Outline

1. Introduction

2. Dynamic Games: introduction
Cartels and collusion in oligopoly

- Single-period non-cooperative Cournot game: unique NE when firms produce higher-output, receive lower profits than if they cooperated (prisoners’ dilemma)
- Can cooperation occur in multi-period ("dynamic") games?
- Equilibrium concept for multi-period games: **Subgame Perfect Equilibrium**
- Finitely-repeated Cournot game
- Infinitely-repeated Cournot game
Example 1: Limit pricing

Consider the following simple dynamic game

<table>
<thead>
<tr>
<th>Entrant</th>
<th>Incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay Out</td>
<td>(P(M), 0)</td>
</tr>
<tr>
<td>Enter</td>
<td></td>
</tr>
<tr>
<td>Fight</td>
<td>(0, -F)</td>
</tr>
<tr>
<td>Don’t fight</td>
<td>(P(C), P(C) - F)</td>
</tr>
</tbody>
</table>
Extensive Form ("Tree") Representation of a Game

- Extensive Form Representation specifies:

1. players in the game.

2. when each player has the move.

3. what each player can do at each of his or her opportunities to move.

4. what each player knows at each of his or her opportunities to move.

5. the payoff received by each player for each combination of moves that could be chosen by the players.
Strategy in dynamic games

Definition
A strategy for a player is a complete plan of action. It specifies a feasible action for the player in every contingency in which the player might be called to act.

What are strategies in limit pricing game?
Limit pricing analysis

- One way to analyze this game, is to “flatten” the tree into a game matrix. Look at Nash equilibrium.
- What are NE?
- But what if entrant enters?
- Some Nash equilibria seem unpalatable. Need stronger equilibrium concept.
- **Subgame Perfect equilibrium**: removes all non-credible threats, including incumbent’s threat to counter entry with “limit pricing”
A subgame is the part of the multi-period game that starts from any given node onwards.

A subgame perfect equilibrium is a strategy profile, from which, no player can receive a higher payoff in any subgame. That is, each player’s SPE strategy must be a best-response in any subgame.

All SPE are NE, not all NE are SPE.
Limit pricing, redux

- Subgames?
- What are SPE?
- Backwards induction: eliminate all non-BR actions from player 2's subgame
Sequential Version of BoS: Pat moves first

**Strategic / Normal Form Representation**

<table>
<thead>
<tr>
<th></th>
<th>Op-Op</th>
<th>Op-Fi</th>
<th>Fi-Op</th>
<th>Fi-Fi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opera</td>
<td>(2,1)</td>
<td>(2,1)</td>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Fight</td>
<td>(0,0)</td>
<td>(1,2)</td>
<td>(0,0)</td>
<td>(1,2)</td>
</tr>
</tbody>
</table>

**Extensive Form / Game Tree**

```
Chris
  /    
Op-Op  Op-Fi  Fi-Op  Fi-Fi
  |      |     |     |     |
Pat   Opera Fight Opera Fight
  (2,1) (0,0) (0,0) (1,2)```

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Dynamic Games

Sequential Version of BoS: Pat moves first

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Repeated Cournot Game

2-firm Cournot quantity-setting game. Relevant quantities are:

- NE profits $\pi^* = \frac{(a-c)^2}{9b}$
- Cartel profits $\pi^j = \frac{(a-c)^2}{8b}$
- Firm 1 cheats on firm 2: $\pi^x = \pi_1(BR_1(q_j^2)) = \frac{9(a-c)^2}{64b}$
- Prisoners’ dilemma analogy:

<table>
<thead>
<tr>
<th>Firm 2 → cheat</th>
<th>Firm 1 ↓ cheat</th>
<th>Firm 2 → cartel</th>
<th>Firm 1 ↓ cartel</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a-c)^2/9b</td>
<td>(a-c)^2/9b</td>
<td>9(a-c)^2/64b</td>
<td>3(a-c)^2/32b</td>
</tr>
<tr>
<td>3(a-c)^2/32b</td>
<td>9(a-c)^2/64b</td>
<td>(a-c)^2/8b</td>
<td>(a-c)^2/8b</td>
</tr>
</tbody>
</table>
Finitely-repeated Cournot Game 2

2-period Cournot game

- Firm 1 chooses quantities \((q_{11}, q_{12})\)
- Firm 2 chooses quantities \((q_{21}, q_{22})\)

- What are the subgames?
- What are SPE: solve backwards
- Second period: unique NE is (cheat, cheat)
- First period: (cheat, cheat) \(\implies\) unique SPE is ((cheat, cheat), (cheat, cheat))
- What about ((cartel, cartel), (cartel, cartel))?
- What about ((cartel, cheat), (cartel, cheat))?
- What about
  - Firm 1 plays (cartel; cheat if cheat, cartel if cartel)
  - Firm 2 plays (cartel; cheat if cheat, cartel if cartel) ???
- What about 3 periods? \(N\) periods?
What if the 2-firm Cournot game is repeated forever? Are there SPE of this game in which both firms play “cartel” each period?

- Discount rate $\delta \in [0, 1]$, which measures how “patient” a firm is.
- The “discounted present value” of receiving $10$ both today and tomorrow is $10 + \delta 10$.
- If $\delta = 1$, then there is no difference between receiving $10$ today and $10$ tomorrow.
- Geometric series property: $x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}$. 

$$x + \delta x + \delta^2 x + \cdots + \delta^n x + \cdots = \frac{x}{1-\delta}.$$
Let $q^i$ denote the cartel (joint profit-maximizing) quantity.

**Proposition:** If the discount rate is “high enough”, then these strategies constitute a SPE of the infinitely-repeated Cournot game:

1. In period $t$, firm 1 plays $q_{1t} = q^j$ if $q_{2,t-1} = q^j$.
2. Play $q^*$ if $q_{2,t-1} \neq q^j$.

Firm 1 cooperates as long as it observes firm 2 to be cooperating. Once firm 2 cheats firm 1 produces the Cournot-Nash quantity every period hereafter: **Nash reversion** (or “grim strategy”).

Show that these strategies constitute a SPE by finding conditions such that they prescribe best-response behavior for firm 1 given that firm 2 is following this strategy also in each subgame.
Consider firm 1 (symmetric for firm 2). There are two relevant subgames for firm 1.

1. After a period in which cheating (either by himself or the other firm) has occurred. Proposed strategy prescribes playing $q^*$ forever, given that firm 2 also does this. This is NE of the subgame: playing “$q^*$ forever” is a best-response to firm 2 playing “$q^*$ forever”. This satisfies SPE conditions.
2. After a period when no cheating has occurred.

- Proposed strategy prescribes cooperating and playing $q^j$, with discounted PV of payoffs $\pi_j/(1 - \delta)$.

- The best other possible strategy is to play $BR_1(q^j_2) \equiv q^x_1$ this period, but then be faced with $q_2 = q^*$ forever. This yields discounted PV $= \pi^x + \delta(\pi^*/(1 - \delta))$.

- In order for $q_j$ to be NE of this subgame, require $\pi_j/(1 - \delta) > \pi^x + \delta(\pi^*/(1 - \delta))$ (profits from cooperating exceed profits from deviating). This is satisfied if $\delta > 9/17$.

Therefore, the Nash reversion specifies a best response in both of these subgames if $\delta > 9/17$ (“high enough”). In this case, Nash reversion constitutes a SPE.
One-Shot Deviation Principle

It is enough to check one-shot deviations to check for SPNE

1. If there is no profitable one-shot deviation, then there is no profitable finite sequence of deviations
   - Last deviation can’t be profitable . . .

2. If there is no profitable finite deviation, then there is no profitable infinite sequence of deviations
   - Suppose that there are no one-shot profitable deviations from a strategy $p_i$, but that there is a stage $t$ and a history $h_t$ such that player $i$ could improve his payoff by using a different strategy $\hat{p}_i$ in the subgame starting at $h_t$.

   - Since payoff is a discounted sum and feasible profits are bounded, distant payoffs can’t matter much. Thus, there is a $t'$ such that the strategy $p'_i$ that coincides with $\hat{p}_i$ at all stages before $t'$ and agrees with $p_i$ at all stages from $t'$ on must improve on $p_i$ in the subgame starting at $h_t$. But this contradicts the fact that no finite sequence of deviations can make no improvement at all.
Nash reversion is but one example of strategies which yield cooperative outcome in an infinitely-repeated Cournot game.

In general, firm 1 need not punish firm 2 forever to induce it to cooperate; after firm 2 deviates, just produce $q^*$ for long enough so that it never pays for firm 2 to ever deviate. These “carrot and stick” strategies used to interpret price wars.

In general, the Folk Theorem says that, if the discount rate is “high enough”, an infinite number of SPE exist for infinite-horizon repeated games, which involve higher payoffs than in the single-period Nash outcome.

$\delta$ “high enough”: punishments must be severe. If $\delta$ too low, firm 2 prefers higher profits from cheating now, undeterred from lower future profits from firm 1’s punishment.
Infinitely-repeated Cournot Game 5

Generally: threats of punishment must be *credible* — it must be a firm’s best-response to punish when it detects cheating. Industry and/or firm characteristics can make punishments more credible:

1. **Flexible capacity:** punishment may involve a large hike in quantity, this must be relatively costless.

2. **Good monitoring technology:** cheating must be detected rather quickly. Trade journals facilitate collusion? Fewer number of firms?

3. **Demand uncertainty foils detection of cheating:** is low profits due to lower demand or cheating?

4. **Homogeneous products:** so firm 1 can hurt firm 2 by producing more...