1. Suppose we have a market with two firms. The inverse demand function is given by
\( p = a - Q \), where \( Q = q_1 + q_2 \). Suppose that the cost function of firm \( i \) is given by
\( c(q_i) = c_i \cdot q_i \), and \( 0 < c_1 < c_2 \).

(a) What is the Cournot best reaction function for each firm? Note that the problem is not symmetric!

(b) What are the Cournot equilibrium quantities price and market shares?

(c) Suppose now that firms compete via Bertrand (price) competition. What is the equilibrium price quantities and market shares.

2. In many markets, e.g., liquor in Rhode Island, firms are allowed to advertise but not allowed to mention prices or the existence of a sale. Suppose we have a good with a total demand function \( Q = D(p) \). Suppose that there are two firms, each with the same constant marginal cost, \( c \). Assume that if neither firm advertises then half of the market would go to each firm. However, if firm \( i \) and \( j \) do advertise, spending \( a_i \) and \( a_j \) respectively, then the proportion \( \frac{a_i}{a_i + a_j} \) of the market visits firm \( i \) only, and \( \frac{a_j}{a_i + a_j} \) of the market visits firm \( j \) only. That is consumers do not ”shop around” they only go to one firm.

(a) Write down the profit maximization problem for firm \( i \).

(b) What is the equilibrium of the game where firms each first choose advertising simultaneously, and then choose prices? Can you interpret the equilibrium prices and advertising levels?

(c) Suppose now that the number of firms, \( n \), increases. What happens to prices, the level of advertising in the market, and profits in the limit as \( n \) increases?
3. Consider a market with two firms with constant marginal cost, where firm i’s demand function is given by $q_i = a - bp_i + dp_j$ where $p_i$ is its own price and $p_j$ is its rivals price. Assume that $a, b, c, d$ are positive and that $a > c$. Suppose that this market is a Bertrand Game (i.e. price competition).

(a) Write down firm i’s profit function.
(b) What is firm i’s reaction or best response function?
(c) What are the Bertrand equilibrium prices?