PS/Ec 172, Homework 6
Due Wednesday, May 18th

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Reserve prices.* Sathwick and Ruthwick would both like to buy an item owned by Kristin. Sathwick and Ruthwick's valuations are chosen independently from the uniform distribution on $[0, 1]$, and each is known only to himself.
   (a) *15 points.* What is Kristin's expected revenue from a second price auction?

   (b) *30 points.* Kristin now introduces a *reserve price* $b_r \in [0, 1]$: if the maximum bid is under b_r then the auction is canceled, no one gets the item and no one pays. Otherwise, the winner pays the maximum of b_r and the loser's bid. What is her expected revenue, as a function of b_r?

   (c) *5 points.* What is the maximal expected revenue she can get?

(2) *Bundling.* Torin walks into a store with the intention of buying a loaf of bread and a stick of butter. His valuations for the two items are chosen independently from the uniform distribution on $[0, 1]$. Esther, the store owner, has to set the prices. We assume that Torin will buy for any price that is lower than his valuation.
   (a) *10 points.* Assume first that Esther sets a price b_l for the loaf and b_s for the stick. What is her expected revenue, as a function of b_l and b_s?

   (b) *5 points.* What is the maximal expected revenue she can get?

   (c) *30 points.* Esther now decides to *bundle*: she sets a price b_b for buying both items together, and does not offer each one of them separately. That is, she offers Torin to either buy both for b_b, or else get neither. What is her expected revenue, as a function of b_b?

   (d) *5 points.* What is the maximal expected revenue she can get now?

   (e) *Bonus question (5 points).* Assume now that Esther sets three different prices: b_l for the loaf, b_s for the stick, and b_b for both, so that Torin can choose if to buy just the loaf (for b_l), just the stick (for b_s), or both (for b_b). Assume that he will choose to buy whichever items maximize his utility, which is his value for the bought items minus the price paid. What is the maximal expected revenue she can get now?

(3) *Bonus question: The incredible casino.* A casino has a sequence of slot machines $(M_1, M_2, \ldots)$. Each machine requires the gambler to swipe her

Omer Tamuz. Email: tamuz@caltech.edu.

credit card, and has a single button. After swiping the card and pressing the button, machine M_n credits the gambler \$1 with probability $1 - 1/n^2$, and otherwise charges her n^2 dollars.

(a) *3 points.* What is the gambler's expected revenue when using machine M_n?

(b) *7 points.* Kim gambles once at each machine, in order: M_1, M_2, M_3, etc. Explain why, with probability one, her revenue will tend to infinity.
Hint: use the Borel-Cantelli lemma. You can read about it on Wikipedia: `http://en.wikipedia.org/wiki/Borel-Cantelli_lemma`.