Due Tuesday, May 15 (In class, at the beginning of lecture)

Instructions.
> Take home
> Time limit 4 hours (will probably need less time)
> Open notes, textbooks, and any web reference resources desired.
> Computing aids like calculators and matlab are allowed.

Important.
1) Please check NOW that all of your bluebooks are named
2) If something is unclear in the exam, please state your interpretation of the question, and answer the question under those assumptions.
1 (30 points)

Consider a basic competitive market with two types of consumers: there are 100 consumers of type-H and 100 consumers of type-L.

Consumers of type-H have an experienced utility function given by

$$ U_H(q,e) = 10\log(q) + e $$

and they are endowed with $10 dollars of good $e$.

Consumers of type-L have an experienced utility function given by

$$ U_L(q,e) = 5\log(q) + e $$

and they are also endowed with $10 dollars of good $e$.

On the production side, the $q$ good is produced by 100 identical competitive firms that have a technology described by a cost function

$$ c(q) = \frac{1}{30}q^2. $$

Each of these firms is owned by one of the H type consumers (who is entitled to all of their profits).

A (5 points) Compute the aggregate demand and aggregate supply functions in the $q$ market.

B (5 points) Compute the market equilibrium price, aggregate quantity, as well as the amount produced by each firm, and the amount consumed by each consumer.

C (5 points) What are the profits of each firm in equilibrium?

D (5 points) What is the amount of utility derived by each type of consumer at the equilibrium allocation? (Note: You can provide your answer in terms of the log(.) function)

E (5 points) What is social surplus in equilibrium?

F (5 points) What does the First Welfare Theorem imply about the size of the deadweight loss in equilibrium?

2 (30 points)

Suppose that the government decides to intervene in the previous economy, and instead of having allocations determined by the market, it implements the following policy:

1. It forces each company to sell to the government 20 units of the $q$ good at a price of $1$ p/unit.

2. It finances the policy with an equal lump-sum tax on every consumer.

3. It gives an equal amount of the good to every consumer (regardless of type).
In addition, the government does not allow consumers to trade with each other, or to buy additional units from firms, after it hands out the allocations: they must consume exactly the amount of $q$ that they are given.

**A (2 points)** What is the lump-sum tax that every consumer pays to finance the policy?

**B (3 points)** What are the profits of each firm at the new policy?

**C (5 points)** What is the consumption utility of each type of consumer under the new policy? (Note: You can provide your answer in terms of the log(.) function)

**D (5 points)** Given how the policy impacts their well-being, are high-types in favor or against the policy? What about low-types?

**E (5 points)** What is the deadweight loss generated by this policy? (Note: You can provide your answer in terms of the log(.) function)

**F (5 points)** Suppose that, instead of giving an equal amount of the good to each consumer, the government distributes the $2,000 bought in a way that maximizes total consumption utility. How much would each H-type and L-type consumer get in this case?

**G (5 points)** What is the deadweight loss under the assumptions in part F?

**3 (40 points)**

Finally consider a version of the policy described at the beginning of question 2 in which, after distributing 10 units of the $q$-good to each subject, the government allows them to trade freely with each other (but no further production or trade with firms is allowed).

**A (5 points)** What is the endowment of goods $q$ and $e$ for each consumer at the pre-trade allocation generated by the government policy? (Hint: Don’t forget to take into account the profits of the firms)

**B (5 points)** What is the marginal utility of consuming good $q$ for the two types of consumers at this pre-trade allocation?

**C (5 points)** Is the pre-trade allocation Pareto optimal? Why?

Now look at the post-policy market for the good $q$. Note that in this market there are no firms, and that each consumer is free to buy or sell units to each other, but that they cannot end up consuming a negative amount of $q$.

In analyzing this market, let $ed(p)$ denote the excess demand function of the consumer for good $q$ given that the market price for the good is $p$. This excess demand is equal to the amount that he needs to buy (if positive) or sell (if negative) to achieve his desired level of consumption. If we denote the final desired level of consumption by $d(p)$, then

$$ed(p) = d(p) - 10.$$
D (5 points) Write down the maximization problem that a consumer of each type faces in this market?

E (5 points) What is the excess demand function for each type of consumer?

F (5 points) Compute the market equilibrium price and the amount consumed by each type of consumer in equilibrium. (Hint: As always, market equilibrium occurs at a price at which aggregate supply = aggregate demand)

G (5 points) Is the DWL generated by this policy bigger or smaller than the one that you computed in question 2E? (Hint: You may answer this question numerically, but you can also answer it using basic economic principles)

H (5 points) What would have been the DWL under this policy if the government had bought 1500 units at a price of $1 p/unit, but the policy was identical otherwise? (Hint: Use economic reasoning to answer the question quickly. There is no need to compute anything).

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