leftarrow \text{A2} \]

\[ \nu_{i,t} \equiv v_{i,t} - \mathbb{E} [v_{i,t}] \] (mean over trials)

\[ \Rightarrow P [y_{ij,t} = 1 \mid \mathbb{E} [v_{i,t}], \mathbb{E} [v_{j,t}]] = \]

\[ P [\mathbb{E} [\tilde{v}_{ij,t}] > \tilde{v}_{ij,t} + \tilde{\eta}_{ji,t} \mid \mathbb{E} [v_{i,t}], \mathbb{E} [v_{j,t}]] \]

\[ \text{assume} \tilde{v}_{ij,t} \equiv v_{i,t} - v_{j,t} \sim \text{iid } N \left(0, \sigma^2_{\tilde{v}}\right) \quad \leftarrow \text{A3} \]

\[ \Rightarrow P [y_{ij,t} = 1 \mid \mathbb{E} [v_{i,t}], \mathbb{E} [v_{j,t}]] = \Phi \left(\frac{\mathbb{E} [\tilde{v}_{ij,t}]}{\sigma_{\tilde{\eta}+\tilde{v}}}\right), \text{ where } \sigma^2_{\tilde{\eta}+\tilde{v}} = \sigma^2_{\tilde{\eta}} + \sigma^2_{\tilde{v}} \]

\[ ^4 \text{Item-pair independence follows from the binary choice setup: realizations for different item-pairs must occur on different trials} \]
Stage 1
Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth

Purpose: identify the areas of the brain which encoded the subject’s subjective values, $v_{i,t}$

Stage 2
Subjects passively viewed 20 consumer items, one at a time

Purpose: measure the subjective value of these items

Stage 3
Subjects made all possible binary choices over this set of items in an incentive compatible manner

Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner
Stage 1

Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth.

Purpose: identify the areas of the brain which encoded the subject’s subjective values, $v_{i,t}$.

medial Prefrontal Cortex (mPFC)

Stage 2

Subjects passively viewed 20 consumer items, one at a time.

Purpose: measure the subjective value of these items.

Stage 3

Subjects made all possible binary choices over this set of items in an incentive compatible manner.

Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner.

Find area where this difference is statistically significant.
Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth.

Purpose: identify areas of the brain which encoded the subject’s subjective values, $v_{i,t}$

Stage 2: Subjects passively viewed 20 consumer items, one at a time.

Purpose: measure the subjective value of these items

Stage 3:

Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner.

All items were presented 12 times in random order to each subject. On 20 randomly selected trials (which were excluded from analysis), subjects were asked whether they preferred the item they had just seen or a randomly selected amount of money (from $1 to $10). Subjects were told that one of these question trials would be randomly realized at the end.
Stage 1

Subjects passively viewed the outcome of a series of small lotteries over changes to their wealth.

Purpose: identify the areas of the brain which encoded the subject’s subjective values, $v_{i,t}$.

Stage 3

Subjects made all possible binary choices over this set of items in an incentive compatible manner.

Purpose: compare neural measurements of subjective values and the likelihood of choice outside the scanner.

Each possible binary comparison was presented twice (switching the left-right location on each repetition).

The result of one of these choices was randomly selected for realization.

The choices of subjects were largely consistent (mostly transitive and non-random).

Choices were highly idiosyncratic across subjects.

The goal of this experiment is to determine whether subjective value measured in the absence of choice can be used to predict later choices.
Stage 2 $\rightarrow v_{im}, \ i = 1, \ldots, 20, \ m = 1, \ldots, 11$ for each subject

$\rightarrow$ rank $\tilde{v}_i = \frac{1}{11} \sum_{m=1}^{11} v_{im}$ to order the items

Compare to Stage 3: prediction rate is $59 \pm 1\%$ (i.e., in $59 \pm 1\%$ of trials subjects chose according to this ordering) $\rightarrow$ not much!
Stage 2 \( \rightarrow v_{im}, i = 1, \ldots, 20, m = 1, \ldots, 11 \) for each subject

\[ \text{rank } \bar{v}_i = \frac{1}{11} \sum_{m=1}^{11} v_{im} \text{ to order the items} \]

Compare to Stage 3: prediction rate is 59 \( \pm \) 1\% (i.e., in 59 \( \pm \) 1\% of trials subjects chose according to this ordering) \( \rightarrow \) not much!

Can do better!

- segregate prediction accuracy according to the rank-distance in neural activity between two items

\[ \Rightarrow \text{ordering of subjective values can predict choice outcomes} \]
Analysis

- Stage 2 → $v_{im}$, $i = 1, \ldots, 20$, $m = 1, \ldots, 11$ for each subject
  $\rightarrow$ rank $\bar{v}_i = \frac{1}{11} \sum_{m=1}^{11} v_{im}$ to order the items
- Compare to Stage 3: prediction rate is $59 \pm 1\%$ (i.e., in $59 \pm 1\%$ of trials subjects chose according to this ordering) → not much!
- Can do better!
  - segregate prediction accuracy according to the rank-distance in neural activity between two items

$\Rightarrow$ ordering of subjective values can predict choice outcomes

Q: Is subjective value a cardinal quantity? $\Rightarrow$ NRUM
\[
P [y_{ij,t} = 1 \mid \nu_{i,t}, \nu_{j,t}] = \Phi \left( \frac{\tilde{v}_{ij,t}}{\sigma_\eta} \right) \quad \text{vs}\]

\[
P [y_{ij,t} = 1 \mid \mathbb{E} [\nu_{i,t}], \mathbb{E} [\nu_{j,t}]] = \Phi \left( \frac{\mathbb{E} [\tilde{v}_{ij,t}]}{\sigma_\eta + \nu} \right)
\]

Do not observe \(\nu_{i,t}\) on the trial \(t\) in which choice was made.
\[ P[y_{ij,t} = 1 \mid v_{i,t}, v_{j,t}] = \Phi \left( \frac{\tilde{v}_{ij,t}}{\sigma_{\tilde{\eta}}} \right) \text{ vs} \]

\[ P[y_{ij,t} = 1 \mid \mathbb{E}[v_{i,t}], \mathbb{E}[v_{j,t}]] = \Phi \left( \frac{\mathbb{E}[\tilde{v}_{ij,t}]}{\sigma_{\tilde{\eta}} + \tilde{\nu}} \right) \]

Do not observe \( v_{i,t} \) on the trial \( t \) in which choice was made

To get \( \mathbb{E}[\tilde{v}_{ij,t}] \):

**Blood-Oxygenation Level Dependent (BOLD) signal**

\[ B_{i,m} = a + \gamma v_{i,m} + \mu_{i,m}, \quad \mu_{i,m} \sim \text{iid } \mathcal{N}(0, \sigma_\mu^2) \]

measurement error

\[ \bar{B}_i = a + \gamma \bar{v}_i + \bar{\mu}_i \] (average over \( m \))

\[ \tilde{B}_{ij} = \gamma \tilde{v}_{ij} + \tilde{\mu}_{ij} \] (take difference)

Note: Orderings of \( B_{i,m} \) and \( v_{i,m} \) coincide
Ignoring Measurement Error

\[ P \left[ y_{ij,t} = 1 \mid \mathbb{E} [v_{i,t}], \mathbb{E} [v_{j,t}] \right] = \Phi \left( \frac{\mathbb{E} [\tilde{v}_{ij,t}]}{\sigma_{\tilde{\eta} + \tilde{\nu}}} \right) \]

\[ \mathbb{E} [\tilde{v}_{ij,t}] : \quad \tilde{B}_{ij} = \gamma \tilde{v}_{ij} + \tilde{\mu}_{ij} \]

Probit model:

\[ P \left( y_{ij,t} = 1 \mid \tilde{B}_{ij} \right) = \Phi \left( \frac{\gamma^{-1} \tilde{B}_{ij}}{\sigma_{\tilde{\eta} + \tilde{\nu}}} \right) \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>No Constant</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\gamma^{-1}}{\sigma_{\tilde{\nu} + \tilde{\eta}}} )</td>
<td>0.24 (0.10)</td>
<td>0.24 (0.10)</td>
</tr>
<tr>
<td>( c )</td>
<td>-0.01 (0.08)</td>
<td>-0.01 (0.08)</td>
</tr>
</tbody>
</table>
\[
    y_{ij} = \begin{cases} 
    0, & y_{ij,1} = y_{ij,2} = 0 \\
    1, & y_{ij,1} + y_{ij,2} = 1 \\
    2, & y_{ij,1} = y_{ij,2} = 1 
    \end{cases}
\]

\[
    P(y_{ij} = 0) = \left(1 - \Phi\left(\frac{\gamma^{-1} \tilde{B}_{ij}}{\sigma \tilde{\eta} + \tilde{\nu}}\right)\right)^2 \\
    P(y_{ij} = 1) = 2 \left(1 - \Phi\left(\frac{\gamma^{-1} \tilde{B}_{ij}}{\sigma \tilde{\eta} + \tilde{\nu}}\right)\right) \Phi\left(\frac{\gamma^{-1} \tilde{B}_{ij}}{\sigma \tilde{\eta} + \tilde{\nu}}\right) \\
    P(y_{ij} = 2) = \Phi^2\left(\frac{\gamma^{-1} \tilde{B}_{ij}}{\sigma \tilde{\eta} + \tilde{\nu}}\right)
\]

\[
    \Rightarrow P(y_{ij} = 0) < P(y_{ij} = 2) < P(y_{ij} = 1) \quad \text{for small positive } \tilde{B}_{ij}
\]

Data: too few \textit{once} choices when \( \tilde{B}_{ij} \) is small (ordered Probit model)

\[
    \Rightarrow \text{need to account for measurement error: } \tilde{B}_{ij} = \gamma \tilde{v}_{ij} + \tilde{\mu}_{ij}
\]

Intuition: small \( \tilde{v}_{ij} \) for which \textit{once} is most likely might correspond to large \( \tilde{B}_{ij} \) due to measurement error
Accounting for Measurement Error

\[
P \left[ y_{ij,t} = 1 \mid \mathbb{E} [v_i, t], \mathbb{E} [v_j, t] \right] = \Phi \left( \frac{\mathbb{E} [\tilde{v}_{ij, t}]}{\sigma_{\tilde{\eta} + \tilde{\nu}}} \right)
\]

\[
\mathbb{E} [\tilde{v}_{ij, t}] : \tilde{B}_{ij} = \gamma \tilde{v}_{ij} + \tilde{\mu}_{ij}
\]

\[
P \left[ y_{ij,t} = 1 \mid \tilde{B}_{ij}, \tilde{\mu}_{ij} \right] = \Phi \left( \frac{\gamma^{-1}(\tilde{B}_{ij} - \tilde{\mu}_{ij})}{\sigma_{\tilde{\eta} + \tilde{\nu}}} \right)
\]

\[
\mu_{i,m} \sim \text{iid } \mathcal{N} \left( 0, \sigma_{\mu}^2 \right) \Rightarrow \tilde{\mu}_{ij} \sim \mathcal{N} \left( 0, \sigma_{\tilde{\mu}}^2 = \frac{2}{11} \sigma_{\mu}^2 \right)
\]

Random-effects Probit model:

\[
P \left[ y_{ij,1}, y_{ij,2} \mid \tilde{B}_{ij} \right] = \int_{-\infty}^{+\infty} \frac{e^{-\tilde{\mu}_{ij}^2 / 2\sigma_{\tilde{\mu}}^2}}{\sqrt{2\pi}\sigma_{\tilde{\mu}}} \left[ \prod_{t=1}^{2} P \left[ y_{ij,t} \mid \tilde{B}_{ij}, \tilde{\mu}_{ij} \right] \right] d\tilde{\mu}_{ij}
\]
Accounting for Measurement Error

Random-effects Probit model:

\[
P \left[ y_{ij,1}, y_{ij,2} \mid \tilde{B}_{ij} \right] = \int_{-\infty}^{+\infty} \frac{e^{-\tilde{\mu}_{ij}^2/2\sigma_{\tilde{\mu}}^2}}{\sqrt{2\pi}\sigma_{\tilde{\mu}}} \left[ \prod_{t=1}^{2} P \left[ y_{ij,t} \mid \tilde{B}_{ij}, \tilde{\mu}_{ij} \right] \right] d\tilde{\mu}_{ij}
\]

\[
P \left[ y_{ij,t} = 1 \mid \tilde{B}_{ij}, \tilde{\mu}_{ij} \right] = \Phi \left( \frac{\gamma^{-1}(\tilde{B}_{ij} - \tilde{\mu}_{ij})}{\sigma_{\tilde{\eta}+\tilde{\nu}}} \right)
\]

Caveats:

1. \( \tilde{B}_{ij} \) and \( \tilde{\mu}_{ij} \) are not independent: \( \text{Cov} \left( \tilde{B}_{ij}, \tilde{\mu}_{ij} \right) = 2\text{Var} \left[ \tilde{\mu}_{i} \right] = \frac{2}{11} \sigma_{\tilde{\mu}}^2 \)

\( \Rightarrow \) RE Probit estimate of \( \frac{\gamma^{-1}}{\sigma_{\tilde{\eta}+\tilde{\nu}}} \) will be biased towards zero

2. \( \tilde{\mu}_{ij} \) are not independent over choice pairs: \( \text{Cov} \left( \tilde{\mu}_{ij}, \tilde{\mu}_{ij'} \right) = \text{Var} \left[ \tilde{\mu}_{i} \right] \)

\( \Rightarrow \) RE Probit estimate of standard errors will be biased towards zero

\( \Rightarrow \) use multi-way clustering techniques (Cameron et al., 2011)
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Probit</th>
<th>RE Probit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Constant</td>
<td>Constant</td>
</tr>
<tr>
<td>$\frac{1}{\gamma} - \frac{1}{\sigma_{\tilde{N} + \tilde{V}}}$</td>
<td>0.24 (0.10)</td>
<td>0.24 (0.10)</td>
</tr>
<tr>
<td>$c$</td>
<td>-0.01 (0.08)</td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma_{\mu}^2}{\gamma^2 \sigma_{\tilde{N} + \tilde{V}}^2}$</td>
<td></td>
<td>22.36 (3.49)</td>
</tr>
</tbody>
</table>
Subject specific RE Probit

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>P-Val</th>
<th>Coeff</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>P-Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>0.03</td>
<td>1.14</td>
<td>0.98</td>
<td>$\gamma_1^{-1}$</td>
<td>-1.17</td>
<td>1.07</td>
<td>0.27</td>
</tr>
<tr>
<td>$c_2$</td>
<td>-0.15</td>
<td>1.25</td>
<td>0.91</td>
<td>$\gamma_2^{-1}$</td>
<td>0.66</td>
<td>2.89</td>
<td>0.82</td>
</tr>
<tr>
<td>$c_3$</td>
<td>-0.07</td>
<td>1.27</td>
<td>0.95</td>
<td>$\gamma_3^{-1}$</td>
<td>-3.25</td>
<td>2.36</td>
<td>0.17</td>
</tr>
<tr>
<td>$c_4$</td>
<td>-0.34</td>
<td>1.17</td>
<td>0.77</td>
<td>$\gamma_4^{-1}$</td>
<td>10.14</td>
<td>2.90</td>
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<tr>
<td>$c_5$</td>
<td>0.08</td>
<td>1.22</td>
<td>0.95</td>
<td>$\gamma_5^{-1}$</td>
<td>1.39</td>
<td>0.57</td>
<td>0.02</td>
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<td>$c_6$</td>
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<td>$\gamma_6^{-1}$</td>
<td>-3.23</td>
<td>2.50</td>
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<td>$c_7$</td>
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<td>$\gamma_7^{-1}$</td>
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<td>$c_8$</td>
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<td>0.73</td>
<td>$\gamma_8^{-1}$</td>
<td>10.39</td>
<td>3.53</td>
<td>0.00</td>
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<tr>
<td>$c_9$</td>
<td>-0.18</td>
<td>1.18</td>
<td>0.88</td>
<td>$\gamma_9^{-1}$</td>
<td>4.98</td>
<td>2.38</td>
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<tr>
<td>$c_{10}$</td>
<td>0.69</td>
<td>1.24</td>
<td>0.58</td>
<td>$\gamma_{10}^{-1}$</td>
<td>5.01</td>
<td>1.39</td>
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<tr>
<td>$c_{11}$</td>
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<td>1.23</td>
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<td>$\gamma_{11}^{-1}$</td>
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<td>$c_{12}$</td>
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<tr>
<td>$\sigma^2_{\tilde{\mu}}$</td>
<td>20.49</td>
<td>3.46</td>
<td></td>
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</tbody>
</table>

Note: $\sigma_{\tilde{\eta} + \tilde{\nu}} = 1$

- significant reduction of observations
- six $\gamma_s^{-1}$ are significant and positive / six $\gamma_s^{-1}$ are not significantly different from zero
Neural Random Utility Model

\[ u_{i,t} = \nu_{i,t} + \eta_{i,t} \]

Subjective value (observable)

Choice stochasticity

Choice

\[ f(X_i) + \omega_i + \nu_{i,t} \]

Observable attributes

Measurement error

Assumption of stability

NRUM

\[ u_{i,t} = Ev_{i,t} + \nu_{i,t} + \eta_{i,t} \]

The prediction based on NRUM:

- Simulate $y_{s,ij,1}, y_{s,ij,2}$ using

$$P \left[ y_{s,ij,t} = 1 \mid \tilde{B}_{s,ij} \right] = \Phi \left( \frac{\gamma_s^{-1} \tilde{B}_{s,ij}}{\sigma \tilde{\eta} + \tilde{\nu}, s} \right)$$

subject RE Probit estimate

- If $y_{s,ij,1} + y_{s,ij,2} = y_{s,ij,1} + y_{s,ij,2}$, then success

Compare to the prediction at chance:

- Data: the frequency of $y_{s,ij,1} + y_{s,ij,2} = 0$ is 46%, $y_{s,ij,1} + y_{s,ij,2} = 1$ is 9%, $y_{s,ij,1} + y_{s,ij,2} = 2$ is 45%
- Percent of correct predictions: $\frac{1}{4} \times 46 + \frac{1}{2} \times 9 + \frac{1}{4} \times 45 \approx 27$

Compare to RUM:

$$P \left[ y_{s,ij,t} = 1 \mid X_i, X_j \right] = \Phi \left( (X_i - X_j)\beta_s \right)$$

'Amazon star' rating & price
Table IV: Choice prediction rates (%) resulting from 1000 simulated samples generated by our estimates. Prediction rates are calculated for both (Pop)ulation and (Sub)ject-based estimates, and prediction rates are shown for the (pop)ulation as a whole and for each (sub)ject. Prediction rates are also calculated using both (A)mazon and (P)rice observables, (P)rice and the (B)OLD measure, and all three predictors. *Amazon ratings were not available for the five lotteries, so choice pairs with the lotteries were excluded for these sets of predictions.
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<table>
<thead>
<tr>
<th></th>
<th>BOLD</th>
<th>Amazon*</th>
<th>Price</th>
<th>A+P*</th>
<th>P+B</th>
<th>A+P+B*</th>
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NRUM just matches the performance of a coarse behavioral model.

Neural value measure can add predictive power to behavioral model.

Table IV: Choice prediction rates (%) results generated by our estimates. Predictions (Pop)ulation and (Sub)ject-based estimates, and predictions for the (pop)ulation as a whole and for each (sub)ject. Prediction rates are also calculated using both (A)mazon and (P)rice observables, (P)rice and the (B)OLD measure, and all three predictors. *Amazon ratings were not available for the five lotteries, so choice pairs with the lotteries were excluded for these sets of predictions.
Main Contribution:

1. An econometric framework for relating neural measurements to choice prediction, the Neural Random Utility Model, was introduced.

2. The comparison of the predictive power of NRUM with established techniques was done based on data from a laboratory experiment:
   - the measured neural activity cardinally encodes valuations and predict choice behavior
   - accounting for measurement error and combining neural data with standard observables improves predictive performance