Dynamic Contract for Electricity Procurement

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The deregulated market for electricity services

Generators — Load Serving Entities — Customers

↑ market

• Wholesale market
  ▶ centralized spot market
  ▶ one-on-one bilateral contract
  ▶ financial contract

• Retail market
  ▶ retailer competition
  ▶ demand response
Why does the wholesale market need bilateral contracts

• Volatile prices
  ▶ inelastic demand
  ▶ physical constraints (generation and transmission)
  ▶ no storage

• Provide stable revenue for new capacities, especially for renewable sources

• Allow large loads direct access to suppliers
The current model of bilateral contract uses a fixed unit price

- An existing contract pays the supplier $\int_0^T \bar{p}D_s ds$

- Perverse incentive
  - encourage excessive supply when the market price is low
  - mis-represent high averaged cost

- stochastic dynamics
  - supply uncertainties (e.g. renewable sources, equipment failures)
  - demand uncertainties
  - grid operations (e.g. transmission scheduling, frequency/voltage control, reactive power)
Desirable features of a dynamic contract

- Allow for stochastic fluctuations
- Account for price manipulation (moral hazard)
- Market responsiveness
Related literature: electricity market design


- Different components: reserve market (Chao et al. 2002), forward market (Murphy et al. 2010), bilateral contract for wind generators (Cai et al. 2011)

- Integrating market and engineering control: Cho et al. (2010), Wang et al. (2011), Mathieu et al. (2013)
Related literature: dynamic contract

- State process: Holmstrom et al. (1987), He (2009), Biais et al. (2010)
- Time horizon and payment: lump-sum (Cvitanic et al. 2009), continuous (Sannikov 2008)
- Information: moral hazard, adverse selection (Sung 2005, Cvitanic et al. 2012), learning (Prat et al. 2013)
- Modeling approach: weak or strong formulation
- Application: corporate finance (DeMarzo et. al 2006), surgeon qualities (Fong 2009)
Contracting environment

- The contract uses the spot price:

\[ P_t = P_0 + \int_0^t \rho (A_s - P_s) ds + \int_0^t \sigma dM_s + \sum_{i=1}^{N_t} L_i \]

- Price is a mean reverting jump diffusion

- Agent (supplier) manipulates price by
  - withholding supplies
  - virtual bids
  - proxy

- Principal (e.g. LADWP) cannot contract on actions
Contract

- Principal pays a direct payment $\int_0^T P_s D_s ds$ and an adjustment fee $C_T$
- Agent’s utility: $U(C_T) + \int_0^T \left( u(P_s D_s) - h(A_s) \right) ds$
- $U(\cdot)$ and $u(\cdot)$ are agent’s utility functions, and $h(a)$ is his cost of performing action $a$.

- Principal offers a take-it-or-leave-it contract $(C_T, A)$
- The recommended action $A_t$ could be a function of the history of prices up to time $t$, i.e. $\{P_s : 0 \leq s \leq t\}$
- $C_T$ could depend on the entire path history of $P$
Notations and variables

- $D = \{D_s : 0 \leq s \leq T\}$ is the deterministic process that represents the amount of electricity demanded by the principal
- $M = \{M_s : 0 \leq s \leq T\}$ is a standard Brownian motion
- $N = \{N_s : 0 \leq s \leq T\}$ is a Poisson process with intensity $\lambda$
- $L_i$ is uniformly distributed over $[-L, L]$
- $J$ denotes the compound Poisson process

$$J_t \triangleq \sum_{i=1}^{N_t} L_i \quad \text{and} \quad \Delta J_t \triangleq J_t - J_{t-},$$

- Action $A_t$ takes value on a compact subset $A \subseteq \mathbb{R}$
- $\rho > 0$ is the rate of reversion to the mean
- $\sigma > 0$ is the scale of the volatility of $M$
Agent’s continuation value

- Let $\mathcal{F}_t$ to represent the information at time $t$
- Fix a contract $(C_T, A)$
- The agent’s continuation value

$$W^A_t \triangleq \mathbb{E}^A_t \left[ U(C_T) + \int_t^T \left( u(P_s D_s) - h(A_s) \right) ds \right]$$

Lemma (Representation)

There exist unique, up to measure zero sets in $[0, T] \times \Omega$, predictable processes $(Z^c, Z^d)$ such that

$$W^A_t = U(C_T) + \int_t^T \left( u(P_s D_s) - h(A_s) \right) ds - \int_t^T Z^c_s \sigma dM_s - \sum_{t<s\leq T} Z^d_s \Delta J_s.$$
The set of incentive compatible contracts

We say $(C_T, A)$ is *incentive compatible* if $A$ is an optimal response to $C_T$ for the agent. Denote the set of all incentive compatible contracts by $C$.

**Proposition (Incentives)**

The following two conditions are equivalent.

1. $(C_T, A)$ is incentive compatible.
2. $A_t(\omega) \in \arg \max_a \{\rho Z^c_t(\omega)a - h(a)\}$, for almost every $(t, \omega) \in [0, T] \times \Omega$. 
Resolution of the agent’s problem

Let $\xi(a)$ (resp. $\eta(z)$) be the solution of $\rho z - h'(a) = 0$ in terms of $z$ (resp. $a$).

1. Given $C_T$, the response $A_t = \eta(Z^c_t)$ is optimal, where $(W, Z^c, Z^d)$ is the unique solution of the BSDE

\[
W_t = U(C_T) + \int_t^T \left( u(P_s D_s) - h(\eta(Z^c_s)) + \rho Z^c_s \eta(Z^c_s) \right) ds \\
- \int_t^T Z^c_s \sigma dB_s - \sum_{t < s \leq T} Z^d_s \Delta J_s.
\]

2. Given $A$, set the payment $C_T = I(\tilde{W}_T)$, where $I(\cdot)$ is the inverse of $U(\cdot)$, and

\[
\tilde{W}_t = W_0 - \int_0^t \left( u(P_s D_s) - h(A_s) \right) ds + \int_0^t \xi(A_s) \sigma dM_s.
\]
Principal’s expected payment

• The principal’s value function is

\[ F(t, p, w) \triangleq \min_{A \in \mathcal{A}} \mathbb{E} \left[ I(W^t_p, W^s_w) + \int_t^T P^s_p D_s ds \right] \]

\[
\begin{align*}
  P^s_p &\triangleq p + \int_t^s \rho(A_v - P_v)dv + \int_t^s \sigma dM_v + \sum_{t<v\leq s} \Delta J_v \\
  W^t_p &\triangleq w - \int_t^s \left[ u(P^t_p D_v) - h(A_v) \right] dv + \int_t^s \xi(A_v) \sigma dM_v.
\end{align*}
\]

• The incentive compatibility constraint is resolved by the choice of the dynamics of \( W^t_p \).

• Solving for a stochastic control problem.
Computing the optimal $A$

- The HJB equation

$$- \partial_t F = \min_a \rho(a - p)\partial_p F \,+ \frac{1}{2}\sigma^2 \partial_{pp} F \,+ (h(a) - u(pD_t)) \partial_w F \,+ \frac{1}{2} \xi(a)^2 \sigma^2 \partial_{ww} F \,+ \xi(a) \sigma^2 \partial_{pw} F \,+ \int_{-L}^{L} \left( F(t, p + dp', w) - F(t, p, w) \right) \frac{\lambda}{2L} dp' + pD_t, $$

with terminal condition $F(T, p, w) = I(w)$, for all $p$ and $w$.

- The optimal policy function $a^*(t, p, w)$ is the minimizer of

$$\rho a \partial_p F \,+ h(a) \partial_w F \,+ \frac{1}{2} \xi(a)^2 \sigma^2 \partial_{ww} F \,+ \xi(a) \sigma^2 \partial_{pw} F \quad (1)$$
Properties of the optimal policy

• The state variables \((p, w)\) summarize all the useful information; the agent’s incentive is not affected by the path it takes to get to \((p, w)\).

• In general, \(a^* \neq min\{A\}\);
  ▶ with linear utilities, \(a^* = min\{A\}\)
  ▶ risk aversion and participation constraint
  ▶ balancing present and future compensation

• The principal pays for the cost of \(h(a)\) indirectly, through future price \(P_t\) and continuation value \(W_t\)
Theorem (Optimal contract)

Let $a^*(t, p, w)$ be the minimizer in (1), and the agent is paid $C_T = I(W_T)$, where

$$
\begin{align*}
P_t &= P_0 + \int_0^t \rho \left( a^*(s, P_s-, W_s) - P_s \right) ds + \int_0^t \sigma dM_s + \sum_{0 < s \leq t} \Delta J_s \\
W_t &= R + \int_0^t \left( h(a^*(s, P_s-, W_s)) - u(P_s D_s) \right) ds \\
&\quad + \int_0^t \xi(a^*(s, P_s-, W_s)) \sigma dM_s.
\end{align*}
$$

(2)

Then, the contract $(C_T, A) = (C_T, \{a^*(t, P_t-, W_t) : 0 \leq t \leq T\})$ is incentive compatible for the agent, and optimal for the principal among all incentive compatible contracts that deliver an initial expected value of at least $W_0$ to the agent.
Properties of the contract

- Keep track of $P_t$ to find the minimizing balance between $P_tD_t$ and $C_T$; not necessary if there is no restriction on when payment is made.

- Keep track of agent’s continuation value $W_t$ to provide incentive; not necessary in first-best contract or with linear utilities.

- No full insurance: $C_T$ needs to be random to provide incentives.

- Income effect: $\partial_{ww}F > 0$, due to concavity of $U$ and $u$.

- Sensitivity to market fluctuation:
  - $\xi > 0$; in the same direction as the price movement.
  - $\xi' > 0$; higher action $a \rightarrow$ higher volatility.
An implementation

1. The agent is asked to perform $A$.

2. By time $t$, the agent is paid $\int_0^t P_s D_s ds$

3. By time $t$, the agent is also promised

$$R - \int_0^t \left( u(P_s D_s) - h(A_s) \right) ds + \int_0^t \xi(A_s) \sigma dM_s$$

Note that $P$ is observable, and $M$ is the noise term that can be deduced from $P$. Hence, we can calculate the promised payment.
Qualitative lessons

- We do not know $U, u, \text{ or } h$. We could assume that $u$ and $U$ are linear, but the incentive device $\xi$ depends crucially on the form of $h$. 
- Contracted to pay either $\int \bar{p}D_sds$ or $\int P_sD_sds$ is not optimal.
- The calculation of the adjustment fee must consider: averaged cost and market manipulation.
- Adverse selection is not considered here.
- The reward/punishment is balanced by two opposing effect:
  - consistently higher $P_t \rightarrow$ lower fee
  - market fluctuation trending upward $\rightarrow$ higher fee.
Qualitative lessons, cont.

- Assume linear utilities

- \( R \): Learn the ballpark figure of a fair value of the contract

- \( \int_0^T h(\min\{A\})ds \): Estimate the cost of participating in the market, potentially from the information in the virtual bid markets, or let it be a small fraction of \( R \), or set it to the level of expected subsidies

- \( \xi \): Calibrate a fixed number \( \xi \) so that with respect to historical data, the realized contract value does not fluctuate too much
Conclusion

• Dynamic bilateral contract needs to account for stochastic fluctuations and be market responsive

• Market price aggregates market information and acts as a performance measure

• Payment should be contingent on market conditions (in particular, $P$)

• Subsidies should be sensitive on price volatilities.
Further research

- Numerical procedure and illustration
- Extension
  - control jumps
  - averaged cost is not known, i.e. the reservation value $R$ is private information (adverse selection)
- A dynamic equilibrium pricing model to study the interaction between the centralized and bilateral market
- Retail market design
Reduction to a stochastic control problem

- Principal’s constrained optimization problem is

$$\min_{(C_T,A) \in \mathcal{C}} \mathbb{E} \left[ C_T + \int_0^T P_s D_s ds \right] \text{ subject to } W_0^A \geq R.$$ 

- Using the incentive proposition,

$$\min_{A,W_0 \geq R} \mathbb{E} \left[ I(W_T) + \int_0^T P_s D_s ds \right], \text{ and }$$

$$W_t = W_0 - \int_0^t (u(P_s D_s) - h(A_s)) ds + \int_0^t \xi(A_s) \sigma dM_s.$$ 

- We ignore the control $W_0$ because we can show that any incentive compatible contract with $W_0 > R$ is not optimal for the principal.
Verification of the main theorem

Lemma (1)

The contract \((C_T, A)\) is incentive compatible for the agent, and his initial expected utility is \(R\).

Lemma (2)

Let the contract \((\tilde{C}_T, \tilde{A})\) be incentive compatible and delivers an initial expected utility \(W_0\) to the agent. Then, the contract \((I(\tilde{W}_T), \tilde{A})\) is also incentive compatible and delivers \(W_0\), where

\[
\tilde{W}_t = W_0 + \int_0^t \left[ h(\tilde{A}_s) - u(\tilde{P}_s D_s) \right] ds + \int_0^t \xi(\tilde{A}_s) \sigma dM_s.
\]

Further, the principal does not strictly prefer \((\tilde{C}_T, \tilde{A})\) to \((I(\tilde{W}_T), \tilde{A})\).
Lemma (3)

If a contract \((\bar{C}_T, \bar{A})\) is incentive compatible that delivers an initial expected utility \(R\) to the agent, then the principal’s initial payment is at least \(F(0, P_0, R)\), where \(F(\cdot)\) is the solution to (1). That is,

\[
\mathbb{E} \left[ \bar{C}_T + \int_0^T \bar{P}_s D_s ds \right] \geq F(0, P_0, R),
\]

where \(\bar{P}_t = P_0 + \int_0^t \rho(\bar{A}_s - \bar{P}_s)ds + \int_0^t \sigma dM_s + \sum_{0 < s \leq t} \Delta J_s\). 

Verification of the main theorem, cont.

Lemma (4)

Under the contract \((C_T, A)\), the principal’s initial expected payment is \(F(0, P_0, R)\).

Lemma (5)

Let \((\tilde{C}_T, \tilde{A})\) be an incentive compatible contract that deliver an initial expected value \(\tilde{W}_0\) to the agent. If \(\tilde{W}_0 > R\), then

\[
\mathbb{E} \left[ \tilde{C}_T + \int_0^T \tilde{P}_s D_s ds \right] > \mathbb{E} \left[ C_T + \int_0^T P_s D_s ds \right].
\]

In other words, the principal never wants to offer an incentive compatible contract that gives the agent an initial value higher than \(R\).
Verification of the main theorem, cont.

• The contract is incentive compatible by Lemma 1.

• Set $Z^d = 0$ by Lemma 2.

• Principal starts $W_0 = R$ by Lemma 5.

• Lemma 3 and 4 say that among all incentive compatible contracts in which the agent gets an initial expected payoff $R$, $(C_T, A)$ is at least as good to the principal as any others.