PS/EC 172, HOMEWORK 4
DUE WEDNESDAY, APRIL 27TH

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *20 points.* Solve exercise 3.4 in section "Knowledge in terms of sigma-algebras" of the lecture notes.

(2) *20 points.* Construct an example of a knowledge space with two players, a finite set of states of the world, an event A and a state of the world ω such that $\omega \in K_1 A$, $\omega \in K_2 A$, $\omega \in K_1 K_2 A$, $\omega \in K_2 K_1 A$, but $\omega \notin K_1 K_2 K_1 A$.

(3) *20 points.* Recall the knowledge space associated with the hats riddle, at the point in time right after the initial declaration that there is at least one red hat. For $n = 4$ players, calculate the common knowledge sigma-algebra.

(4) *20 points.* Consider a finite knowledge space. Prove that if the common knowledge sigma-algebra Σ_C is trivial (i.e., equal to $\{\emptyset, \Omega\}$) and if some $A \subseteq \Omega$ is common knowledge at some $\omega \in \Omega$, then $A = \Omega$. (Hint: you can use anything that is proved in the lecture notes.)

(5) *20 points.* Please offer any suggestions for improvement of this course. These can include the lectures, TA office hours, problem sets, etc. You may also share any other thoughts or feelings. Please write this part on a separate piece of paper attached to the rest of the solution. You may put your name on it, or leave it anonymous, in which case I will not know that you wrote it, although the TAs will (so you can get credit for it).

TAs: please give full credit to any answer consisting of at least one coherent and relevant paragraph. Also, please collect these answers separately and pass them to me.

(6) *Bonus question.* A prisoner escapes to $\mathbb{Z}^2$ on Sunday. Every day he must move either one up (i.e., add $(0,1)$ to his location) or one to the right (add $(1,0)$), except on Saturdays, when he must rest. The detective can, once a day, check one element of $\mathbb{Z}^2$ and see if the prisoner is there. If she finds him then she wins. He wins if she never finds him.

Formally, the prisoner's strategy is an element

$$(z, f) \in \mathbb{Z}^2 \times \{(1,0), (0,1), (0,0)\}^{\mathbb{N}}$$

such that $f(n) = (0,0)$ whenever $n \equiv 0 \mod 7$, and $f(n) \in \{(1,0), (0,1)\}$ otherwise. The detective's strategy is a sequence $\{z_n\}_{n \in \mathbb{N}}$ with $z_n \in \mathbb{Z}^2$.

The prisoner's current location when using strategy (z, f) is

$$\ell_n = z + \sum_{k=1}^{n-1} f(k).$$

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The detective wins if $\ell_n = z_n$ for some n. The prisoner wins otherwise.

(a) *10 points.* Show that the detective has a winning strategy.

(b) *10 points.* Show that if we remove the requirement that the prisoner rests on Saturdays then the detective does not have a winning strategy.