Please take three hours to complete this exam. Collaboration is not allowed, except on the bonus question, which you can discuss with others, given that this discussion takes place in public conversations on Piazza. Also, there is no time limit for completing the bonus question.

You may consult the lecture notes, your own notes or any textbook, but not solutions to previous problem sets. There is no need to prove any statements that you make, unless this is explicitly requested (e.g., “explain...”). The exam is due at noon on Friday, June 3rd, and is to be handed in at the instructor’s office, Baxter 213, from 9:30am that day.
(1) A student takes an exam. With probability \( p \) she knows the correct solution, and with probability \( 1 - p \) she does not. The student knows whether or not she knows the correct solution, but the teacher does not (he does know \( p \)).

The student has to choose whether to submit the correct solution (which she can only do if she knows it), submit a nonsensical solution, or submit a blank solution.

After receiving the solution from the student, the teacher immediately sees whether or not the form was left blank. If the form is blank he gives the student 40 points. If the form is not blank, he can decide whether or not to check the solution. If he checks he learns whether the solution is correct or not, and gives the student either 100 or 0 points, accordingly. If he decides not to check then he gives the student 100 points.

If the teacher gives 40 points or 0 points he gets no utility. If he gives 100 points he gains a unit of utility if the solution is correct, and loses a unit of utility if the solution is incorrect. The teacher furthermore loses 0.1 units of utility if he decides to check the solution. The utility for the student is the number of points she gets. Note that the student does not suffer any penalty for the exertion involved in writing a correct solution or making up a nonsensical one.

In the questions below a blank answer will be given 40% of the specified number of points.

(a) 10 points. Assume that the teacher checks all (non-blank) solutions.
   What is the student’s best response? What is the teacher’s best response to this best response?

(b) 10 points. Assume \( p = 1/2 \), and that the teacher checks no solutions and gives 100 to all non-blank ones. What is the student’s best response? What is the teacher’s best response to this best response?

(c) 15 points. Find all values of \( p \) for which there is an equilibrium in which the teacher never checks and gives 100 to all non-blank solutions, and in which the student never hands in a blank solution.

(d) 20 points. Find an equilibrium of this game for the case that \( p = 1/2 \).
   Hint: the teacher should check with some probability \( \sigma \), and if the student does not know the correct solution then she should submit a nonsense one with some probability \( \tau \).
(2) Consider the following game played by \( n \) players who are sitting in a circle. Each player chooses one of two actions: \( X \) or \( Y \). The players make this choice simultaneously. The payoff to a player is 0 if she chooses the same action as the person on her right, and 1 otherwise.

(a) 10 points. Let \( n \) be even. Find a pure Nash equilibrium or explain why none exist.

(b) 10 points. Let \( n \) be odd. Find a pure Nash equilibrium or explain why none exist.

(c) 10 points. Find a completely mixed Nash equilibrium for those values of \( n \) for which no pure one exists. What is the expected utility to each player?

(d) 15 points. For those values of \( n \) for which no pure Nash equilibria exist, find a correlated equilibrium in which the expected utility to each player is \( 1 - 1/n \).

Hint: Imagine a benevolent social planner who (privately) chooses a random number (from some distribution), and according to the outcome tells each player (in private) which action to choose; each player learns only the action that she is to play. This is done in such a way that each player has no incentive to disobey, assuming the rest are also obedient.
(3) *Bonus question.* Choose two integers \(x, y \in \{0, 1, 2, \ldots, 10\}\). Denote by \(\bar{x}\) the average choice for \(x\) made by all the students taking this exam.

You may discuss this question in public Piazza conversations.

(a) As in the midterm, you will get \(x\) points if \(\bar{x} \leq 5.99\). Otherwise you will get nothing.

(b) You will additionally receive \(5e^{-(y-\bar{x})^2}\) points.