(1) Solve exercise 3.4 in section “Knowledge in terms of sigma-algebras” in the lecture notes.

(2) Consider the following example that appears in Geanakoplos and Polemarchakis (1982); they write that “This example was provided by Robert Aumann in exchange for a Kosher dinner.”

Let $\Omega = \{1, 2, \ldots, 16\}$ and $\Sigma = 2^\Omega$. There are two players, with $\mu_1 = \mu_2 = \mu$ being the uniform distribution $\mu(\omega) = 1/16$. Player 1’s initial information partition is

$$\{1, 2, 3, 4\}, \{5, 6, 7, 8\}, \{9, 10, 11, 12\}, \{13, 14, 15, 16\}.$$  

Player 2’s partition is

$$\{1, 2, 3, 4, 5\}, \{6, 7, 8, 9, 10\}, \{11, 12, 13, 14, 15\}, \{16\}.$$  

Let $A = \{1, 6, 11, 16\}$.  

There are discrete time periods $t = \{1, 2, 3, \ldots\}$. In each time period, the players simultaneously announce to each other their posterior probabilities for $A$. Thus, if $\omega = 1$, then in the first period player 1 announces $1/4$, and player 2 announces $1/5$.

Assuming $\omega = 1$, calculate their posteriors at each time period and show that eventually their posteriors become common knowledge.

(3) Recall the graph defined in the lecture notes whose connected components are $C(\omega)$. Show that the collection $\{C(\omega) : \omega \in \Omega\}$ is a partition of $\Omega$ that generates the sigma-algebra $\Sigma_C$.  

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