Lecture 3: Oligopolistic competition

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Mattt Shum
HSS, California Institute of Technology

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Outline

1. Static Games of Complete Information
   - Basic Definitions
   - Normal Form Representation
   - Example: The Prisoners’ Dilemma

2. Mixed Strategies

3. Oligopoly models

4. Capacity constraints in the Bertrand model
Oligopoly Models

- **Oligopoly**: interaction among small number of firms
- **Conflict of interest**:
  - Each firm maximizes its own profits, *but*...
  - Firm j’s actions affect firm i’s profits
  - Example: price war
  - PC: firms are small, so no single firm’s actions affect other firms’ profits
  - Monopoly: only one firm

- **Game theory**: mathematical tools to analyze situations involving conflicts of interest
- Two game-theoretic models of oligopolistic behavior in homogeneous good markets
  1. quantity-setting *Cournot* model
  2. price-setting *Bertrand* model
- Start by introducing some game theory
Games

- Game: Model of interacting decision-makers
- Focus today on Static Games of Complete Information

- Players choose their actions simultaneously (without knowledge of the others’ choices) (Static)
- Players receive payoffs that depend on the combination of actions chosen (Games)
- Each player’s payoff function is common knowledge among the players (Complete Information)
Normal Form Representation of Games

- We can completely describe a game using a *normal form representation*. Specifies:

1. Players $i \in N = \{1, \ldots, n\}$
2. Strategies available to each player $s_i \in S_i$
3. Payoff received by each player for each combination of strategies that could be chosen by the players $u_i(s_1, \ldots, s_n)$

- Normal-form representation of an $n$ player game $G$ is $G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\}$
Example: The Prisoners’ Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Quiet</th>
<th>Fink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiet</td>
<td>(2,2)</td>
<td>(0,3)</td>
</tr>
<tr>
<td>Fink</td>
<td>(3,0)</td>
<td>(1,1)</td>
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</table>
Nash equilibrium

- A player’s **best response strategy** specifies the payoff-maximizing (optimal) move that should be taken in response to a set of strategies played by the other players:

\[
\forall s_i \in S_i : s_i \in BR(s_{-i}) \iff u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \quad \forall s_i \in S_i.
\]

- A **Nash Equilibrium** is a profile of strategies such that each player’s strategy is an optimal response to the other players’ strategies.

**Definition**

A strategy profile \( s^* = \{s_1^*, \ldots, s_n^*\} \) is a **Nash Equilibrium** of the game \( G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\} \) if for each player \( i \),

\[
u_i(s^*_i, s^*_{-i}) \geq u_i(s_i, s^*_{-i}) \quad \forall s_i \in S_i
\]

- Each player’s NE strategy is a best-response strategy to his opponents’ NE strategies:

\[
\forall i : s^*_i \in BR(s^*_{-i}).
\]
Nash equilibrium: Prisoner’s dilemma

- What are strategies?
- What is $BR_1(fink)$? $BR_1(quiet)$?
- Why isn’t $(quiet, quiet)$ a NE?
Moral Hazard in Teams

- Two workers, $i = 1, 2$, each can “work” ($s_i = 1$) or “shirk” ($s_i = 0$).
- Total team output $Q = 4(s_1 + s_2)$ is shared equally between the two workers.
- Each worker incurs private cost 3 when working and 0 when shirking.
- The strategic form representation of this game is thus:

```
    Work    Shirk
  Work (1,1) (-1,2)
  Shirk (2,-1) (0,0)
```

, a Prisoner’s Dilemma
Mutually Correct Expectations

Consider another example, which is traditionally called the “*Battle of Sexes*”. While in the PD the main issue is whether or not the players will cooperate, in BoS the players agree that it is better to cooperate than not to cooperate, but disagree about the best outcome.

Here both (Opera, Opera) and (Fight, Fight) are N.E.

The theory of Nash equilibrium is silent on which equilibrium we should expect to see when they are many, yet the players are assumed to correctly forecast which one it will be.
Introduction

Example: The Prisoners' Dilemma

A Coordination Game

- Pat and Chris, from our BoS game, now agree on the more desirable outcome.

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<table>
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<tr>
<td>Opera</td>
<td>(2*,2*)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Fight</td>
<td>(0,0)</td>
<td>(1*,1*)</td>
</tr>
</tbody>
</table>
```

- The Pareto inferior outcome (Fight, Fight) is a NE

- Apple vs IBM
Pure Conflict: Matching Pennies

- Players would like to outguess the other (poker: how much to bluff, football: run or pass, battle: attack by land or by sea)

- No Nash equilibria*

- Game has a Nash Equilibrium * in mixed strategies
A mixed strategy for player $i$ is a probability distribution over the (pure) strategies in $S_i$.

**Definition**

Suppose $S_i = \{s_{i1}, \ldots, s_{iK}\}$. Then a mixed strategy for player $i$ is a probability distribution $p_i = \{p_{i1}, \ldots, p_{iK}\}$, where $0 \leq p_{ik} \leq 1 \forall k$ and $\sum_k p_{ik} = 1$.

In Matching Pennies a mixed strategy for player $i$ is the probability distribution $(q, 1-q)$, where $q$ is the probability of playing Heads, $1-q$ is the probability of playing Tails, and $0 \leq q \leq 1$.

The mixed strategy $(0, 1)$ is simply the pure strategy Tails, and the mixed strategy $(1, 0)$ is the pure strategy Heads.
Nash Equilibrium in Mixed Strategies

Consider a two player game. The expected payoff of player \( i \) for a mixed strategy profile \( p_1, p_2 \) is

\[
v_i(p_1, p_2) = \sum_j \sum_k p_{1j} p_{2k} u_i(s_{1j}, s_{2k})
\]

Definition

\((p_1^*, p_2^*)\) is a Nash equilibrium of the game \( G = \{\Delta S_1, \Delta S_2; u_1, u_2\} \) if and only if each player’s equilibrium mixed strategy is a best response to the other player’s equilibrium mixed strategy:

\[
v_1(p_1^*, p_2^*) \geq v_1(p_1, p_2^*) \quad \forall p_1 \in \Delta S_1
\]

and

\[
v_2(p_1^*, p_2^*) \geq v_1(p_1^*, p_2) \quad \forall p_2 \in \Delta S_2
\]
Nash Equilibrium in Mixed Strategies

Note from definition of $v_i(p_1, p_2)$ that given the distribution of strategies played by his opponent,

1. player $i$ must be indifferent among all the pure strategies that he plays with positive probability, and that

2. these pure strategies are best responses to the mixed strategies played by his opponent.

Interpret $p_i$ not as coin flipping, but as $j$’s beliefs, and $i$’s beliefs about $j$’s beliefs and so on
Examples

- Matching Pennies
- BoS
First we focus on games in which each player only moves once — static games.

- **Players:** 2 identical firms
- **Strategies:** firm 1 set $q_1$, firm 2 sets $q_2$
- Inverse market demand curve: $p = a - bQ = a - b(q_1 + q_2)$.
- Constant marginal costs: $C(q) = cq$
- **Payoffs** are profits, as a function of strategies:
  \[ \pi_1 = q_1(a - b(q_1 + q_2)) - cq_1 = q_1(a - b(q_1 + q_2) - c). \]
  \[ \pi_2 = q_2(a - b(q_1 + q_2)) - cq_2 = q_2(a - b(q_1 + q_2) - c). \]
Cournot quantity-setting model 2

- Firm 1: \( \max_{q_1} \pi_1 = q_1(a - b(q_1 + q_2) - c) \).
- FOC: \( a - 2bq_1 - bq_2 - c = 0 \rightarrow q_1 = \frac{a-c}{2b} - \frac{q_2}{2} \equiv BR_1(q_2) \).
- Similarly, \( BR_2(q_1) = \frac{a-c}{2b} - \frac{q_1}{2} \).
- Symmetric, so in a Nash equilibrium firms will produce same amount so that \( q_1 = q_2 \equiv q^* \).
- Symmetric NE quantity \( q^* \) satisfies \( q^* = BR_1(q^*) = BR_2(q^*) \) \( \Rightarrow q^* = \frac{a-c}{3b} \).
- Graph: NE at intersection of two firms’ BR functions.
- Equilibrium price: \( p^* = p(q^*) = \frac{1}{3} a + \frac{2}{3} c \)
- Each firm’s profit: \( \pi^* = \pi_1 = \pi_2 = \frac{(a-c)^2}{9b} \)
- Straightforward extension to \( N \geq 2 \) firms.
Prisoner’s dilemma flavor in Nash Equilibrium of Cournot game

- If firms cooperate: \( \max_q = 2q(a - b(2q) - c) \rightarrow q^j = \frac{(a-c)}{4b} \)
- \( p^j = \frac{1}{2}(a + c) \), higher than \( p^* \).
- \( \pi^j = \frac{(a-c)^2}{8b} \), higher than \( \pi^* \).
- But why can’t each firm do this? Because NE condition is not satisfied: \( q^j \neq BR_1(q^j) \), and \( q^j \neq BR_2(q^j) \). Analogue of (don’t confess, don’t confess) in prisoner’s dilemma.
- What if we repeat the game? Possibility of punishment for cheating.
More generally, the Cournot problem is:

$$\max q_1 \pi_1(q_1, q_2) = q_1 P(q_1 + q_2) - C_1(q_1)$$  \hspace{1cm} (1)$$

with first-order condition

$$\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = P(q_1 + q_2) - C'_1(q_1) + q_1 P'(q_1 + q_2) = 0$$  \hspace{1cm} (2)$$

This can be rearranged to yield an expression for the Cournot markup (or Lerner index):

$$L_1 \equiv \frac{P - C'_1}{P} = \frac{\alpha_1}{\varepsilon} \quad \alpha_1 = q_1/Q; \varepsilon = Q'P/Q = P/(P'Q).$$

Lerner index is proportional to the firm’s market share and inversely proportional to the elasticity of demand.
Lerner equation still holds with \( N \geq 2 \) firms

Note that Cournot competition yields

\[
\sum \alpha_i L_i = \frac{1}{\varepsilon} \sum \alpha_i^2 = \frac{K_H}{\varepsilon}
\]

where \( K_H \) is the Herfindahl index, a common measure of market concentration. (Varies between 0 and 1000, with higher values traditionally interpreted as higher market power).

This supports the idea that more concentrated markets will have more important departures from marginal cost (though not necessarily lower welfare: welfare is enhanced if low cost firms gain market share at the expense of high cost firms)
Bertrand price-setting model

- Players: 2 identical firms
- Firm 1 sets \( p_1 \), firm 2 sets \( p_2 \)
- Market demand is \( q = \frac{a}{b} - \frac{1}{b} p \). \( C(q) = cq \).
- Recall: products are homogeneous, or identical. This implies that all the consumers will go to the firm with the lower price:

\[
\begin{align*}
\pi_1 &= \begin{cases} 
(p_1 - c)(\frac{a}{b} - \frac{1}{b} p_1) & \text{if } p_1 < p_2 \\
\frac{1}{2}(p_1 - c)(\frac{a}{b} - \frac{1}{b} p_1) & \text{if } p_1 = p_2 \\
0 & \text{if } p_1 > p_2
\end{cases}
\end{align*}
\]  

(3)

- Firm 1’s best response:

\[
BR_1(p_2) = \begin{cases} 
p_2 - \epsilon & \text{if } p_2 - \epsilon > c \\
c & \text{otherwise}
\end{cases}
\]  

(4)

- NE: \( p^* = BR_1(p^*) = BR_2(p^*) \)
  Unique \( p^* = c \)! The “Bertrand paradox”.
Oligopoly models

**Cournot vs. Bertrand**

- Recall: with homogeneous products, firms are “price takers”. Bertrand outcome coincides with competitive outcome.
- Contrast with Cournot results. Presents a conundrum for policy analysis of oligopolisti industries. Is “two enough” for competition?
- Some resolutions:
  - Capacity constraints: one firm can’t supply the whole market
  - Differentiated products
  - Consumer search
Now add capacity constraints to the model: $k_1, k_2$. Zero production costs if produce under capacity; cannot produce above.

Assume capacities are “small” relative to the market. For inverse demand $P = 1 - Q$, assume that $k_1 + k_2 \leq 1$.

Let $R(k_2) = (1 - k_2)/2$ denote firm 1’s best response to production of $k_2$ by firm 2, at zero production costs. (“monopolist output on residual demand curve”: illustrate).
Case 1: small capacities

- Assume capacities are small, so that $k_1 \leq R(k_2)$. Similarly, $k_2 \leq R(k_1)$.
- **Claim**: equilibrium price $p^* = 1 - k_1 - k_2$
  - $p_1 = p_2 < p^*$: cannot produce more
  - $p_1 = p_2 > p^*$: at least one firm cannot sell to capacity; this firm should undercut slightly, and sell capacity
  - $p_i < p_j$ not feasible: either (i) firm $i$ is capacity constrained, and wants to raise its price; or (ii) $p_i$ is firm $i$’s monopoly price and supplies entire demand, but then firm $j$ is making zero profit and should undercut.
- Note that $p^*$ is above marginal cost of zero
- Suggestive: what if you “endogenize” capacity, and add on initial capacity investment stage?
- Best-response behavior: each firm chooses $k_i$ to maximize $k_i(1 - k_i - k_{-i})$. Symmetric solution is ???
Case 2: Large capacities

- For the rest, assume $k_1 = k_2 = k$ for convenience.
- Now assume $k > R(k)$. Is $p = 1 - 2k$ an equilibrium?
- No—firm 1 (say) can deviate by setting $q_1 = R(k) < k$, resulting in higher price.
- Consider the following sequence of events:
Edgeworth cycles

- Start out at above equilibrium with $p_1 = p_2 = 1 - 2k$ (for convenience). Both firms producing at capacity $q_1 = q_2 = k$.
- Firm 1: deviate and raise price to $p^h = (1 - k) - R(k)$. Sell $R(k)$ units.
- Firm 2: has two choices:
  1. Slightly undercut by setting $p_2 = p^h - \epsilon$, and sell $k$.
  2. Be monopolist on residual demand of $p_2 = (1 - R(k)) - q_2$.
Assume Option 1 yields more profits. So $p_2 = p^h - \epsilon$, and $q_2 = k$.
- Firm 1: has same two choices. Again, assume option 1 is better. So $p_1 = p_2 - \epsilon$, $q_1 = k$.

...  

- At some point, option 2 will dominate; assume this is for firm 1. Then it sets $p_1 = p^h$ and $q^1 = R(k)$ and cycle starts again.
- “Edgeworth cycle”
Edgeworth cycles

- Not an equilibrium phenomenon. In fact, this model has no pure-strategy equilibria.
- (But have mixed strategy equilibria)
- Can be shown to be equilibrium path of dynamic game (Maskin-Tirole)
- Note crucial assumption that $k > R(k)$, so that capacity not too small. In earlier case, if $k \leq R(k)$ then pure strategy equilibrium can exist, with $p_1 = p_2 = 1 - 2k$.
- Empirical evidence.
Summary

- Nash equilibrium: a strategy profile in which each player’s NE strategy is a best-response to opponents’ best-response strategies.
  2-player case: \( s_1 = BR_1(s_2), \ s_2 = BR_2(s_1) \)

- Cournot: noncooperative quantity-choice game.
- Bertrand: noncooperative price-setting game.
  - Bertrand paradox: when goods are homogeneous, firms are price-takers!
  - Price competition with capacity constraints.