Ma144A, Homework 8
Due by Noon on Friday, December 1st

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) Prove that for $0 < p \leq 1$ the law of $\tilde{E}$ is absolutely continuous relative to the law of $E$. (See the lecture notes chapter on percolation for the definitions of these random variables.)

(2) Let $G$ be the 3-regular tree. This is the unique connected graph with no cycles in which each vertex is adjacent to three edges. Show that for any $p$ s.t. $p_c < p < 1$ it holds under $\mathbb{P}_p$ that, w.p. 1, there are infinitely many infinite connected components in the $p$ percolation on $G$.

(3) Bonus. Show that $(Z_k)_{k \in \mathbb{Z}}$ is stationary and ergodic (see the lecture on the simplex of invariant measures for the definition).

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