PS/Ec 172
Final Exam

Please take three hours to complete this exam. Collaboration is not allowed, except on the bonus question, which you can discuss with others, given that this discussion takes place in public conversations on Piazza. Also, there is no time limit for completing the bonus question.

You may consult the lecture notes, your own notes or any textbook, but not solutions to previous exams. There is no need to prove any statements that you make, unless this is explicitly requested (e.g., “explain...”). The exam is due at 2pm on Friday, March 17th, and is to be handed in at the instructor’s office, Baxter 213, from 10am that day.

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(1) The Chandler Cafeteria has started offering *escargot*. Gabby and Akshay are eager to try it, but both are afraid that it is awful. A-priori, there is a 10% chance that it is awful (\(A\)) and a 90% chance that it is good (\(G\)).

There are time periods \(t \in \{0, 1, 2, \ldots \}\), and in each time period they each have to simultaneously decide whether to eat (\(E\)) it or not (\(N\)). Once one of them has decided to eat it, the quality of the escargot is revealed to both and never changes.

The utility for taking action \(a\) at period \(t\) with escargot of quality \(q\) is

\[
  u_t(a, t, q) = \begin{cases} 
  0 & \text{if } a = N \\
  1 & \text{if } a = E \text{ and } q = G \\
  -40 & \text{if } a = E \text{ and } q = A
  \end{cases}
\]

The total utility is \((1 - \delta) \sum_{t=0}^{\infty} \delta^t u_t\) for \(\delta = 9/10\).

(a) 5 points. Explain why in any equilibrium in which Gabby eats in the first period, her expected utility is the same. Calculate this utility.

(b) 5 points. Explain why in every equilibrium at least one of the players eventually eats, with positive probability.

(c) 10 points. Find a pure Nash equilibrium in which Gabby’s expected utility is higher than Akshay’s.

(d) 15 points. Find a symmetric mixed Nash equilibrium.

(e) 15 points. Find a symmetric correlated equilibrium in which the players’ expected utilities are higher than in the symmetric mixed Nash equilibrium.
A student takes an exam. With probability $p$ she knows the correct solution, and with probability $1 - p$ she does not. The student knows whether or not she knows the correct solution, but the teacher does not (he does know $p$).

The student has to choose whether to submit the correct solution (which she can only do if she knows it), submit a nonsensical solution, or submit a blank solution.

After receiving the solution from the student, the teacher immediately sees whether or not the form was left blank. If the form is blank he gives the student 40 points. If the form is not blank, he can decide whether or not to check the solution. If he checks he learns whether the solution is correct or not, and gives the student either 100 or 0 points, accordingly. If he decides not to check then he gives the student 100 points.

If the teacher gives 40 points or 0 points he gets no utility. If he gives 100 points he gains a unit of utility if the solution is correct, and loses a unit of utility if the solution is incorrect. The teacher furthermore loses 0.1 units of utility if he decides to check the solution. The utility for the student is the number of points she gets. Note that the student does not suffer any penalty for the exertion involved in writing a correct solution or making up a nonsensical one.

In the questions below a blank answer will be given 40% of the specified number of points.

(a) 5 points. Assume that the teacher checks all (non-blank) solutions. What is the student’s best response? What is the teacher’s best response to this best response?

(b) 10 points. Assume $p = 1/2$, and that the teacher checks no solutions and so gives 100 to all non-blank ones. What is the student’s best response? What is the teacher’s best response to this best response?

(c) 15 points. Find all values of $p$ for which there is an equilibrium in which the teacher never checks and gives 100 to all non-blank solutions, and in which the student never hands in a blank solution.

(d) 20 points. Find an equilibrium of this game for the case that $p = 1/2$. Hint: the teacher should check with some probability $\sigma$, and if the student does not know the correct solution then she should submit a nonsense one with some probability $\tau$. 

(3) **Bonus question.** Choose two integers $x, y \in \{1, 2, \ldots, 10\}$. Denote by $\bar{x}$ the average choice for $x$ made by all the students taking this exam.

You may discuss this question in public Piazza conversations.

(a) As in the midterm, if $\bar{x} = 1$ you will get 5.5 bonus points. You will get $x$ points if $\bar{x} < 4$. Otherwise you will get nothing.

(b) You will additionally receive $4e^{-(y-\bar{x})^2}$ bonus points.