Auctions: problems

1. Which random variables are the expectations taken over in (all from Milgrom and Weber (1982)):
   - The proof on the top of p. 1101?
   - The series of equations on p. 1106?

2. Implement the Guerre, Perrigne, and Vuong (2000) procedure for an IPV auction model:
   - Generate 1000 valuations $x \sim U[0, 1]$. Recall (as derived in lecture notes) the equilibrium bid function in this case is
     \[ b(x) = \frac{N - 1}{N} \cdot x. \]
   - For 500 of the valuations, split them into 125 4-bidder auctions. For each of these valuations, calculate the corresponding equilibrium bid.
   - For the other 500 valuations, split them into 100 5-bidder auctions. For each of these valuations, calculate the corresponding equilibrium bid.
   - For each $b_i$, compute the estimated valuation $\tilde{x}_i$ using the GPV equation:
     \[
     \frac{1}{g(b_i)} = (N_i - 1) \frac{x_i - b_i}{G(b_i)}
     \]
     \[ \iff x_i = b_i + \frac{G(b_i)}{(N_i - 1)g(b_i)} \]
     (where $N_i$ denotes the number of bidders in the auction that the bid $b_i$ is from).

   In computing the $G$ and $g$ functions, try
   1. Epanechnikov kernel ($\mathcal{K}(u) = \frac{3}{4}(1 - u^2)\mathbf{1}(|u| \leq 1)$)
   2. Uniform kernel ($\mathcal{K}(u) = \frac{1}{2}\mathbf{1}(|u| \leq 1)$).

   Also, try different bandwidths $h \in \{0.5, 0.1, 0.05, 0.01\}$.

   For each case, plot $x$ vs. $\tilde{x}$. Can you comment on performance of the procedure for different bandwidth values?
   - Compute and plot the empirical CDF’s for the estimated valuations $\tilde{x}_i$, separately for $N = 4$ and $N = 5$. 
3. Consider an example of a common-value model with conditionally independent signals, drawn from Matthews (1984). Namely

⇒ Pareto-distributed common values: \( v \sim g(v) = \alpha v^{-(\alpha + 1)} \), with support \( v \in [1, +\infty) \).

⇒ Conditionally independent signals: \( x|v \sim U[0, v] \).

⇒ Equilibrium bidding strategy:

\[
b(x) = \left[ \frac{N - 1 + \max(1, x)^{-N}}{N} \right] \cdot \left( \frac{N + \alpha}{N + \alpha - 1} \right) \cdot \max(1, x) \tag{1}
\]

So do the following:

- Simulate the common values \( v_t \) i.i.d. from \( G(v) \), for \( t = 1, 225 \) (225 auctions).

- For each auction \( t = 1, 125 \), generate 4 signals each, where \( x_{it} \sim U[0, v_t] \), for \( i = 1, \ldots, 4 \), and \( t = 1, \ldots, 125 \).

  Then for each signal \( x_{it} \), generate the corresponding equilibrium bid \( b_{it} \) for a 4-bidder auction, using Eq. (1).

  For each bid \( b_{it} \), pick out the maximum among rivals’ bids in auction \( t \): \( b^*_{it} \equiv \max_{j \neq i} b_{jt} \).

  For each bid in the simulated 4-bidder auctions, recover the corresponding pseudovalue \( \xi(b_{it}, N_t) \), using Eq. (10) from auction lecture notes.

- For each auction \( t = 126, 225 \), generate 5 signals each, where \( x_{it} \sim U[0, v_t] \), for \( i = 1, \ldots, 5 \), and \( t = 126, \ldots, 225 \).

  As above, generate the corresponding \( b_{it}, b_{it}^* \) for each signal.

  Then, for each bid in these 5-bidder auctions, recover the pseudovalue \( \xi(b_{it}, N_t) \).

- Compute and plot the empirical CDF’s for the estimated pseudovalues \( \xi(b_{it}, N_t) \), separately for \( N_t = 4 \) and \( N_t = 5 \).

References


\( ^{1} \)To simulate from any non-uniform CDF, use the “inverse-quantile” procedure. Generate \( w \sim U[0, 1] \), then transform \( v = G^{-1}(w) \). The random variable \( v \sim G(v) \).