TURNOUT AND POWER SHARING

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Abstract. We compare turnout under proportional power sharing electoral systems and winner take all elections. The effect of such institutional differences on turnout depends on the distribution of voter preferences. If the two parties have relatively equal support, turnout is higher in a winner-take-all system; the result is reversed when there is a clear underdog. We report findings from a laboratory experiment that was designed and conducted to explore this theoretical hypothesis and several other secondary hypotheses that are also implied by the theoretical model. The results are broadly supportive of the theoretical predictions on comparative turnout, the partial underdog compensation effect, and the competition effect.

Keywords: Voter turnout; Electoral systems; Power Sharing; Underdog Effect

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1. Introduction

Voter participation is an essential component of democracy, and changes in the level of electoral participation may affect the political positioning of the competing parties, electoral outcomes and ultimately public policy. At the same time, the level of electoral participation, electoral outcomes, political parties and other aspects of the political landscape are all endogenous and widely believed to be consequences of the electoral rules. A key property of electoral systems is the degree of proportionality in translating votes to seats. Many conjectures have been offered about whether or not more proportional systems lead to higher turnout and why. Heuristic arguments have been offered on both sides, but this article provides the first theoretical and experimental analysis of the complex relationship between proportionality and turnout, focusing mainly on the question of how equilibrium turnout is jointly affected by a combination of the relative size of the competing parties and the electoral rules.

The laboratory experiment, while a stylized version of elections in the field, enjoys an important advantage over empirical or historical studies, by virtue of eliminating potential confounding factors that have challenged such studies, and led to an ambiguous mix of findings. These confounding effects include the measurement of competitiveness, properly controlling for social/cultural factors, endogeneity of the choice of electoral system, isolating the effects of district magnitude or multimember districts, and taking into account institutional variations in government formation, to name a few.

A claim that is essentially folk wisdom in the political science literature on comparative politics is that proportionality increases turnout, and, in particular, proportional systems will produce more turnout than single member plurality (winner take all) systems.[1] Selb (2009), for example, leads off his introduction with a sweeping statement: "There is wide agreement among scholars that the proportionality of electoral systems...is positively associated with voter participation." Regardless of this wide agreement, however, if one looks more closely it turns out that, in fact, the results are quite mixed. There are some glaring exceptions that are

[1]See, for example, Powell (1980, 1986), Crewe (1981), Jackman (1987) and Jackman and Miller (1995), Blais and Carthy (1990) and Franklin (1996). There have also been statements suggesting that this claim is implied by theoretical results (see e.g. Bowler et al. (2001) and Cox (1999)), but the "theory" referred to consists of informal arguments based on casual theorizing. There have also been informal theoretical arguments on the other side: Powell (1980) argues that for several reasons SMP systems are more transparent than PR systems, which may boost turnout. Some possible factors have been argued both ways. For example, it was initially argued (e.g., Gosnell 1930) that PR may produce higher turnout because it leads to more political parties.

dismissed with idiosyncratic explanations, and without such exclusion the evidence for this central claim is weak.[2] Acemoglu (2005) argues that cultural and idiosyncratic characteristics that are difficult to control for make it difficult to assess the causal effects of institutional differences, since institutions are themselves endogenous.[3] Blais (2006) in his turnout survey concludes that "many of the findings in the comparative cross-national research are not robust, and when they are, we do not have a compelling microfoundation account of the relationship."

Where does this leave us? The empirical findings are mixed and the published theoretical claims are mostly nonexistent or based on informal intuitions. First of all, this paper provides a formal theoretical model that (a) offers a rationale for why one should a priori expect cross sectional empirical studies to lead to mixed findings and (b) identifies clear conditions under which proportional systems will lead to more (or less) turnout than winner take all systems. Second, this paper offers data from a laboratory experiment that provides exactly the kind of controls that are missing in the empirical studies. The experiment itself is specifically designed to test whether predictions falling out of the theoretical characterization are supported by data.

The theoretical results we present here on the comparison between electoral participation in winner-take-all systems and power sharing systems[4] are an abridged version of what appears in more detail in our working paper (Herrera et al. 2013), most of which was contained in the earlier working paper of Herrera and Morelli (2008)[5]. The key insight is that the ranking of turnout in the two systems depends on the expected closeness of the election. While closeness has been conjectured to play an important role in winner take all elections, until now little

[2]Switzerland is the most prominent exception of a PR system with low turnout. Evidence from Latin America also runs counter to folk wisdom. New Zealand (prior to switching to PR) offers another counterexample. They switched to PR and turnout declined. Blais (2000 & 2006) points out how the result in Blais and Carthy (1990) relies entirely on the treatment of New Zealand as a deviant case.

[3]Putnam et al. (1983) make a similar point, as does Boix (2000).

[4]We use the term power sharing rather than proportional representation because the mapping from vote shares to power shares can be in principle proportional even in systems that strictly speaking do not use PR electoral rules. The relative power of the majority party for a given election outcome varies with the degree of separation of powers, the organization of chambers, the assignment of committee chairmanships and institutional rules on agenda setting, allocation of veto powers, and obviously electoral rules. See Lijphart (1999) and Powell (2000) for a comprehensive analysis of the impact of political institutions on what they call degree of proportionality of influence, which is basically our vote-shares to power-shares mapping. Electoral rules determine the mapping from vote shares to seat shares in a legislature, whereas the other institutions determine the subsequent mapping from seat shares to power shares across parties.

[5]See for instance: http://www.cer.ethz.ch/education/morelli_3.pdf

has been known theoretically about the effect of closeness in proportional systems. The main theoretical result is that in large elections a winner take all system induces higher turnout if and only if the election is expected to be close; power sharing systems induce higher turnout than winner take all systems in less competitive races. We base our analysis in this paper on the costly rational voting model (see e.g. Ledyard (1984) and Palfrey and Rosenthal (1985)) under population uncertainty[6], extending the analysis to the proportional influence or proportional power sharing system.[7]

This main finding follows from an important intermediate result that we prove in this paper, *the partial underdog compensation effect.* Generally in turnout models with large numbers of voters, there is an underdog compensation effect, namely the supporters of the candidate who is expected to lose will have higher turnout rates than the supporters of the favored candidate. The modifier "partial" indicates that the amount by which underdog supporters vote more relative to the favorite's supporters does not fully compensate for the ex ante advantage of the favorite. Therefore, in a winner-take-all system, when preferences are not evenly split the partial underdog compensation preserves the ex-ante leading party as the ex-post leading party in equilibrium. The expected winning margin remains high, leading to pivot probabilities that decline very rapidly in N, the expected number of voters, strongly discouraging participation. In a proportional power sharing system the expected marginal benefit of a single vote is proportional to the marginal change in the vote share determined by that vote, rather than pivot probabilities, and therefore a less competitive election (i.e. a higher expected winning margin) does not affect the incentives to vote as much. In contrast, if the two candidates are equally favored ex ante, then the pivot probabilities decline very slowly, on the order of the square root of N, which implies turnout in winner take all elections to be relatively higher.

Several theoretical papers have assumptions that made them obtain for winner-take-all elections a *full* underdog compensation effect in the limiting case of large electorates: the

[6]Viewing the size of the electorate as a random variable (see Myerson 1998 and 2000) has the advantage of simplifying the computations without altering the incentives driving the results. See also Krishna and Morgan (2011) for a similar model but with common values. Numerical computations we performed with fixed population sizes confirm however that all our comparative results do not depend on population uncertainty.

[7]Elsewhere (Herrera et al. 2013), we show that the main theoretical finding is shared by a wide range of non-instrumental turnout models, such as group mobilization models (see e.g. Morton (1987, 1991), Cox and Munger (1989), Uhlaner (1989), and Shachar and Nalebuff (1999)) and ethical voting (see e.g. Coate and Conlin (2004) and Feddersen and Sandroni (2006)). In that paper the main results are extended to allow for multiple parties as well.

theoretical claim is that in pivotal voting models the expected vote shares of the two parties are *equal* independent of the distribution of partisan preferences in the population. Our paper shows that this result depends on what one assumes about the distribution of voting costs. In particular, full underdog compensation can occur when the distribution of voting costs is degenerate (Goeree and Grosser 2007, Taylor and Yildirim 2010), or is bounded below by a strictly positive minimum voting cost (Krasa and Polborn 2009), or is identical for voters from different parties. In contrast, the original Palfrey-Rosenthal (1985) model rules out the first two of these cases and explicitly allows for different distributions of voting costs for the two parties. The full underdog compensation result is, in fact, not a general property of pivotal voting models in large electorates. With a finite number of voters, typically the underdog compensation effect is only partial, and in rare cases can even be reversed. In the limiting case, one can get less than full underdog compensation in several ways. As one extreme, if the two parties have supporters with heterogeneous costs of voting with a distribution of voting costs with lower bound of support less than or equal to zero, then the underdog compensation effect is always partial or zero in the limit. The intuition for this is easy to see in the extreme case where some positive fraction of voters have zero or negative voting costs. In this case, for large elections they are the only ones who vote, and there is no free rider problem among these voters because they get direct utility (or zero cost) from voting, hence there is zero underdog compensation in large elections. In the intermediate case that we study in this article, with the continuous distribution of voting costs that has non-negative support and a positive density at zero, voters supporting the underdog turn out in higher percentages than voters that support the favorite, but the underdog compensation is only *partial*. Hence, the party with higher ex-ante support is always expected to win large elections, albeit by a smaller margin of victory than the ex-ante support advantage (e.g. the opinion polls) would predict.

If one moves away from the convenient population uncertainty world (convenient in terms of tractability) it is straightforward to derive *exact* equilibrium conditions to be used for numerical computation of the predictions for any known number of voters. Hence it is possible to test the comparative results in the laboratory. The relevance of the comparative findings is put to a test in a large study of 1900 laboratory elections, where we experimentally control and manipulate the proportionality parameter, voting costs and the competitiveness of the election. Since Levine and Palfrey (2007) already provide a preliminary set of data with winner-take-all rules, we adopted the same treatments even in the new proportional experiments, so that the data could be pooled together. The experimental results confirm the theoretical predictions of the general model, as well as other predictions on the closeness effects that come out specifically from the known-population model computations.

1.1. **Related Literature.** Our modeling strategy is related to a body of literature that applies the Poisson game approach to model strategic voting in large elections, including recent contributions by Bouton (2012), Bouton and Gratton (2013), Krishna and Morgan (2011), McMurray (2012). The closest of these to our paper is Castanheira (2003), which examines turnout in large winner take all elections in the context of one-dimensional spatial competition between two candidates. He obtains results by making the voters a continuum, i.e. swapping the incremental benefit of a vote with a derivative. He also partially extends his main results about the rate of convergence to zero turnout in winner-take-all elections to the case where there can be a "mandate effect", which he models with a linearized weighting function, similar to Stigler (1972).

A recent related theoretical study by Kartal (2013) addresses the differences in turnout across electoral systems, using a somewhat different approach and with a different focus. Following the approach of Palfrey and Rosenthal (1985), that paper does not adopt the Poisson games framework, assumes no uncertainty about the number of voters in the election, and the analysis is primarily focused on welfare comparisons across different systems with endogenous turnout. A similar result to ours on the partial compensation effect is obtained. A later study by Faravelli and Sanchez-Pages (2012) also compares turnout and welfare with majority and with general power sharing rules with endogenous turnout. They obtain results only in a neighborhood of perfectly even elections ($q=1/2$) or perfectly biased elections ($q=1$).[8] The Herrera and Morelli (2008) working paper contained many of these comparative results, and also established "model robustness" by proving that the main theoretical results comparing power sharing with winner take all also hold in several alternative theoretical approaches that allow for non-instrumental motives for voting and when there are more than two parties.

Earlier experimental evidence (see Schram and Sonnemans (1996)) suggests turnout is higher in a majoritarian system than in proportional representation, but the experimental design featured only the case of perfect symmetry in the ex-ante supports for the two parties. Thus, their finding that the winner take all elections display higher turnout is consistent with our theoretical results. However, our theory also predicts that the turnout ordering will be reversed if the parties' ex-ante supports are sufficiently asymmetric. Levine and Palfrey (2007)

[8]See also Faravelli et al (2013) for other extensions.

conduct an experiment with a very similar design to ours, but only looks at winner take all elections. Recent experimental findings related to ours can also be found in Kartal (2011). That experiment implements a different, piecewise linear, version of proportional representation, and the design mainly addresses issues of representation and efficiency. The experimental design in the present paper contrasts a purely proportional representation system with winner take all, and focuses on the effect of relative party size on turnout in proportional versus winner take all systems. In addition, we run some robustness treatments with different cost distributions, including some treatments in which a *reverse* underdog compensation effect is predicted and observed.

The paper is organized as follows. Section 2 contains the analysis of our rational voter model, comparing the properties of proportional power sharing and winner-take-all systems. Section 3 describes the experimental design, procedures, and data analysis, the results of which are broadly supportive of the theory in terms of comparative statics in small elections, and where one can see the general findings of the theory further confirmed. Section 4 offers some concluding remarks and describes potential paths of future research.

2. The Voter Turnout Model

Two parties, A and B, compete for power. Citizens have strict political preferences for one or the other, chosen exogenously by Nature. We denote by $q \in (0,1)$ the preference split, i.e. the chance that any citizen is assigned (by Nature) a preference for party A (thus $1-q$ is the expected fraction of citizens that prefer party B).[9] Without loss of generality, we assume that $q \leq 1/2$, so that A is the underdog party with smaller ex-ante support, and B is the favorite party. The indirect utility for a citizen of preference type i, $i = A, B$, is increasing in the share of power that party i has. For normalization purposes, we let the utility from "full power to party i" equal 1 for type i citizens and 0 for the remaining citizens.

Beside partisan preferences, the second dimension along which citizens differ is the cost of voting. Citizen i's cost of voting, c_i, is drawn from a distribution with infinitely differentiable pdf $f(c)$, strictly positive on the support $[0, \overline{c}]$, with $\overline{c} > 1/2$.[10]

[9] For the case in which q is not fixed, see Myatt (2012).

[10] Cost draws are independent. The assumption that $\overline{c} > 1/2$ is made for technical reasons, to avoid dealing with boundary cases. One could also allow for the support to include negative voting costs, which trivially implies a zero compensation effect in large electorates.

We assume that the size of the electorate is finite but uncertain.[11] The number n of citizens who are able to vote in a given election is distributed as a Poisson distribution with mean N:

$$n \sim \frac{e^{-N}(N)^n}{n!}$$

Citizens have to choose to vote for party A, party B, or abstain. If a share α of A types vote for A and a share β of B types vote for B, the expected turnout $T = q\alpha + (1-q)\beta$.

We compare the above equilibrium conditions in two electoral systems that differ on the benefit side: a winner-take-all system (M) and a proportional power sharing system (P). Under system M, if the vote share for A is $V > 1/2$, then party A has full power and each A supporter gets utility 1. The payoffs are reversed if $V < 1/2$, and every citizen gets a payoff of 1/2 if $V = 1/2$. Under P, for any V, an A supporter's payoff is V and a B supporter's payoff is $1 - V$.[12] For each system we characterize the Bayesian equilibria of the game, which take the form of a pair of cutoff thresholds, (c_α, c_β), one for each party. That is, each A supporter with a cost below a threshold c_α votes for type A, each B supporter with a cost below c_β votes for B. All other citizens abstain. So on aggregate, the expected proportion of type A citizens who vote is $\alpha = F(c_\alpha)$ and the expected proportion of type B citizens who vote is $\beta = F(c_\beta)$.

In an equilibrium strategy profile (c_α, c_β), the expected marginal benefit of voting, B (which will be characterized by different expressions for the P and M systems) equals the cutoff cost of voting (indifference condition for the citizen with the highest cost among the equilibrium voters).[13] Hence the equilibrium conditions can be written as:

$$B^A(\alpha,\beta) = F^{-1}(\alpha), \qquad B^B(\alpha,\beta) = F^{-1}(\beta) \tag{1}$$

[11]Our results hold more generally: we use this specification for convenience only in the limit equilibrium results that will follow.

[12]By convention, let $V = 1/2$ in the event nobody votes.

[13]If we drop the assumption that $\bar{c} > 1/2$ and N is sufficiently small, then the cutoff costs may be on the boundary, at $\bar{c}$, for one or both parties, in which case the equilibrium condition would be an inequality. The assumption of $\bar{c} > 1/2$ avoids these uninteresting cases.

In the M system B is equal to the probability of making or breaking a tie times the payoff from doing so $(1/2)$. Hence:

$$\begin{aligned} B_M^A &= \sum_{k=0}^{\infty}\left(\frac{e^{-qN\alpha}\left(Nq\alpha\right)^k}{k!}\right)\left(\frac{e^{-(1-q)N\beta}\left(\left(1-q\right)N\beta\right)^k}{k!}\right)\frac{1}{2}\left(1+\frac{\left(1-q\right)N\beta}{k+1}\right) \\ B_M^B &= \sum_{k=0}^{\infty}\left(\frac{e^{-qN\alpha}\left(Nq\alpha\right)^k}{k!}\right)\left(\frac{e^{-(1-q)N\beta}\left(\left(1-q\right)N\beta\right)^k}{k!}\right)\frac{1}{2}\left(1+\frac{qN\alpha}{k+1}\right) \end{aligned}$$

Equating the benefit side to the cost side we obtain a system of two equations in (α,β) (the M system henceforth).

In the P system B is equal to the expected increase in vote share if one votes instead of abstains. Hence:

$$B_P^A=\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\left(\frac{e^{-qN\alpha}\left(qN\alpha\right)^a}{a!}\right)\left(\frac{e^{-(1-q)N\beta}\left(\left(1-q\right)N\beta\right)^b}{b!}\right)\left(\frac{a+1}{a+b+1}-\frac{a}{a+b}\right)\right)$$

$$B_P^B=\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\left(\frac{e^{-qN\alpha}\left(qN\alpha\right)^a}{a!}\right)\left(\frac{e^{-(1-q)N\beta}\left(\left(1-q\right)N\beta\right)^b}{b!}\right)\left(\frac{b+1}{a+b+1}-\frac{b}{a+b}\right)\right)$$

With P, unlike the M system, there is a double summation because an A supporter, for instance, has an impact on the electoral outcome not only in the event of a tied election ($a=b$ and $a=b-1$), but for all realizations of a and b. In the P system voters always have some impact on the electoral outcome albeit very small, whereas in the M system voters have a large impact only in a pivotal event and zero impact otherwise. We first establish two basic results that hold for all expected electorate sizes, N, and then provide a full characterization of the limiting equilibrium properties for very large N. All proofs are in the Appendix.

2.1. **Common Properties of Equilibrium for all N.** We first establish existence of equilibrium and the partial underdog effect.

Proposition 1. *The following properties hold in both M and P systems for any N.*

(1) *Existence:* There exists an equilibrium (α,β) for any q.
(2) *Partial Underdog Compensation:* In any equilibrium (α,β) we have:

$$q<1/2 \quad \Longrightarrow \quad \alpha>\beta, \quad q\alpha<(1-q)\beta$$

As we show in the proof in the appendix, the underdog compensation holds for general power sharing functions[14], with the property that for any given ex-post vote realization (a,b),

[14] The Poisson uncertainty assumption we use in the proof is not crucial: this result holds without population uncertainty as well.

an additional vote for the underdog gives the underdog a higher marginal gain than an additional vote for the leader gives the leader. The underdog compensation being just partial implies that the minority party has a higher expected turnout *rate* but lower expected total turnout in terms of the expected *number* of voters, so the party with more ex-ante support remains the more likely winner of the election. As a consequence, we have a balanced election with a 50% expectation of victory from each side *only* when $q = 1/2$. With homogeneous costs, or assuming that all voters have strictly positive costs, the result would be different, namely a full underdog compensation and a 50% chance of victory regardless of the ex-ante preference split q. The partial underdog compensation effect also plays an important role for the comparative turnout results, to which now we turn.

2.2. **Limit Equilibrium Properties of Elections.** We consider the limit case directly, rather than establishing results that hold for sufficiently large N. For technical reasons, we assume voters base their calculus on a first order approximation of the limiting pivot probabilities. We will refer to the equilibrium based on this assumption as a *limit equilibrium*.[15] We assume that the parameters of the model remain fixed as N varies.[16]

To study the asymptotic properties of equilibrium turnout, we first establish that, in any equilibrium, large N necessarily implies large average turnout level for each party $(N\alpha, N\beta)$.

Lemma 2. *Any equilibrium solution (α_N, β_N) to the M and P system has the following properties*

$$\lim_{N\to\infty} N\alpha_N = \lim_{N\to\infty} N\beta_N = \infty, \qquad \lim_{N\to\infty} \alpha_N = \lim_{N\to\infty} \beta_N = 0, \qquad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} \in (0, \infty)$$

The next proposition establishes five additional results about turnout in a limit equilibrium. First, it establishes uniqueness and fully characterizes in closed form the limit equilibrium turnout rates for the M system if F is weakly concave.[17] Second, it establishes uniqueness and fully characterizes in closed form the limit equilibrium turnout rates for the P system . Third, it establishes the size effect, that turnout declines in N. Fourth, it establishes that for elections

[15]A similar approach is taken elsewhere to analyze strategic voting in large electorates (for example, Myatt 2007).

[16]Fixing all the model parameters when taking limits is standard in the literature, but not necessarily innocuous. For example, in some cases the value of a winning outcome might plausibly increase in the size of the electorate.

[17]F weakly concave is strong but by no means a necessary condition. It suffices that $xF^{-1}(x)$ is weakly convex in a neighborhood of 0, a weaker but less intuitive condition.

that are expected to be close turnout in the M system, T_M, is higher than turnout in the P system, T_P, and otherwise turnout is higher in the P system. Fifth, it shows that the underdog compensation effect is always greater in the P system, and strictly greater if $q < 1/2$.

Proposition 3. *If voters use the asymptotic pivot probabilities in the limiting case of arbitrarily large elections, then:*

(1) *There exists a unique limit equilibrium in the M system if F is weakly concave, characterized by:*

$$q\alpha_M \left(F^{-1}\left(\alpha_M\right)\right)^2 = (1-q)\,\beta_M \left(F^{-1}\left(\beta_M\right)\right)^2$$

(2) *There exists a unique limit equilibrium in the P system, characterized by*

$$q\alpha_P \left(F^{-1}\left(\alpha_P\right)\right) = (1-q)\,\beta_P \left(F^{-1}\left(\beta_P\right)\right)$$

(3) *Size Effect.* For any q, the equilibrium expected total turnout, T, declines in the expected size of the electorate, N.

(4) *Comparative Turnout.*

$$\begin{aligned} T_M &> T_P \qquad \text{if } q = 1/2 \\ T_P &> T_M \qquad \text{if } q \neq 1/2 \end{aligned}$$

(5) *Comparative Underdog Compensation:*

$$\frac{1-q}{q} = \left(\frac{\alpha_P}{\beta_P}\right)^{n+1} = \left(\frac{\alpha_M}{\beta_M}\right)^{2n+1}$$

where $n \geq 1$ is the lowest integer for which $\frac{d^n F^{-1}}{dx^n}|_{x=0} \in (0,\infty)$.

These theoretical results about the asymptotic properties of turnout in M and P systems provide a sharp comparison of how turnout in the two systems varies with q. Of particular interest with respect to the empirical literature is that turnout is higher under the M system when q is close to $1/2$, but higher under P if q is not close to $1/2$. In the limit, an ex ante 50-50 split of the electorate versus a non 50-50 split, imply very different overall turnout numbers in large elections because the driving force for voter turnout (being pivotal) only arises when the vote split is exactly 50-50 in the M system, whereas, a voter is always pivotal (a little bit) in the P system regardless of the exact vote split. Specifically, under M, when $q = 1/2$ (or if costs are homogeneous) the asymptotic benefit of voting (and hence the asymptotic turnout) declines at rate $N^{-1/2}$ because $q\alpha = (1-q)\beta$. However, if $q\alpha \neq (1-q)\beta$, turnout declines at an *exponential rate,* which is the case whenever $q \neq 1/2$.[18] In contrast, the rate of convergence in the P system is always on the order of N^{-1} independent of q, a rate that is quantitatively *in between* the two rates of convergence in the M system: $N^{-1} \in \left(N^{-1/2}, e^{-N}\right)$.

The above proposition (part 5) also shows that the underdog compensation is larger in the P system than the M system, namely

$$q < 1/2 \quad \Rightarrow \quad \alpha_P/\beta_P > \alpha_M/\beta_M > 1$$

Compared to the P system, in the M system minority voters are always more discouraged to vote relative to majority voters. This result also implies a higher *relative winning margin* in the M system than in the P system for any given preference split q, where the relative winning margin is equal to $\frac{|q\alpha-(1-q)\beta|}{T}$.

To illustrate the results, we turn to a numerical example.

2.3. **Example.** Consider the cost distribution family ($z > 0$): $F(c) = c^{1/z}$ with $c \in [0, 1]$.

This example yields an explicit solution for the P system, i.e.

$$\alpha_P = \left(\frac{1}{N}\frac{(1-q)\,q^{\frac{1}{z+1}}\,(1-q)^{\frac{1}{z+1}}}{\left(q\,(1-q)^{\frac{1}{z+1}} + (1-q)\,q^{\frac{1}{z+1}}\right)^2}\right)^{\frac{1}{z+1}} \quad \beta_P = \left(\frac{1}{N}\frac{q}{1-q}\frac{(1-q)\,q^{\frac{1}{z+1}}\,(1-q)^{\frac{1}{z+1}}}{\left(q\,(1-q)^{\frac{1}{z+1}} + (1-q)\,q^{\frac{1}{z+1}}\right)^2}\right)^{\frac{1}{z+1}}$$

The M system equilibrium has no closed form solution, namely (α_M, β_M) jointly solve

$$\beta_M = \left(\frac{q}{1-q}\right)^{\frac{1}{2z+1}}\alpha_M, \qquad \alpha_M^z = \frac{e^{-N\left(\sqrt{(1-q)\beta_M}-\sqrt{q\alpha_M}\right)^2}}{\sqrt{N}}\left(\frac{\sqrt{q\alpha_M}+\sqrt{(1-q)\,\beta_M}}{4\sqrt{\pi}\,(q\,(1-q)\,\alpha_M\beta_M)^{1/4}}\right)$$

Setting $N = 3000$ and $z = 5$, the numerical solutions to the M system yield a clear illustration of the comparative result of proposition 3. Figure 1 compares expected total turnout, T in the M system (continuous line) and in the P system (dashed line), as a function of q. For q is in the range of $(.45, .55)$ turnout is higher with M, and the opposite is true for more extreme q. Also observe that P is much less responsive to q. As N becomes large, the P curve approaches a horizontal line, and the M curve approaches a spike at $q = 1/2$.

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[18] The two rates of convergence derived above do not depend on the (Poisson) population uncertainty in this model, but hold more generally. See, for example, Chamberlain and Rothschild (1981) and Herrera and Martinelli (2006), which analyze a majority rule election without population uncertainty.

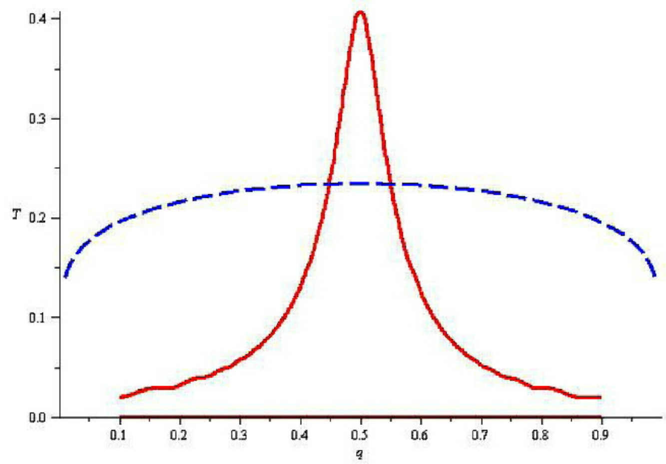

Figure 1: Turnout as a function of q in the M (continuous) and P (dashed) models ($z = 5$, $N = 3000$).

Figure 2 contrasts how the underdog compensation effect, α/β, varies with q in the P system (dashed line) compared to the M system (continuous line). As a reference point, the figure also shows the full underdog compensation curve, as a dotted line. This figure illustrates three features of the equilibrium. First, if an underdog exists ($q \neq 1/2$), the underdog compensation is only partial. Second, the underdog effect is always stronger under P than M, and, third under P, the underdog compensation effect is more responsive to changes in q.

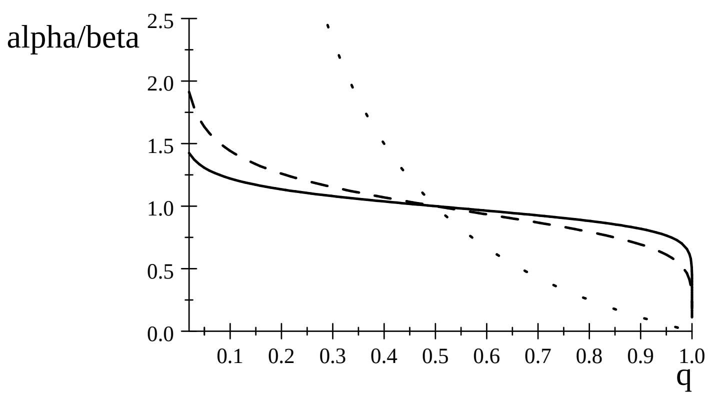

Figure 2: Partial underdog compensation (α/β) as a function of q in P (dashed), M (continuous), and Full underdog compensation (dotted).

3. Experimental Analysis

To examine the comparative theoretical hypotheses about turnout differences under P and M in a way that avoids the measurement and endogeneity problems inherent in much of the empirical literature on turnout, we conducted a large number of small-scale laboratory experiments, based on the model of heterogeneous independent costs, and varying the relative sizes of the underdog and favorite party. Obviously, in our laboratory elections we can have only a finite number of citizens, but the model is easily adapted to this case. Furthermore, for the parameters of the experiment the equilibrium is unique and inherits that same comparative hypotheses about total turnout, underdog effects, and the effect of q as in the Poisson model.[19] The equilibrium conditions for our laboratory implementation of the model, with finite electorates and no population uncertainty[20], are given below. It is straightforward to exactly characterize symmetric Bayesian equilibrium for these finite environments, and comparative statics that are similar to the Poisson model can be computed directly from these exact equilibrium solutions.

In what follows, let N_A denote the number of citizens supporting party A and $N_B = N - N_A$ denote the number of voters with a preference for party B, and assume without loss of generality that $N_A \leq N_B$. As before, a symmetric equilibrium is characterized by two cutoff levels, one for each party, c^α and c^β, with corresponding expected turnout levels equal to $\alpha = F(c_\alpha)$ and $\beta = F(c_\beta)$. The equilibrium conditions are slightly different for the M and P systems, and these are derived next.

3.0.1. *Equilibrium conditions for M.* The expected marginal benefit of voting for a party A citizen equals:

$$B_M^A = \frac{1}{2}\left[\begin{array}{c} \sum_{k=0}^{N_A-1}\binom{N_A-1}{k}\binom{N_B}{k}\alpha^k(1-\alpha)^{N_A-1-k}\beta^k(1-\beta)^{N_B-k}+ \\ \sum_{k=0}^{N_A-1}\binom{N_A-1}{k}\binom{N_B}{k+1}\alpha^k(1-\alpha)^{N_A-1-k}\beta^{k+1}(1-\beta)^{N_B-1-k} \end{array}\right]$$

$$B_M^B = \frac{1}{2}\left[\begin{array}{c} \sum_{k=0}^{\min\{N_A,N_B-1\}}\binom{N_A}{k}\binom{N_B-1}{k}\alpha^k(1-\alpha)^{N_A-k}\beta^k(1-\beta)^{N_B-1-k}+ \\ \sum_{k=0}^{N_A-1}\binom{N_A}{k+1}\binom{N_B-1}{k}\alpha^{k+1}(1-\alpha)^{N_A-1-k}\beta^k(1-\beta)^{N_B-1-k} \end{array}\right]$$

where $\frac{1}{2}$ is the value of making or breaking a tie, $\alpha = F(c_M^\alpha)$, and $\beta = F(c_M^\beta)$. In each expression, the first summation is the probability of your vote breaking a tie, and the second summation is the probability of your vote creating a tie, given turnout rates α and β. The equilibrium conditions for c_M^α and c_M^β are given by $c_M^\alpha = B_M^A$ and $c_M^\beta = B_M^B$.

[19] The one exception is that in one of our treatments we choose parameters that lead to a predicted *reverse* underdog effect.

[20] The reason to consider *known* population size is that the analytical computations with the Poisson game approach apply only to the limiting case of very large electorates, which is not feasible in the laboratory.

3.0.2. *Equilibrium conditions for P.* Given expected turnout rates in the two parties, α and β the expected marginal benefit of voting for a party A citizen is:

$$B_P^A = \sum_{j=0}^{N_A-1} \sum_{k=0}^{N_B} \left[\frac{j+1}{j+1+k} - \frac{j}{j+k} \right] \binom{N_A - 1}{j} \binom{N_B}{k} \alpha^j (1-\alpha)^{N_A-1-j} \beta^k (1-\beta)^{N_B-k}$$

$$B_P^B = \sum_{j=0}^{N_B-1} \sum_{k=0}^{N_A} \left[\frac{j+1}{j+1+k} - \frac{j}{j+k} \right] \binom{N_B - 1}{j} \binom{N_A}{k} \beta^j (1-\beta)^{N_B-1-j} \alpha^k (1-\alpha)^{N_A-k}$$

where $\alpha = F(c_P^\alpha)$, and $\beta = F(c_P^\beta)$. The first term in the summation is the increase in vote share and the second term is the probability of the vote share being equal to $\frac{j}{j+k}$ without your vote, given turnout rates α and β.[21] The equilibrium condition for c_P^α and c_P^β are given by $c_P^\alpha = B_P^A$ and $c_P^\beta = B_P^B$.

3.0.3. *Experimental design and parameters.* All our electorates in the experiment have exactly $N = 9$ voters, with three different N_A treatments: $N_A = 2, 3, 4$. We consider two different distributions of voter costs. In our *low cost* (or, equivalently, *high benefit*) elections c_i is uniformly distributed on the interval [0,.3]. In our *high cost* (or, equivalently, *low benefit*) elections c_i is uniformly distributed on the interval [0,.55]. Table 1 below gives the symmetric equilibrium expected turnout levels (by party and total turnout) for each treatment, rounded

[21] As in the theory section, we denote $\frac{j}{j+k} = .5$ if $j = k = 0$.

to two decimal places.

N_A	N_B	c_{max}	Rule	#Subjects	#Sessions	#Elections	α^*	β^*	$\frac{\alpha^* N_A + \beta^* N_B}{N}$
4	5	.3	M	18	2	100	.60	72	.67
3	6	.3	M	18	2	100	.51	.52	.52
2	7	.3	M	18	2	100	.45	.40	.41
4	5	.55	M	81	9	450	.46	.45	.46
3	6	.55	M	81	9	450	.41	.37	.39
2	7	.55	M	18	2	100	.38	.30	.32
4	5	.3	P	18	2	100	.48	.43	.45
3	6	.3	P	18	2	100	.55	.39	.45
2	7	.3	P	18	2	100	.67	.36	.43
4	5	.55	P	18	2	100	.35	.31	.33
3	6	.55	P	18	2	100	.40	.29	.32
2	7	.55	P	18	2	100	.48	.26	.31

Table 1. Design summary and equilibrium turnout rates.

There are five main theoretical hypotheses comparing turnout in the M and P voting systems in the elections we study. We state these below:

H1 For the larger party, turnout is higher in M than in P. For the smaller party, turnout is higher in M than in P in competitive races, but the reverse is true in lopsided races.

H2 Total expected turnout is higher under M than under P.

H3 The competition effect is reversed for the smaller party in the proportional vote system. That is, for the smaller party, turnout decreases as their share of the electorate increases. Under M, the usual competition effect applies to both parties: elections that are expected to be closer lead to higher turnout.[22]

H4 The competition effect on total expected turnout is negligible in P elections.

H5 In all P elections we study, there is an underdog effect. There is an underdog effect in all M elections, except for reverse underdog effects in the low cost 5-4 and 6-3 M elections.

In the experimental section we will return to these five predictions of the known population model.

[22] In the general model with population uncertainty we were not able to obtain general results on the competition effects, whereas the numerical analysis of the known population case allows for these additional predictions.

3.1. **Procedures.** A total of 171 subjects participated in 1900 elections across 19 sessions. Each session consisted of two parts with 50 nine-voter elections in each part. The parameters were the same in all elections within a part, but in each session exactly one parameter was changed between part I and part II. In all sessions the same voting rule (M or P) was used in all 100 elections. For all of the treatments except for the 7-2 elections, the distribution of voting costs were the same for all 100 elections. Half of these sessions were conducted with part I having 5-4 elections and part II having 6-3 elections. The other half of the sessions reversed the order so the 6-3 elections were in part I and the 5-4 elections in part II.[23] For the 7-2 elections, half the elections in a session were conducted with $c_{\max} = 55$ and half with $c_{\max} = 30$, in both orders. Subjects were informed of the exact parameters (N_A, N_B, $C_{\max}$ and the voting rule) at the beginning of each part. Before each election, each subject was randomly assigned to either group A or group B and assigned a voting cost, drawn independently from the uniform distribution between 0 and $c_{\max}$, in integer increments. Therefore, each subject gained experience as a member of the majority and minority party in both parts of the session. Instructions were read aloud so everyone could hear, and Powerpoint slides were projected in front of the room to help explain the rules. After the instructions were read, subjects were walked through two practice rounds and then were required to correctly answer all the questions on a computerized comprehension quiz before the experiment began. After the first 50 rounds, a very short set of new instructions were read aloud to explain the change of parameters.

The wording in the instructions was written so as to induce as neutral an environment as possible.[24] There was no mention of voting or winning or losing or costs. The labels were abstract. The smaller group was referred to the alpha group (A) and the larger group was referred to as the beta group (B). Individuals were asked in each round to choose X or Y. For the M treatment, if more members of $A(B)$ chose X than members of $B(A)$ chose X, then each member of $A(B)$ received 100 and each member of group $B(A)$ received 0. In case of a tie, each member of each group received the expected value of a fair coin toss, 50. For the P treatment, each voter received a share of 100 proportional to the number of voters in their party that chose X compared to the number of voters in the other party that chose X. The voting cost was implemented as an opportunity cost and was referred to as a "Y bonus". It was added to a player's earnings if that player chose Y instead of X. If a player chose X, that player did

[23]The order was 5-4 followed by 6-3 in both P/55 sessions due to an error in one of the program files for running the experiment.

[24]A sample of the instructions from one of the sessions is in Appendix E.

not receive their Y bonus in that election. Y-bonuses were randomly redrawn in every election, independently for each subject, and subjects were only told their own Y bonus. Bonuses were integer valued and took on values from 0 to 30 in the low cost treatment and 0 to 55 in the high cost treatment. Payoffs were denominated in points that were converted to US dollars at a pre-announced rate.[25] Each subject earned the sum of their earnings across all elections. All decisions took place through computers, using the Multistage experimental software program.[26] The experiments were conducted in 2011, and subjects were registered students at Caltech.[27] Each session lasted about forty five minutes and subjects earned between eleven and seventeen dollars, in addition to a fixed payment for showing up on time.

3.2. **Experimental Results.** Table 2 summarizes the observed turnout rates by treatment. The table reports turnout by party and also total turnout for each experimental treatment. The last three columns give the equilibrium turnout levels. The table (and the ones that follow) reports standard errors clustered at the individual voter level. A statistical comparison of the average turnout rates with the equilibrium turnout rates indicates that the data are quantitatively closely aligned with the theoretical predictions from the pivotal voter model: in 10 out of 12 cases, $\widehat{\alpha}$ is not significantly different from α^*, at the 5% significance level; in 10 out of 12 cases, $\widehat{\beta}$ is not significantly different from β^*, at the 5% significance level; and in 10 out of 12 cases, $\widehat{T}$ is not significantly different from T^*, at the 5% significance level. Besides statistical significance, the differences are generally small in quantitative terms as well: 25 of the 36 turnout rates are within five percentage points of the theoretical rates.

[25] Each point was equal to $.01.

[26] http://multistage.ssel.caltech.edu

[27] Data for the high cost M 5-4 and 6-3 elections are from an earlier study with UCLA students as subjects (Levine and Palfrey 2007), which used the same Multistage software and the same protocol.

N$_A$	N$_B$	c$_{\max}$	Rule	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{T}$	α^*	β^*	T^*
4	5	.3	M	0.622* (.047)	0.636* (.050)	0.630* (.042)	.60	.72	.67
3	6	.3	M	0.513* (.046)	0.520* (.055)	0.518* (.044)	.51	.52	.52
2	7	.3	M	0.490* (.073)	0.360* (.060)	0.389* (.054)	.45	.40	.41
4	5	.55	M	0.479* (.026)	0.451* (.028)	0.464* (.021)	.46	.45	.46
3	6	.55	M	0.436* (.025)	0.399* (.030)	0.411* (.022)	.41	.37	.39
2	7	.55	M	0.330* (.045)	0.284* (.037)	0.294* (.031)	.38	.30	.32
4	5	.3	P	0.547 (.029)	0.486 (.028)	0.513 (.026)	.48	.43	.45
3	6	.3	P	0.547* (.054)	0.465* (.048)	0.492* (.040)	.55	.39	.45
2	7	.3	P	0.600 (.036)	0.421* (.047)	0.461* (.038)	.67	.36	.43
4	5	.55	P	0.362* (.024)	0.370* (.039)	0.367* (.024)	.35	.31	.33
3	6	.55	P	0.477 (.037)	0.305* (.037)	0.362* (.028)	.40	.29	.32
2	7	.55	P	0.515* (.027)	0.320 (.029)	0.363 (.024)	.48	.26	.31

Table 2. Observed turnout rates. Subject-clustered standard error in parenthesis.

*cannot reject theory at 5%

Using these turnout data, we next turn to the five hypotheses generated by the theoretical equilibrium turnout levels. Recall that there are five main theoretical predictions from the pivotal voter model about differences between turnout in the M and P voting systems in the elections we study. We go through each of these briefly below.

H1 **For the larger party, turnout is higher in M than P. For the smaller party, turnout is higher in M than in P in competitive races, but the reverse is true in lopsided races.** We find support for this hypothesis except for the larger party in extreme landslide elections ($N_A = 2$) where turnout rates are slightly higher in P than M. The two cases where the sign is not correct, the difference is not statistically different from 0. For smaller parties for one of the intermediate case between highly competitive races and lopsided races (6-3) the sign is not consistent with theory, but the empirical difference (-0.04) is not significantly different from 0 nor from the theoretically predicted difference (.01). Thus the theory is supported in four out of six paired comparisons. The difference is not statistically significant at the 5% level in the two exceptions. See columns 4 and 5 of Table 3.

N_A	N_B	$c_{\max}$	$\widehat{\alpha}_M - \widehat{\alpha}_P$	$\widehat{\beta}_M - \widehat{\beta}_P$	$\widehat{T}_M - \widehat{T}_P$
4	5	.3	.075 (.055)$^{+}$	.150 (.057)$^{+*}$	.117 (.048)$^{+*}$
3	6	.3	-.033 (.070)$^{+}$	.055 (.072)$^{+}$	.026 (.059)$^{+}$
2	7	.3	-.110 (.080)$^{+}$	-.061 (.075)	-.072 (.065)$^{+*}$
4	5	.55	.117 (.035)$^{+*}$	.081 (.047)$^{+}$	.097 (.031)$^{+*}$
3	6	.55	-.040 (.044)	.094 (.047)$^{+*}$	049 (.035)$^{+}$
2	7	.55	-.185 (.052)$^{+*}$	-.036 (.047)	-.069 (.039)

Table 3. H1, H2: Voting Rule Effect. Subject-clustered standard error in parenthesis.

$^{+}$Correct sign. *Significant at 5% level or better.

H2 **Total turnout is higher under M than under P.** We find support for this hypothesis except for the extreme landslide elections ($N_A = 2$) low cost elections, where turnout rates are slightly higher in P than M. However, the $N_A = 2$ low-cost elections are the one exception where turnout is predicted to be higher in P than M. Thus the theory is supported in five out of six paired comparisons. Three of the five differences are statistically significant. The difference is not statistically significant in the one exception. See the last column of Table 3.

H3 **The competition effect is reversed for the smaller party under P. That is, for the smaller party, turnout decreases as their share of the electorate increases. Under P for the majority party, as well as under M for both parties, the usual competition effect applies. The competition effect on total turnout applies to both P and M elections: i.e., more competitive elections lead to higher turnout.** We measure the competition effect as the difference in turnout between the 5-4 and 6-3 elections, the difference in turnout between the 6-3 and 7-2 elections, and the difference between the 5-4 and 6-3 elections. The sign is correctly predicted in all but three cases (33 out of 36 comparisons). In both exceptions, the differences are not significantly different from 0. See Table 4.

Comparison	$c_{\max}$	M	P
$\widehat{\alpha}_{5/4} - \widehat{\alpha}_{6/3}$	0.30	.109 (.037)$^{+*}$	.001 (.057)
$\widehat{\alpha}_{6/3} - \widehat{\alpha}_{7/2}$	0.30	.023 (.085)$^{+*}$	-.053 (.064)$^{+}$
$\widehat{\alpha}_{5/4} - \widehat{\alpha}_{7/2}$	0.30	.132 (.086)$^{+}$	-.052 (.046)$^{+}$
$\widehat{\alpha}_{5/4} - \widehat{\alpha}_{6/3}$	0.55	.043 (.028)$^{+}$	-.114 (.041)$^{+*}$
$\widehat{\alpha}_{6/3} - \widehat{\alpha}_{7/2}$	0.55	.106 (.051)$^{+*}$	-.038 (.045)$^{+}$
$\widehat{\alpha}_{5/4} - \widehat{\alpha}_{7/2}$	0.55	.149 (.051)$^{+*}$	-.152 (.036)$^{+*}$
$\widehat{\beta}_{5/4} - \widehat{\beta}_{6/3}$	0.30	.116 (.033)$^{+*}$	.021 (.046)$^{+}$
$\widehat{\beta}_{6/3} - \widehat{\beta}_{7/2}$	0.30	.160 (.080)$^{+*}$	.044 (.066)$^{+}$
$\widehat{\beta}_{5/4} - \widehat{\beta}_{7/2}$	0.30	.276 (.077)$^{+*}$	.065 (.054)$^{+}$
$\widehat{\beta}_{5/4} - \widehat{\beta}_{6/3}$	0.55	.053 (.025)$^{+*}$	.065 (.029)$^{+*}$
$\widehat{\beta}_{6/3} - \widehat{\beta}_{7/2}$	0.55	.114 (.047)$^{+*}$	-.015 (.047)
$\widehat{\beta}_{5/4} - \widehat{\beta}_{7/2}$	0.55	.167 (.046)$^{+*}$	.050 (.048)$^{+}$
$\widehat{T}_{5/4} - \widehat{T}_{6/3}$	0.30	.112 (.028)$^{+*}$	.021 (.037)$^{+}$
$\widehat{T}_{6/3} - \widehat{T}_{7/2}$	0.30	.129 (.069)$^{+}$	.031 (.055)$^{+}$
$\widehat{T}_{5/4} - \widehat{T}_{7/2}$	0.30	.241 (.067)$^{+*}$	.052 (.045)$^{+}$
$\widehat{T}_{5/4} - \widehat{T}_{6/3}$	0.55	.053 (.020)$^{+*}$	.004 (.026)$^{+}$
$\widehat{T}_{6/3} - \widehat{T}_{7/2}$	0.55	.117 (.037)$^{+*}$	-.001 (.037)
$\widehat{T}_{5/4} - \widehat{T}_{7/2}$	0.55	.169 (.037)$^{+*}$	.003 (.034)$^{+}$

Table 4. H3 Competition Effect. Subject-clustered standard error in parenthesis.

$^{+}$Correct sign. *Significant at 5% level or better.

H4 **The competition effect on total expected turnout is larger in the M elections than the P elections.** This is exactly what we find in the data. The sign is correctly predicted in all six cases, and the differences are statistically different from 0 in four of the six cases. See the last column of Table 5.

Comparison	$c_{\max}$	*M*	*P*	*M − P*
$\widehat{T}_{5/4} - \widehat{T}_{6/3}$	0.30	.112 (.028)	.021 (.037)	.091 (.046)$^{+*}$
$\widehat{T}_{6/3} - \widehat{T}_{7/2}$	0.30	.129 (.069)	.031 (.055)	.098 (.087)$^{+}$
$\widehat{T}_{5/4} - \widehat{T}_{7/2}$	0.30	.241 (.067)	.052 (.045)	.189 (.080)$^{+*}$
$\widehat{T}_{5/4} - \widehat{T}_{6/3}$	0.55	.053 (.020)	.004 (.026)	.048 (.035)$^{+}$
$\widehat{T}_{6/3} - \widehat{T}_{7/2}$	0.55	.117 (.037)	-.001 (.037)	.118 (.052)$^{+*}$
$\widehat{T}_{5/4} - \widehat{T}_{7/2}$	0.55	.169 (.037)	.003 (.034)	.166 (.050)$^{+*}$

Table 5. H4: Competition Effect M vs. P. Subject-clustered standard error in parenthesis.

$^{+}$Correct sign. *Significant at 5% level or better.

H5 **In all P elections we study, there is an underdog effect. There is an underdog effect in all M elections, except for the predicted reverse underdog effects in the low cost 5-4 and 6-3 M elections.** All but one of our underdog hypotheses have support in the data. We find that in all P elections there is an underdog effect, with one exception where the difference is less than one percentage point ($\widehat{\alpha} = .362$, $\widehat{\beta} = .370$) and not statistically significant. That one exception is the 5-4 high cost treatment, where theory predicts the smallest underdog effect (less than four percentage points). In the M elections, all predicted underdog and reverse underdog effects are observed in the data. (eleven of twelve comparisons).

Comparison	$c_{\max}$	*M*	*P*
$\widehat{\alpha}_{5/4} - \widehat{\beta}_{5/4}$	0.30	-.013 (.051)$^{+}$	.061 (.027)$^{+*}$
$\widehat{\alpha}_{6/3} - \widehat{\beta}_{6/3}$	0.30	-.007 (.058)$^{+}$	.082 (.066)$^{+}$
$\widehat{\alpha}_{7/2} - \widehat{\beta}_{7/2}$	0.30	.130 (.083)$^{+}$	.179 (.057)$^{+*}$
$\widehat{\alpha}_{5/4} - \widehat{\beta}_{5/4}$	0.55	.028 (.035)$^{+}$	-.007 (.047)
$\widehat{\alpha}_{6/3} - \widehat{\beta}_{6/3}$	0.55	.038 (.037)$^{+}$	.172 (.052)$^{+*}$
$\widehat{\alpha}_{7/2} - \widehat{\beta}_{7/2}$	0.55	.046 (.057)$^{+}$	.195 (.036)$^{+*}$

Table 6. H5: Underdog effect. $\widehat{\alpha} - \widehat{\beta}$. Subject-clustered standard error in parenthesis.

$^{+}$Correct sign. *Significant at 5% level or better.

Thus, the comparative statics are correctly predicted by theory in 66 out of 72 paired comparisons. In none of the 6 exceptions are the differences significantly different from 0, and in most cases not statistically different from the exact quantitative theoretical difference. Overall, 33 of the 66 correctly predicted signed differences are significantly different from 0 (at the 5% level, using a two-tailed test and clustered standard errors). To illustrate in a single figure how close the equilibrium turnout rates are to the equilibrium turnout rates, Figure 3 presents a scatter plot of the observed vs. equilibrium turnout rates. A perfect fit of the data to the theory would have all the points lined up along the 45% degree line. A simple OLS regression of the observed turnout on equilibrium turnout, using the 36 points in the graph gives a slope of .815, an intercept of .097 and an R-squared equal to .871. The theoretical model slightly underestimates turnout when the model prediction is below 50% and slightly over-estimates turnout when the model prediction is over 50%, consistent with the findings of Levine and Palfrey (2007) on their much larger data set for plurality elections. Levine and Palfrey (2007) show that the Logit QRE model can account fairly well for these over and underpredictions of the Bayesian Nash Equilibrium Model. The same is true for our data. The QRE estimation results are reported in Appendix B.

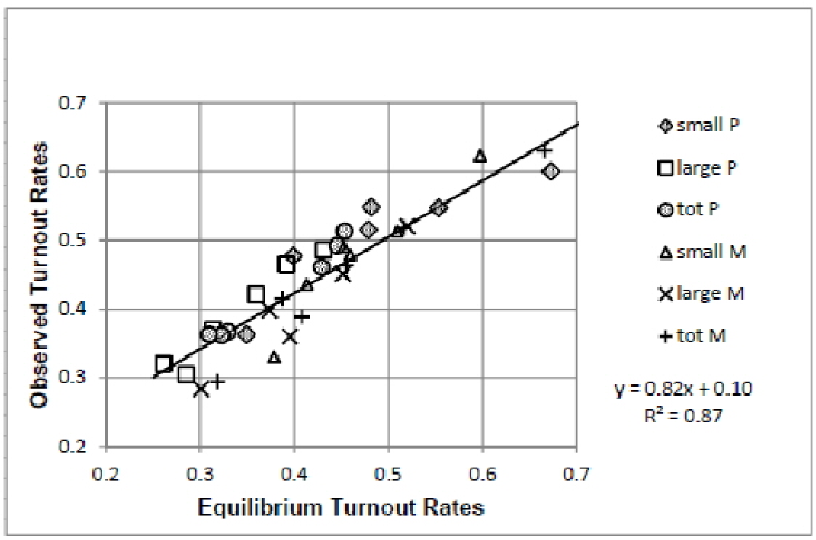

Figure 3: Scatter plot of observed vs. equilibrium turnout rates.

4. Concluding Remarks

Turnout depends on the degree of proportionality of influence in the institutional system in a clear way: higher turnout in a winner-take-all system than in a proportional power sharing system when the population is evenly split in terms of partisan preferences, and the opposite when one party's position has a clear majority of support. This provides a theoretical basis to expect a mixed bag of results from empirical studies that fail to adequately measure or control for the interaction of the electoral rules and the relative strength of parties in the electorate. The fact that in a winner-take-all system the underdog cannot win a large election when partisan preferences are not evenly split strongly discourages turnout, but with proportional power sharing there is no absolute winner, some competition remains even when preferences are relatively lopsided, and therefore the effect of relative party size on turnout is small. The theoretical results are robust to a wide range of alternative assumptions about the voting game and about the rationality of voters (Herrera et al. 2013)).

The laboratory experiment allows a clean test of this interaction effect using small electorates, and additional treatments are included that check for robustness and allow us to examine several secondary hypotheses that emerge from the equilibrium turnout model. Our prediction that for the larger party, turnout is higher in a winner-take-all system than in a proportional power sharing system was confirmed by the experimental analysis, as well as most of the other secondary hypotheses concerning differences in the competition and underdog effects. Our design allows us to examine the prediction that the competition effect can be reversed for the smaller party in the proportional system. That is, for the smaller party, if the underdog compensation effect is strong enough, turnout may actually decrease as their share of the electorate increases. With a winner-take-all system, the usual competition effect applies to both parties: elections that are expected to be closer lead to higher turnout. These theoretical predictions, as well as the hypothesis that competition effect on total expected turnout is negligible in a proportional system, are supported in the data from the experiment.

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Appendix A: Proofs

Proof of Proposition 1 . .

Existence.

We present the proof for the M system. The proof for P is similar.

Fix N and q. The pair of equilibrium conditions for the M system can be written in terms of the "cost cutpoints" of the two parties:

$$c_A = \sum_{k=0}^{\infty} \left(\frac{e^{-qN\alpha}[NqF(c_A)]^k}{k!} \right) \left(\frac{e^{-(1-q)N\beta} \left((1-q)\, NF(c_B) \right)^k}{k!} \right) \frac{1}{2} \left(1 + \frac{(1-q)\, NF(c_B)}{k+1} \right) \equiv \beta^A(c_A, c_B)$$

$$c_B = \sum_{k=0}^{\infty} \left(\frac{e^{-qN\alpha} \left(NqF(c_A) \right)^k}{k!} \right) \left(\frac{e^{-(1-q)N\beta} \left((1-q)\, NF(c_B) \right)^k}{k!} \right) \frac{1}{2} \left(1 + \frac{qNF(c_A)}{k+1} \right) \equiv \beta^A(c_A, c_B)$$

Because $\overline{c} > 1/2$, β^A and β^B are continuous functions of c_A, c_B from $[0, \overline{c}]^2$ into itself, and $[0, \overline{c}]^2$ is a compact convex subset of R^2.[28] Therefore, by Brouwer’s theorem there exists a fixed point $(c_A^*,\, c_B^*)$, which satisfies both equations and is an equilibrium.

Underdog Effects.

Subtracting the two equilibrium conditions (1) we have for the M system

$$F^{-1}(\alpha) - F^{-1}(\beta) = \sum_{k=0}^{\infty} \left(\frac{e^{-qN\alpha} \left(Nq\alpha \right)^k}{k!} \frac{e^{-(1-q)N\beta} \left((1-q)\, N\beta \right)^k}{k!} \right) \left(N \frac{(1-q)\,\beta - q\alpha}{k+1} \right)$$

Hence, comparing the signs for both sides, we have one of three possibilities:

$$\begin{array}{lcl} \alpha > \beta & \Longrightarrow & q\alpha < (1-q)\,\beta \Longrightarrow q < 1/2 \\ \alpha < \beta & \Longrightarrow & q\alpha > (1-q)\,\beta \Longrightarrow q > 1/2 \\ \alpha = \beta & \Longrightarrow & q\alpha = (1-q)\,\beta \Longrightarrow q = 1/2 \end{array}$$

which partitions the whole parameter space, hence proves the result.

Similarly, for the P system we have

$$F^{-1}(\alpha) - F^{-1}(\beta) = \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left(\frac{e^{-qN\alpha} \left(qN\alpha \right)^a}{a!} \frac{e^{-(1-q)N\beta} \left((1-q)\, N\beta \right)^b}{b!} \right) W(a, b)$$

[28] The assumption $\overline{c} > 1/2$ guarantees that the range of these functions is contained in $[0, \overline{c}]^2$. Existence also holds more generally for any $\overline{c} > 0$, with only minor changes in the proof to account for the possibility that $\overline{c}$ is the cutpoint (i.e., 100% turnout) for one or both parties.

where

$$W(a,b) := \frac{b-a}{(a+b)(a+b+1)}$$

Note that $W(a,b) = -W(b,a)$. If $\alpha > \beta$ the RHS needs to be positive, so we must have $q\alpha < (1-q)\beta m$ for the Poisson weights: suppose that $q\alpha \geq (1-q)\beta m$ then the vote outcomes $b > a$ occur with lower or equal probability than the symmetric outcomes $a > b$ so the RHS would be negative or zero.

Likewise, if $\alpha = \beta$ the RHS needs to be zero so we must have equal Poisson weights $q\alpha = (1-q)\beta$. The rest of the argument is identical to the one for the M system.

This proof just hinges on the symmetry of $W(a,b)$ and on the fact that

$$W(a,b) > 0 \quad \text{if} \quad a < b$$

so, in general, the partial underdog effect holds whenever a symmetric power sharing function $V(a,b)$ (as e.g. the proportional one we analyze here: $V = \frac{a}{a+b}$) has the property that an additional vote for the underdog has a higher marginal impact for the underdog than an additional vote for the leader has for the leader, that is:

$$\begin{aligned} W(a,b) &:= (V(a+1,b) - V(a,b)) - (V(b+1,a) - V(b,a)) > 0 \\ \text{if} \quad a &< b \end{aligned}$$

$\square$

Proof of Lemma 2. .

M System.

We first show that for the M system

$$\lim_{N\to\infty} \alpha_N = \lim_{N\to\infty} \beta_N = 0$$

Define the modified Bessel functions of the first kind, see Abramowitz and Stegun (1965), as

$$I_0(z) := \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^k}{k!} \frac{\left(\frac{z}{2}\right)^k}{k!}, \qquad I_1(z) := \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^k}{k!} \frac{\left(\frac{z}{2}\right)^{k+1}}{(k+1)!}$$

Defining

$$x := qN\alpha, \quad y := (1-q)N\beta, \quad z := 2\sqrt{xy}$$

then the benefits of voting $\left(B_M^A, B_M^B\right)$ can be written as

$$B_M^A = \frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{e^{-x}x^k}{k!}\right)\left(\frac{e^{-y}y^k}{k!}\right)\left(1+\frac{y}{k+1}\right) = \frac{e^{-x}e^{-y}}{2}\left(I_0\left(2\sqrt{xy}\right) + \sqrt{\frac{y}{x}}I_1\left(2\sqrt{xy}\right)\right)$$

$$B_M^B = \frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{e^{-x}x^k}{k!}\right)\left(\frac{e^{-y}y^k}{k!}\right)\left(1+\frac{x}{k+1}\right) = \frac{e^{-x}e^{-y}}{2}\left(I_0\left(2\sqrt{xy}\right)+\sqrt{\frac{x}{y}}I_1\left(2\sqrt{xy}\right)\right)$$

For large z the modified Bessel functions are asymptotically equivalent and approximate to, see Abramowitz and Stegun (1965)[29]

$$I_0\left(z\right) \simeq I_1\left(z\right) \simeq \frac{e^z}{2\pi z}$$

For any exogenously fixed $(\alpha,\beta) \in (0,1]^2$ x and y go to infinity as N goes to infinity, so we can approximate the benefits of voting for large N as

$$B_M^A \simeq e^{-x-y+2\sqrt{xy}}\frac{\sqrt{x}+\sqrt{y}}{4\sqrt{\pi}\sqrt{\sqrt{xy}}}\frac{1}{\sqrt{x}}, \qquad B_M^A \simeq e^{-x-y+2\sqrt{xy}}\frac{\sqrt{x}+\sqrt{y}}{4\sqrt{\pi}\sqrt{\sqrt{xy}}}\frac{1}{\sqrt{y}} \tag{2}$$

As a consequence for any given $(\alpha,\beta) \in (0,1]^2$ the benefits of voting vanish as N grows, namely

$$\lim_{N\to\infty} B_M^A(\alpha,\beta) = 0, \qquad \lim_{N\to\infty} B_M^B(\alpha,\beta) = 0$$

Now consider (α,β) as endogenous, i.e. solutions to the system

$$B_M^A(\alpha,\beta) = F^{-1}(\alpha), \qquad B_M^B(\alpha,\beta) = F^{-1}(\beta)$$

Since F and F^{-1} are increasing and continuous with $F\left(0\right) = 0$, then $B_M^A(\alpha,\beta) = F^{-1}(\alpha)$ implies $\lim_{N\to\infty}\alpha_N = 0$. Likewise, we have $\lim_{N\to\infty}\beta_N = 0$.

Next, we show that

$$\lim_{N\to\infty} N\alpha_N = \lim_{N\to\infty} N\beta_N = \infty, \qquad \lim_{N\to\infty}\frac{\alpha_N}{\beta_N} \in (0,\infty)$$

Suppose $\lim_{N\to\infty} N\alpha_N < \infty$ and $\lim_{N\to\infty} N\beta_N < \infty$, then

$$\lim_{N\to\infty} B_M^A(\alpha_N,\beta_N) > 0$$

and any solution to $B_M^A(\alpha,\beta) = F^{-1}(\alpha)$ would imply $\lim_{N\to\infty}\alpha_N > 0$, which contradicts $\lim_{N\to\infty}\alpha_N = 0$.

Suppose $\lim_{N\to\infty} N\alpha_N = \infty$ and $\lim_{N\to\infty} N\beta_N < \infty$, then $\lim_{N\to\infty}\frac{\alpha_N}{\beta_N} = \infty$ which implies (using a Taylor expansion of F^{-1} on the numerator and the denominator around zero) that $\lim_{N\to\infty}\frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)} = \frac{\alpha\left(F^{-1}(0)\right)'+\alpha^2/2\left(F^{-1}(0)\right)''+\ldots}{\beta(F^{-1}(0))'+\beta^2/2(F^{-1}(0))''+\ldots} = \infty$.

For all N we have

$$\frac{B_M^A(\alpha_N,\beta_N)}{B_M^B(\alpha_N,\beta_N)} = \frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)}$$

[29] $X\left(z\right) \simeq Y\left(z\right)$ means that $\lim_{z\to\infty}\frac{X(z)}{Y(z)} = 1$.

Taking the limit on one side we have

$$L := \lim_{N\to\infty} \frac{B_M^A(\alpha_N,\beta_N)}{B_M^B(\alpha_N,\beta_N)} = \lim_{\frac{x}{y}\to\infty} \frac{I_0\left(2\sqrt{xy}\right) + I_1\left(2\sqrt{xy}\right)\sqrt{\frac{y}{x}}}{I_0\left(2\sqrt{xy}\right) + I_1\left(2\sqrt{xy}\right)\sqrt{\frac{x}{y}}} \leq 1$$

In fact, $L \leq 1$ if $\lim_{\frac{x}{y}\to\infty} \frac{I_1\left(2\sqrt{xy}\right)}{I_0\left(2\sqrt{xy}\right)} = 0$ and $L = 0$ if $\lim_{\frac{x}{y}\to\infty} \frac{I_1\left(2\sqrt{xy}\right)}{I_0\left(2\sqrt{xy}\right)} \in (0,+\infty]$. So we have a contradiction as $L \leq 1$ cannot be equal to $\lim_{N\to\infty} \frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)} = \infty$. The same argument shows that it cannot be the case that $\lim_{N\to\infty} N\alpha_N < \infty$ and $\lim_{N\to\infty} N\beta_N = \infty$.

The above arguments also imply that we cannot have either

$$\lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = 0, \qquad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = \infty$$

P System.

The marginal benefit of voting in the P system has the exact closed form

$$\begin{aligned} B_P^A &= \frac{(1-q)\,\beta}{NT^2} - \left(\frac{\left((1-q)\,\beta\right)^2 - (q\alpha)^2 + (1-q)\,\beta\frac{1}{N}}{2T^2}\right)e^{-NT} \\ B_P^B &= \frac{q\alpha}{NT^2} + \left(\frac{\left((1-q)\,\beta\right)^2 - (q\alpha)^2 - q\alpha\frac{1}{N}}{2T^2}\right)e^{-NT} \end{aligned} \tag{3}$$

Namely, for given (α,β) call the expected number of voters for each party $R := qN\alpha$, $S := (1-q)\,N\beta$, we have

$$B_P^A = e^{-R-S}\sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\frac{R^a}{a!}\right)\left(\frac{S^b}{b!}\right)\left(\frac{a+1}{a+b+1} - \frac{a}{a+b}\right)$$

By differentiating and integrating the summands and inverting the series and integral operators we have

$$\begin{aligned} \sum_{b=0}^{\infty}\frac{S^b}{b!}\frac{a}{a+b} &= \frac{a}{S^a}\sum_{b=0}^{\infty}\int_0^S \frac{d}{dr}\left(\frac{1}{b!}\frac{r^{a+b}}{a+b}\right)dr = \\ &= \frac{a}{S^a}\int_0^S\sum_{b=0}^{\infty}\left(\frac{1}{b!}r^{a+b-1}\right)dr = \begin{cases} \frac{a}{S^a}\int_0^S r^{a-1}e^r\,dr & \text{for } a \geq 1 \\ \\ 1/2 & \text{for } a = 0 \end{cases} \end{aligned}$$

and

$$\sum_{b=0}^{\infty}\frac{S^b}{b!}\frac{a+1}{a+b+1} = \frac{a+1}{S^{a+1}}\int_0^S r^a e^r\,dr$$

By inverting the series and integral operators again in the series over a, we have

$$
\begin{aligned}
B_P^A &= e^{-R-S}\left(\sum_{a=0}^{\infty}\frac{R^a}{a!}\left(\frac{a+1}{S^{a+1}}\int_0^S r^a e^r dr\right)-\sum_{a=1}^{\infty}\frac{R^a}{a!}\left(\frac{a}{S^a}\int_0^S r^{a-1}e^r dr\right)-\frac{1}{2}\right)\\
&= e^{-R-S}\left(\int_0^S\left(\begin{array}{c}\frac{1}{S}\left(\sum_{a=0}^{\infty}\frac{\left(\frac{R}{S}r\right)^a}{a!}+\sum_{a=1}^{\infty}\frac{\left(\frac{R}{S}r\right)^a}{(a-1)!}\right)\\ -\frac{R}{S}\sum_{a=1}^{\infty}\frac{\left(\frac{R}{S}r\right)^{a-1}}{(a-1)!}\end{array}\right)e^r dr-\frac{1}{2}\right)\\
&= e^{-R-S}\left(\frac{1}{S^2}\int_0^S e^{\left(1+\frac{R}{S}\right)r}\left(S-RS+Rr\right)dr-\frac{1}{2}\right)\\
&= \frac{S}{\left(R+S\right)^2}-\frac{e^{-(R+S)}}{\left(R+S\right)^2}\frac{S^2-R^2+S}{2}
\end{aligned}
$$

and by symmetry

$$
B_P^B\left(R,S\right)=B_P^A\left(S,R\right)
$$

We first show that

$$
\lim_{N\to\infty}\alpha_N=\lim_{N\to\infty}\beta_N=0
$$

For any fixed $\alpha>0$ and $\beta>0$, by inspection of the closed form expression (3) we see that $\lim_{N\to\infty}B_P^A(\alpha,\beta)=\lim_{N\to\infty}B_P^B(\alpha,\beta)=0$, so the same argument obtained for the M system applies.

Next, we show that

$$
\lim_{N\to\infty}N\alpha_N=\lim_{N\to\infty}N\beta_N=\infty,\qquad \lim_{N\to\infty}\frac{\alpha_N}{\beta_N}\in(0,\infty)
$$

Summing the two P system equations we have

$$
\frac{1}{NT}\left(1-\frac{e^{-NT}}{2}\right)=F^{-1}\left(\alpha\right)+F^{-1}\left(\beta\right)
$$

Since the RHS goes to zero the LHS will too, which means that NT must go to infinity so we cannot have both $\lim_{N\to\infty}N\alpha_N<\infty$ and $\lim_{N\to\infty}N\beta_N<\infty$. Hence, for N large, since the exponential terms e^{-NT} in (3) vanish faster than the hyperbolic terms, the system approximates to

$$
\frac{\left(1-q\right)\beta}{NT^2}=F^{-1}\left(\alpha\right),\quad \frac{q\alpha}{NT^2}=F^{-1}\left(\beta\right) \tag{4}
$$

Suppose $\lim_{N\to\infty}N\alpha_N=\infty$ and $\lim_{N\to\infty}N\beta_N<\infty$, then $\lim_{N\to\infty}\frac{\alpha_N}{\beta_N}=\infty$ which implies (using a Taylor expansion of F^{-1} on the numerator and the denominator around zero) that $\lim_{N\to\infty}\frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)}=\frac{\alpha\left(F^{-1}(0)\right)'+\alpha^2/2\left(F^{-1}(0)\right)''+\ldots}{\beta(F^{-1}(0))'+\beta^2/2(F^{-1}(0))''+\ldots}=\infty$. From (4) we have

$$
\frac{1-q}{q}\frac{\beta_N}{\alpha_N}=\frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)}
$$

so we reach a contradiction as the above equality cannot hold as $N \to \infty$. The same argument shows that it cannot be the case that $\lim_{N\to\infty} N\alpha_N < \infty$ and $\lim_{N\to\infty} N\beta_N = \infty$.

The above arguments also imply that we cannot have either

$$\lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = 0, \quad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = \infty$$

$\square$

Proof of Proposition 3. .

1. Uniqueness of M System.

For N large, since $\lim_{N\to\infty} N\alpha_N = \lim_{N\to\infty} N\beta_N = \infty$, if the players use asymptotic approximations in their computations, then we can use the asymptotic expression for the modified Bessel functions (2) which yields

$$\sqrt{q\alpha}F^{-1}(\alpha) = \sqrt{(1-q)\,\beta}F^{-1}(\beta)$$

Since the function $\sqrt{\alpha}F^{-1}(\alpha)$ is increasing we can define the function

$$\beta := \beta_M(\alpha)$$

where $\beta_M : [0,1] \longrightarrow [0,1]$ is an increasing and differentiable function with $\beta_M(0) = 0$. The system is reduced to a single equation

$$B_M^A(\alpha, \beta_M(\alpha)) = F^{-1}(\alpha)$$

For uniqueness we need to show that the B_M^A is decreasing in α, namely, renaming $g := \sqrt{x}$ and $h = \sqrt{y}$, that the following quantity is negative:

$$\frac{d}{dg}\left(\frac{e^{-N(h-g)^2}}{\sqrt{N}}\frac{g+h}{4\sqrt{\pi}g\sqrt{hb}}\right) = \frac{e^{-N(h-g)^2}}{\sqrt{N}}\left(-2N(h-g)\frac{d(h-g)}{dg}\frac{g+h}{4\sqrt{\pi}g\sqrt{gh}} + \frac{d}{dg}\left(\frac{g+h}{4\sqrt{\pi}g\sqrt{gh}}\right)\right)$$

For large N this derivative will be negative if and only if

$$\frac{d(h-g)}{dg} = \frac{\sqrt{1-q}}{\sqrt{q}}\frac{d\beta'}{d\alpha'} - 1 > 0$$

where we defined

$$\begin{aligned} \beta' \ &: = \sqrt{\beta}, \quad \alpha' := \sqrt{\alpha} \\ G(\alpha') \ &: = \alpha' F^{-1}\left((\alpha')^2\right) = \sqrt{\alpha}F^{-1}(\alpha) \end{aligned}$$

we have

$$\left(\sqrt{1-q}\right)G(\beta') = (\sqrt{q})\,G(\alpha') \quad \Longrightarrow \quad \frac{\sqrt{1-q}}{\sqrt{q}}\frac{d\beta'}{d\alpha'} = \frac{G'(\alpha')}{G'(\beta')}$$

So we need G' to be increasing

$$G'(\alpha') = \frac{d}{d\alpha}\left(\sqrt{\alpha}F^{-1}(\alpha)\right)\frac{d\alpha}{d\alpha'} = 2\frac{d}{d\alpha}\left(\alpha F^{-1}(\alpha)\right)$$

so it suffices for $\alpha F^{-1}(\alpha)$ to be weakly convex, so it suffices to have $F(\alpha)$ weakly concave.

2. Uniqueness of P System.

If the players use asymptotic approximations in their computations we can ignore the higher order terms in both equations of the system (3) (see proof of Lemma 2), then the system gives the relation

$$q\alpha F^{-1}(\alpha) = (1-q)\,\beta F^{-1}(\beta)$$

Since the function $\alpha F^{-1}(\alpha)$ is increasing we can define

$$\beta := \beta_P(\alpha)$$

where $\beta_P(\alpha) : [0,1] \longrightarrow [0,1]$ is an increasing differentiable function with $\beta_P(0) = 0$. We now reduced the P system to one equation

$$B_P^A := \frac{(1-q)\,\beta_P(\alpha)}{NT^2} = F^{-1}(\alpha)$$

which we now show has one and only one solution.

The cost side $F^{-1}(\alpha)$ is increasing from 0 to 1. Uniqueness comes from the fact that the benefit side decreases in α as its derivative is proportional to

$$\begin{aligned} \frac{dB_P^A}{d\alpha} &\propto \left[\beta_P'(\alpha)\left(q\alpha + (1-q)\,\beta_P(\alpha)\right) - 2\beta_P(\alpha)\left(q + (1-q)\,\beta_P'(\alpha)\right)\right] \\ &= -\left[\left((1-q)\,\beta_P(\alpha) - q\alpha\right)\beta_P'(\alpha) + 2q\beta_P(\alpha)\right] < 0 \end{aligned}$$

because $q\alpha < (1-q)\,\beta$.

3. Size Effects.

For the M system, note that the marginal benefit side B_P^A decreases with N for all α while the cost side remains unchanged. Hence by the implicit function theorem as we increase N we have lower α which implies lower β and in turn lower turnout, formally

$$\begin{aligned} 0 &= \frac{d\left(B_M^A - F^{-1}\right)}{d\alpha}\frac{d\alpha}{dN} + \frac{d\left(B_M^A - F^{-1}\right)}{dN} \\ \frac{d\alpha}{dN} &= -\frac{\frac{dB_M^A}{dN}}{\frac{d\left(B_M^A - F^{-1}\right)}{d\alpha}} < 0 \quad \Longrightarrow \quad \frac{d\beta}{dN} < 0 \Longrightarrow \frac{dT_M}{dN} < 0 \end{aligned}$$

The proofs for the size effect and the underdog compensation effect for the P system is analogous.

4. Comparative Turnouts.

Assuming the cost side $F^{-1}(\alpha)$ is the same in the two systems, it suffices to show that the benefit sides of the equations determining the equilibrium α are ranked.

For any $q \neq 1/2$ we need to show that eventually (i.e. for any N above a given $\overline{N}$) we have

$$B_M^A(\alpha, \beta_M(\alpha)) < B_P^A(\alpha, \beta_P(\alpha)), \qquad \text{for all } \alpha \in (0,1]$$

namely

$$e^{-N\left(\sqrt{q\alpha}-\sqrt{(1-q)\beta_M}\right)^2}\sqrt{N} < \frac{(1-q)\beta_P}{(q\alpha+(1-q)\beta_P)^2}\left(\frac{\sqrt{q\alpha}+\sqrt{(1-q)\beta_M}}{4\sqrt{\pi}\left(q(1-q)\alpha\beta_M\right)^{1/4}}\frac{1}{\sqrt{q\alpha}}\right)^{-1}$$

which is satisfied as LHS above converges to zero, whereas the RHS is a positive constant for all $\alpha \in (0,1]$ because

$$\begin{aligned} \alpha &\in (0,1] \quad\Longrightarrow\quad \beta_P \in (0,1], \quad \beta_M \in (0,1] \\ q &\neq 1/2 \quad\Longrightarrow\quad \sqrt{q\alpha} \neq \sqrt{(1-q)\beta_M(\alpha)} \end{aligned}$$

Hence, eventually we have

$$q \neq 1/2 \quad\Longrightarrow\quad \alpha_M < \alpha_P$$

The symmetry property $\beta(q) = \alpha(1-q)$ (which holds in both the M and P systems) implies

$$q \neq 1/2 \quad\Longrightarrow\quad \beta_M < \beta_P$$

hence

$$q \neq 1/2 \quad\Longrightarrow\quad T_M < T_P$$

For $q = 1/2$ we have $\alpha = \beta$ in both P and M systems. We need to show that eventually

$$B_M^A > B_P^A, \qquad \alpha \in (0,1]$$

namely

$$\frac{1}{\sqrt{N}}\left(\frac{2\sqrt{q\alpha}}{4\sqrt{\pi}}\right)\frac{1}{q\alpha} > \frac{1}{N}\left(\frac{q\alpha}{2(2q\alpha)^2}\right)$$

Rearranging we have

$$\sqrt{N}\left(\frac{1}{2\sqrt{\pi}\sqrt{q\alpha}}\right) > \left(\frac{1}{8q\alpha}\right)$$

which is satisfied as the RHS is a positive constant and the LHS increases to infinity. Hence

$$q = 1/2 \quad\Longrightarrow\quad \alpha_M > \alpha_P \quad\Longrightarrow\quad T_M > T_P$$

5. Comparative Underdog Effects.

Given that for the M system we have

$$q\alpha_M \left(F^{-1}\left(\alpha_M\right)\right)^2 = (1-q)\,\beta_M \left(F^{-1}\left(\beta_M\right)\right)^2$$

and for the P system we have

$$q\alpha_P \left(F^{-1}\left(\alpha_P\right)\right) = (1-q)\,\beta_P \left(F^{-1}\left(\beta_P\right)\right)$$

then

$$\frac{1-q}{q} = \left(\frac{\alpha_P}{\beta_P}\right)^2 \left(\frac{\frac{F^{-1}(\alpha_P)}{\alpha_P}}{\frac{F^{-1}(\beta_P)}{\beta_P}}\right) = \left(\frac{\alpha_M}{\beta_M}\right)^3 \left(\frac{\frac{F^{-1}(\alpha_M)}{\alpha_M}}{\frac{F^{-1}(\beta_M)}{\beta_M}}\right)^2$$

By definition of derivative at zero we have

$$\frac{dF^{-1}}{dx}|_{x=0} = \lim_{x\to 0} \frac{F^{-1}\left(x\right)}{x} \in (0,\infty)$$

For N large, α and β converge to zero both in the M and in the P system so

$$\lim_{N\to\infty} \left(\frac{\frac{F^{-1}(\alpha)}{\alpha}}{\frac{F^{-1}(\beta)}{\beta}}\right) = 1$$

and the result follows. If $\frac{dF^{-1}}{dx}|_{x=0} \in \{0,\infty\}$ then the above limit is indeterminate and the result need not be true. If the function F^{-1} is infinitely differentiable and n is the lowest integer for which

$$\frac{d^n F^{-1}}{dx^n}|_{x=0} \in (0,\infty)$$

then by iterating the procedure we have

$$\lim_{N\to\infty} \left(\frac{\frac{d^{n-1}F^{-1}(\alpha)}{d\alpha^{n-1}}}{\frac{d^{n-1}F^{-1}(\beta)}{d\beta^{n-1}}}\right) = 1$$

so the underdog compensation comparison generalizes to

$$\frac{1-q}{q} = \left(\frac{\alpha_P}{\beta_P}\right)^{n+1} = \left(\frac{\alpha_M}{\beta_M}\right)^{2n+1}$$

□

Appendix B: Results from QRE estimation

Table 7 displays the estimated logit QRE turnout rates, constraining the logit parameter to be the same across all treatments within each voting rule. The estimated value of $\widehat{\lambda}$ is 7 for the the M data and 17 for the P data. There is essentially no change in the estimated QRE turnout rates if $\widehat{\lambda}$ is constrained to be equal in both treatments.

N_A	N_B	c_{max}	Rule	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{T}$	$\alpha^*_{\widehat{\lambda}}$	$\beta^*_{\widehat{\lambda}}$	$T^*_{\widehat{\lambda}}$
4	5	.3	M	0.622* (.047)	0.636* (.050)	0.630* (.042)	0.61	0.65	0.63
3	6	.3	M	0.513* (.046)	0.520* (.055)	0.518* (.044)	0.51	0.52	0.52
2	7	.3	M	0.490* (.073)	0.360* (.060)	0.389* (.054)	0.44	0.42	0.42
4	5	.55	M	0.479* (.026)	0.451* (.028)	0.464* (.021)	0.48	0.45	0.46
3	6	.55	M	0.436* (.025)	0.399* (.030)	0.411* (.022)	0.44	0.40	0.41
2	7	.55	M	0.330* (.045)	0.284* (.037)	0.294* (.031)	0.33	0.28	0.29
4	5	.3	P	0.547 (.029)	0.486 (.028)	0.513 (.026)	0.48	0.44	0.46
3	6	.3	P	0.547* (.054)	0.465* (.048)	0.492* (.040)	0.55	0.40	0.45
2	7	.3	P	0.600 (.036)	0.421* (.047)	0.461* (.038)	0.65	0.37	0.43
4	5	.55	P	0.362* (.024)	0.370* (.039)	0.367* (.024)	0.35	0.32	0.33
3	6	.55	P	0.477 (.037)	0.305* (.037)	0.362* (.028)	0.40	0.29	0.33
2	7	.55	P	0.515* (.027)	0.320 (.029)	0.363 (.024)	0.48	0.27	0.32

Table 7. Observed turnout rates. Clustered standard errors in parenthesis.

The scatter plot of the QRE-estimated turnout rates against the observed turnout rates is given below. Note that the slope has increased from 0.82 to 0.89, the constant term has decreased from 0.10 to 0.07 and the R^2 has increased from .87 to .91.

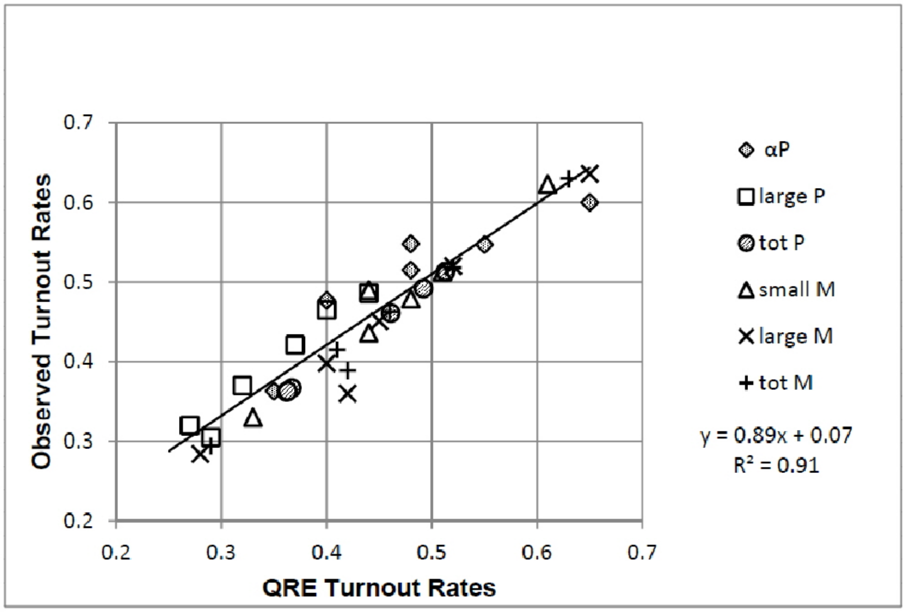

Figure 4: Observed Turnout Rates plotted against QRE-estimated Turnout Rates.

5. Appendix C: Experiment Instructions

Thank you for agreeing to participate in this decision making experiment. During the experiment we require your complete, undistracted attention, and ask that you follow instructions carefully. You may not open other applications on your computer, chat with other students, or engage in other distracting activities, such as using your phone, reading books, etc. You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. The entire experiment will take place through computer terminals, and all interaction between you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If you have any questions after the experiment has begun, raise your hand, and an experimenter will come and assist you.

We will begin with a brief practice session to help familiarize you with the computer interface. The practice rounds will be followed by 2 different paid sessions. Each paid session will consist of 50 rounds. At the end of the last paid session, you will be paid the sum of what you have earned in all rounds of the two paid sessions, plus the show-up fee of \$5.00. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in POINTS. Your DOLLAR earnings are determined by multiplying your earnings in POINTS by a conversion rate. In this experiment, the conversion rate is 0.002, meaning that 100 POINTS is worth 20 cents.

We will now go through two practice rounds to explain the rules for the first part of the experiment, and will explain the screen display. During the practice rounds, please do not hit any keys until I tell you, and when you are prompted by the computer to enter information, please wait for me to tell you exactly what to enter. You are not paid for these practice rounds.

[AUTHENTICATE CLIENTS]

Please pull out your dividers. Please double click on the icon on your desktop that says MULTISTAGE CLIENT. When the computer prompts you for your name, type your First and Last name. Then click SUBMIT and wait for further instructions.

SCREEN 1 (user interface)

[Point out while reading the following.]

You now see the first screen of the experiment on your computer. It should look similar to this screen. Please do not do anything with your mouse yet, until I have finished explaining the screen. [POINT TO PPT SLIDE DISPLAYED ON SCREEN IN FRONT OF ROOM]

Here are the instructions for the first part of the experiment. At the top of the screen will be your id number. Each of you has been assigned to one of two groups, called the ALPHA GROUP and the BETA GROUP. The ALPHA group always has 2 members and the BETA group always has 7 members. The screen informs you which group you will be in and reminds you how many members are in each group.

Each of you will be asked to choose either “X” or “Y” by clicking on a button with the mouse. Please wait and don’t do anything yet.

The sample display in front of the room shows you what the screen looks like for a member of the Alpha group. The screen also tells you what your “Y bonus” is. This is an extra bonus you earn if you choose Y instead of X, independent of what other participants choose.

Your earnings are computed in the following way. It is very important that you understand this, so please listen carefully.

SCREEN 2

[Point while reading.] First suppose you choose X. To compute your earnings, we compare the number of members of your group choosing “X” to the number of members of the other group choosing X. Your payoff is 105 if the number of members in your group choosing X is greater than the number of members of the other group who choose X. Your payoff is 55 if the number of members in your group choosing X is equal to the number of members of the other group who choose X. Your payoff is 5 if the number of members in your group choosing X is fewer than the number of members of the other group who choose X.

Your earnings are computed slightly differently if you choose Y. Specifically, in addition to the above earnings (either 105, 55, or 5) you also earn your Y bonus. This payoff information is displayed in a table on your screen.

The amount of each participant's Y-bonus is assigned completely randomly by the computer at the beginning of each round and is shown in the second line down from the top of the screen. Y-bonuses are assigned separately for each participant, so different participants will typically have different Y-bonuses. What you see up the front is just an example of one participant's Y-bonus. In any given round you will have an equal chance of being assigned any Y-bonus between 0 and 30 points. Your Y-bonus in each round will not depend on your Y-bonus or decisions in previous rounds, or on the Y-bonuses and decisions of other participants. While you will be told your own Y-bonus in each round before making a decision, you will never be told the Y-bonuses of other participants. You will only know that each of the other participants has a Y-bonus that is some number between 0 and 30.

At this time, if your ID number is even, please click on row label Y; if your ID number is odd, please click on the row label X. Once everyone has made their selection, the results from this first practice round are displayed on your screen. It will look like

SCREEN 3

if your choice was X, and

SCREEN 4

if your choice was Y.

This completes the first practice round, and you now see a screen like this. The bottom of the screen contains a history panel. This panel will be updated to reflect the history of all previous rounds. [go over columns of history screen]

At the beginning of every new round you will be randomly re-assigned to new groups, and will have the opportunity to choose between "X" and "Y." In other words, you will not necessarily be in the same group during each round. You will also be randomly reassigned a new Y-bonus at the beginning of each round.

We will now go to a second practice round. When this practice round is over, an online quiz will appear on your screen. Everyone must answer all the questions correctly before we can proceed to the paid rounds. Does anyone have any question?

Please take note of your new group assignment, alpha or beta, since the group assignments are shuffled randomly between each round. Also, please take note of your new Y-bonus, which has been randomly redrawn between the values of 0 and 30.

[SLIDE 4]

GO TO NEXT MATCH

Please make your decision now by clicking on the row label X or Y.

A quiz is now displayed on your screen. Please read each question carefully and select the correct answer. Once everyone has answered all the questions correctly, you may all go on to the second page of the quiz. After everyone has correctly answered the second page of questions, we will begin the first paid session. If you have any questions as you are completing the quiz, please feel free to raise your hand and I will go to your workstation to answer your question.

The first paid session will follow the same instructions as the practice session. There will be a total of 50 rounds in the first paid session. Let me summarize those instructions before we start.

[Go over summary slide.] Are there any questions before we begin the first paid session? [Answer questions.] Please begin. There will be 50 rounds, and then you will receive new instructions. (Play rounds 1 – 50) The first session is now over.

SESSION 2

We will now begin session 2.

[SLIDE 5]

The second paid session will be slightly different from the first session. Let me summarize those rules before we start. Please listen carefully. The rules are the same as before with only one exception. In each round of this session, you will have an equal chance of being assigned a Y-bonus between 0 and 55 points. Again, our Y-bonus in each round will not depend on your Y-bonus or decisions in previous rounds, or on the Y-bonuses and decisions of other participants. While you will be told your own Y-bonus in each round before making a decision, you will never be told the Y-bonuses of the other participants. You will only know that each of the other participants has a Y-bonus that is some number between 0 and 55. You may choose "X" or "Y".

There will be 50 rounds in this second session. After each round, group assignments will be randomly reshuffled and everyone will be reassigned a new Y-bonus. Therefore, some rounds you will be in the Alpha group and other rounds you will be in the Beta group. In either case, everyone is told which group they are in and what their private Y-bonus is, before making a choice of X or Y.

Are there any questions before we begin the second paid session? (no quiz) Please Begin. (Play rounds 1 – 50)

Session 2 is now over. Please record your total earnings in dollars for the experiment on your record sheet. After you have recorded your earnings, click the 'ok' button. We cannot pay anyone until everyone has recorded their earnings AND clicked the ok button. Please remain seated and you will be called up one by one according to your ID number to have your recorded earnings amount checked against our own record. Please wait patiently and do not talk or use the computers.