PS/Ec 172, Set 2
Due Tuesday, January 24th

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Cournot competition.* The Cournot competition game is described in the lecture notes.
   (a) *20 points.* Find a symmetric pure Nash equilibrium of the Cournot competition game, as described in Exercise 2.7 of the lecture notes.

(2) *Mixed Nash equilibria.* Consider the following version of Battle of the Sexes.

	M	O
M	$3,2$	$0,0$
O	$0,0$	$2,4$

   (a) *10 points.* Find the pure Nash equilibria of this game. What are the players' utilities in each equilibrium?

   (b) *20 points.* Find all mixed Nash equilibria of this game. What is the expected utility in each game?

(3) *Elimination of weakly dominated strategies.* In this problem we will show that eliminating weakly dominated strategies can change the set of pure Nash equilibria. This is in contrast to what happens when eliminating strictly dominated strategies, which does not change the set of pure equilibria (see Theorem 2.9 in the lecture notes).

   In the following game the additional strategy A was added to matching pennies.

	H	T	A
H	$1,0$	$0,1$	$2,0$
T	$0,1$	$1,0$	$1,0$
A	$1/2,0$	$0,1$	$2,2$

   (a) *10 points.* Show that this game has a pure Nash equilibrium.

   (b) *10 points.* What are the weakly dominated strategies?

   (c) *10 points.* Iteratively remove the weakly dominated strategies. What is the resulting game? What are its pure Nash equilibria?

(4) *The surprise quiz.* A teacher and a student play the following game. The teacher gives a surprise quiz on one of the five days of the work week. The student, who does not know the material, will fail if he does not review the material right before the quiz, but only has time to study on one day. Thus each player's set of the strategies is the set of five days of the work week.

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The student's utility is one if he and the teacher chose the same day, and zero otherwise. The teacher's utility is one minus the student's.

(a) *10 points.* Show that this game does not have a pure Nash equilibrium.

(b) *10 points.* Find a mixed Nash equilibrium for this game.

(c) *Bonus question (1 point).* Show that if there are infinitely many days then there does not exist a mixed Nash equilibrium. Why does this not violate Nash's Theorem?