On behavioral complementarity and its implications

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Complementary goods
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Coffee and Sugar
Tea and Lemon
Peanut butter and Jelly
Cheese and Wine
Beer and Pretzels
Gin and Tonic

Matters for IO models and practical problems.
(e.g. firms’ pricing policy).
Explain consumer behavior: choices of \textit{consumption bundles} 
\[ x \in X \subseteq \mathbb{R}^n \]
Consumer Theory

Postulate a *utility function*

\[ u : X \rightarrow \mathbb{R} \]

Consumer behaves as if

\[
\max \quad u(x) \\
x \in \{ y \in X : p \cdot y \leq m \}
\]
Consumer Theory

\[ y : p \cdot y \leq p \cdot x \]
Problem: we don’t observe $u$; it’s a theoretical construct.
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Consider data $(x^k, p^k)$, $k = 1, \ldots, K$ on prices and consumption. Say that $u$ rationalizes the data if, for all $k$,

$$y \neq x^k \text{ and } p^k \cdot y \leq p^k \cdot x^k \Rightarrow u(y) < u(x^k)$$
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Can we rationalize anything choosing the appropriate $u$?
Two observations: \((x^1, p^1), (x^2, p^2)\)
Consider data \((x^k, p^k), k = 1, \ldots, K\).

**Theorem**

The following are equivalent

1. The data can be rationalized.
2. The data satisfy GARP.
3. The data can be rationalized by a weakly monotonic, continuous, and concave utility.
Back to complementary goods
How do choices between Coffee and Sugar and between Coffee and Tea differ?
(choices → testing)

How do preferences differ?
(preferences → modeling)
Complementarity in utility (Edgeworth-Pareto)

\[ u(x) - u(x \wedge y) \leq u(x \lor y) - u(y) \]
Complementarity in utility (Edgeworth-Pareto)

\[ u(x) - u(x \land y) \leq u(x \lor y) - u(y) \]
Complementarity in utility (Edgeworth-Pareto)

\[ u(x) - u(x \land y) \leq u(x \lor y) - u(y) \]
Theorem (Chambers & Echenique (2006))

Data, \((x^k, p^k), k = 1, \ldots, K\), can be rationalized if and only if it can be rationalized by a supermodular utility.
Behavioral complementarity

The standard notion of complementarities based on utility has no implication for (observable) choices.

So we study a *behavioral* notion.
Behavioral complementarity

\[ \downarrow \text{price of coffee} \Rightarrow \uparrow \text{demand for sugar.} \]

“Behavioral” \(\neq\) condition on preferences.
Behavioral complementarity
Behavioral complementarity

Essentially a property of *pairs of goods*.

Ex: Coffee, Tea and Sugar.
Use more sugar for tea than for coffee.
What if have \( n \) goods?


We need separability in preferences (and possibly aggregation).

e.g. heating and housing.
A demand is a function $D : \mathbb{R}^{2+} \times \mathbb{R}_+ \rightarrow \mathbb{R}^2$ s.t.

- $p \cdot D(p, I) = I$ (Walras Law).
- $\forall t > 0, D(tp, tl) = D(p, I)$ (Homogeneity of degree zero)

A demand function is *rational* if it can be *rationalized* by a weakly monotonic utility.
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i.e. $\exists$ w. mon. $u$ with

$$D(p, I) = \arg\max\{x : p \cdot x \leq I\} u(x)$$
Demand: $D(p, I)$ (Nominal Income).

$D$ satisfies *complementarity* if

$p \leq p' \Rightarrow D(p', I) \leq D(p, I)$.

Demand: $D(p, p \cdot \omega)$ (Endowment Income)

$D$ satisfies *(weak) complementarity* if, for every $p, \omega$ there is a $p'$ such that

$$[D_1(p', p' \cdot \omega) - D_1(p, p \cdot \omega)] [D_2(p', p' \cdot \omega) - D_2(p, p \cdot \omega)] \geq 0$$
Results (vaguely)

- Necessary and sufficient condition for observed demand to be consistent with complementarity (testable implications).
- Necessary and sufficient condition (within domains) for preferences to generate complements in demand.
- Differences in Nom. Income vs. Endowment Income.
Nominal Income – Testable Implications

Expenditure data: \((x, p) (x', p')\)
(Samuelson (1947), Afriat (1967), etc.)
Nominal Income – Testable Implications
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\[ \text{So } x \lor x' \notin B \lor B' \]
Nominal Income – Testable Implications
Necessary condition 1: \( x \lor x' \in B \lor B' \)
Nominal Income – Testable Implications
Nominal Income – Testable Implications
Necessary condition 2: a strengthening of WARP.
Nominal Income

By homogeneity, $D(p/I, 1) = D(p, I)$.  
So fix income $I = 1$ and write $D(p)$. 
Note: $B \vee B'$ is budget with $p \wedge p'$ and $I = 1$. 
Nominal Income

An *observed demand function* is a function $D : P \rightarrow \mathbb{R}^2_+$

- $P \subseteq \mathbb{R}^2_+$ is finite
- $p \cdot D(p) = 1$
Let $D : P \rightarrow \mathbb{R}^2_+$ be an observed demand.

**Theorem (Observable Demand)**

$D$ is the restriction to $P$ of a rational demand that satisfies complementarity iff $\forall p, p' \in P$

1. $(p \land p') \cdot (D(p) \lor D(p')) \leq 1$.

2. If $p' \cdot D(p) \leq 1$ and $p'_i > p_i$ then $D(p'_j) \geq D(p)_j$ for $j \neq i$. 
Nominal Income

Theorem (Continuity)

Let $D : \mathbb{R}^2_+ \to \mathbb{R}^2_+$ be a rationalizable demand function which satisfies complementarity. Then $D$ is continuous. Furthermore, $D$ is rationalized by an upper semicontinuous, quasiconcave and weakly monotonic utility function.
Nominal Income

Theorem (Continuity)

Let \( D : \mathbb{R}^2_{++} \rightarrow \mathbb{R}^2_+ \) be a rationalizable demand function which satisfies complementarity. Then \( D \) is continuous. Furthermore, \( D \) is rationalized by an upper semicontinuous, quasiconcave and weakly monotonic utility function.

Remark

We extend the data to a demand, and then find it’s rational. Difference from Afriat’s approach of constructing a utility.
Theorem 1

Suppose we observe \((p, x)\) and \((p'', x'')\).
Find demand at prices \(p'\) (extend demand).
Theorem (Observed Demand)
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Let $u$ be a $C^2$ utility.

$$m(x) = \frac{\partial u(x)/\partial x_1}{\partial u(x)/\partial x_2}$$

is the marginal rate of substitution of $u$ at an interior point $x$. 
Let $D$ a demand w/ interior range and monotone increasing, $C^2$, and strictly quasiconvex rationalization.

**Theorem (Smooth Utility)**

*D satisfies complementarity iff*

$$\frac{\partial m(x)/\partial x_1}{m(x)} \leq -\frac{1}{x_1} \quad \text{and} \quad \frac{\partial m(x)/\partial x_2}{m(x)} \geq \frac{1}{x_2}$$
Theorem (Smooth Utility)
Nominal Income

Separability: \( u(x, y) = f(x) + g(y) \).
Then complementarity iff \( f \) and \( g \) more concave than log.

Expect. util.: \( \pi_1 U(x) + \pi_2 U(y) \).
Then complementarity iff RRA \( \geq 1 \).

Analogous result for subst. due to Wald (1951) and Varian (1985).
Endowment Income

$D$ satisfies *complementarity* if, for all $(p, \omega)$ and all $p'$,

$$
[D_1(p', p' \cdot \omega) - D_1(p, p \cdot \omega)] [D_2(p', p' \cdot \omega') - D_2(p, p \cdot \omega)] \geq 0.
$$

(1)

$D$ satisfies *weak complementarity* if, for every $(p, \omega)$, there is one price $p' \neq p$ satisfying (1).
Endowment Income

Let $D$ be a rational demand.

**Theorem (Endowment Model)**

The following are equivalent:

1. $D$ satisfies complementarity.
2. $D$ satisfies weak complementarity.
3. $\exists$ cont. strictly monotone, $f_i : \mathbb{R}_+ \rightarrow \mathbb{R} \cup \{\infty\}, i = 1, 2$, at least one of which is everywhere real valued ($f_i(\mathbb{R}_+) \subseteq \mathbb{R}$), s.t.

\[
    u(x) = \min \{ f_1(x_1), f_2(x_2) \}
\]

is a rationalization of $D$. 
Endowment Income

\[ \omega_p x = \omega' \]
Endowment Income
Conclusions: