Collaboration on homework is encouraged, but individually written solutions are required. Please name all collaborators and sources of information on each assignment. Any such named source may be used.

(1) *Chicken.* Consider the game of Chicken, as parametrized by \(a, b, c\), with \(a, b, c > 0\) and \(a > b\):

\[
\begin{array}{c|cc}
 & Y & D \\
\hline
Y & a, a & 0, b \\
D & b, 0 & -c, -c \\
\end{array}
\]

(a) Calculate the set of (mixed and pure) Nash equilibria for every possible set of parameters.

(b) For which values of \(a, b, c\) (with \(a, b, c > 0\) and \(a > b\)) does there exist a symmetric correlated equilibrium with total expected utility larger than that of any equilibrium? In this game, a correlated equilibrium is symmetric if the probabilities of \((Y, D)\) and \((D, Y)\) are the same.

(2) *Two player, two strategy zero-sum games.* Consider the following zero-sum game. The entry in each matrix is the utility of player 1, and \(a, b, c, d \in \mathbb{R}\) are some numbers.

\[
\begin{array}{c|cc}
 & S & T \\
\hline
X & a & b \\
Y & c & d \\
\end{array}
\]

(a) Give a necessary and sufficient condition on \(a, b, c\) and \(d\) such that there exists a mixed NE in which both strategies are not pure.

(b) Under this condition, how many mixed equilibria can the game have?

(3) *The Kripke S5 system as a characterization of knowledge.* Let \(\Omega\) be finite. Show that if a map \(L: 2^\Omega \to 2^\Omega\) satisfies the Kripke S5 system axioms, then it is the knowledge operator for some type: there exists a type space \(T\) and a function \(t: \Omega \to 2^\Omega\) such that \(L\) is equal to the associated knowledge operator given by

\[
KA = \{\omega : P(\omega) \in A\},
\]

where \(P(\omega) = t^{-1}(t(\omega))\).

Hint: let \(T = 2^\Omega\) and let \(t(\omega)\) be the intersection of all the sets \(A \in 2^\Omega\) such that \(\omega \in LA\).

(4) *Bonus question.* A prisoner escapes to the number line. He chooses some \(n \in \mathbb{Z}\) to hide on the zeroth day. He also chooses some \(k \in \mathbb{Z}\), and every day hides at a number that is \(k\) higher than in the previous day. Hence on day \(t \in \{0, 1, 2, \ldots\}\) he hides at \(n + k \cdot t\).

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Every day the detective can check one number and see if the prisoner is there. If he is there, she wins. Otherwise she can check again the next day.

Formally, the game played between the prisoner and the detective is the following. The prisoner’s strategy space is \( \{(n, k) : n, k \in \mathbb{Z}\} \), and the detective’s strategy space is the set of sequences \((a_0, a_1, a_2, \ldots)\) in \( \mathbb{Z} \). The detective wins if \( a_t = n + k \cdot t \) for some \( t \). The prisoner wins otherwise.

Prove that the detective has a winning strategy.