Due-SENIORS: Tuesday, June 5, 3:00 pm (location: Jenny Niese’s office, 350 Baxter)

Due-NON-SENIORS: Friday, June 8, 3:00 pm (location: Jenny Niese’s office, 350 Baxter)

Instructions-SENIORS:
> You only need to solve problems 1-4.
> Each problem is worth 25 points
> No extra credit will be given for solving problem 5.

Instructions-NON-SENIORS:
> You need to solve four out of the five problems.
> You choose which ones.
> Each problem is worth 25 points.
> No extra-credit will be given for solving a fifth problem.
> If you submit solutions to more than four problems, a randomly selected one will not be graded.

Instructions-ALL:
> Take home
> Time limit 5 hours (will probably need less time)
> Open notes, textbooks, and any web reference resources desired.
> Computing aids like calculators and mathematical software are allowed.

Important:
1) Please check NOW that all of your bluebooks are named
2) If something is unclear in the exam, please state your interpretation of the question, and provide an answer under those assumptions.
Problem 1

Consider the problem of a consumer with a utility function from consumption given by
\[10\log(q) + e\]
where \(q\) denotes the amount of the good that they purchase in the market, and \(e\) denotes all of their other consumption (measured in $, and potentially negative). Subjects have $100 of exogenous income. The market price for the good is fixed at \(p = $1\) per unit.

Compute the amount that the consumer wants to buy in each of the following situations.

A (6 points) The consumer receives a ‘buying incentive’ from the government of $2 for every unit bought.

B (6 points) If the consumer buys \(q\) units of the good he receives a total cash rebate of \(p\log(q)\) dollars.

C (6 points) Warren Buffet has agreed to give the consumer one unit of \(q\) for every unit that he buys in the market.

D (7 points) The government imposes a non-linear tax on consumption given by \(q^2\).

Problem 2

Consider a simple market with linear aggregate demand given by
\[P(Q) = p^{max} - mQ.\]
There is a single firm in the market with marginal costs of \(c^{mar}\) and zero fixed costs.

Suppose that the government charges the producer a per-unit tax of \(\tau\) per-unit sold, and that the tax revenue is returned to the consumers using an identical lump-sum transfer.

A (6 points) Compute the equilibrium quantity and price in the market as a function of the size of the tax.

B (6 points) Compute the deadweight loss in the market as a function of the size of the tax.

C (6 points) Compute the marginal impact of an increase in the tax on the total well-being of the consumers. (Hint: Don’t forget to take into account that the tax revenue goes back to the consumers)

D (7 points) What level of the tax \((\tau \geq 0)\) maximizes aggregate consumer welfare?
Problem 3

Consider a competitive market with the following characteristics.

First, aggregate demand is insensitive to price: consumers always demand 100 units of the good, regardless of the price.

Second, there are 20 identical firms in the market. Each firm uses labor as its sole input of production, and its production function has the form

$$F(L) = L^{1/2},$$

where $L$ is the amount of labor that it hires, and $F(L)$ is the amount of labor that it produces.

For the rest of the problem, suppose that the price per unit of labor for the firm is $1.

A (3 points) What is the cost of producing $q$ units of the good?

B (2 points) Derive and plot the marginal cost, average variable cost, and average total cost curves.

C (3 points) What is the short-run equilibrium in this market?

D (2 points) What are the equilibrium profits for each firm?

Now suppose that the government introduces a firm lump-sum tax of $100 per company. In the short-run, all of the firms in the market are required to pay the tax, regardless of their level of production. In the long-run, only firms that sell a positive amount (i.e., are in the market) have to pay the tax. Suppose also that the production technology in the market is easy to copy, so there are an infinite number of potential entrants.

E (5 points) How does the introduction of the tax affect the short-run equilibrium in the market?

F (3 points) What are the firm’s profits at the new short-run equilibrium?

G (5 points) What is the long-run equilibrium of the market? (Please provide the number of firms as well the equilibrium price and quantity produced by each firm)

H (2 points) What are the firm’s profits at the long-run equilibrium?

Problem 4

Consider an economy with 100 identical consumers that have no exogenous income, but earn income from selling their labor to the firms. The consumers utility function is given by

$$2aq^{1/2} - \frac{\beta}{2}l^2 + e,$$

where $q$ is a consumption good purchased in the market, and $l$ is the amount of labor that they sell to the firms.
The economy also has 100 identical firms with a production function that allows the to transform labor into the $q$—good according to the following production function

$$F(l) = 2Al^{1/2}$$

where $l$ denotes the amount of labor that they hire, and $F(l)$ denotes the amount of output produced.

Let $p$ denote the market price of the $q$—good and $w$ denote the market wage rate.

**A** (6 points) Compute the labor demand function and supply function for the $q$—good for each firm, as a function of $p$ and $w$.

**B** (6 points) Compute the labor supply function and demand function for the $q$—good for each consumer, as a function of $p$ and $w$.

**C** (7 points) Compute the equilibrium $w$ and $p$ as a function of the parameters $\alpha$, $\beta$, and $A$. (Note: You may express the solution in terms of powers of the prices; e.g., $p^n = f(\alpha, \beta, A)$ to make things easier).

**D** (6 points) What is the impact on the equilibrium $w$ and $p$ of an increase in the parameter $A$? $\alpha$? $\beta$?

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**Problem 5**

Consider an economy with 100 identical consumers and 100 identical firms. Each consumer has preferences given by

$$100\log(q) + e$$

where $q$ denotes the amount of the good that they purchase in the market, and $e$ denotes all of their other consumption (measured in $\$, and potentially negative). Subjects have $1000 of exogenous income.

Suppose that consuming the good $q$ generates pollution that is not harmful to humans, but interferes with the technology used to produce the good $q$. In particular, assume that each unit of $q$ leads to 1/100 units of pollution. Let $T$ denote the total amount of pollution. The cost function of each firm is given by

$$c(q|T) = 2Tq.$$  

**A** (5 points) Compute the competitive market equilibrium and the associated level of pollution.

**B** (2 points) What are the marginal social benefit (MSB) and the marginal social cost (MSC) at the market equilibrium allocation?

**C** (5 points) Compute the Pareto optimal level of $q$.

**D** (2 points) What are the marginal social benefit (MSB) and the marginal social cost (MSC) at the optimal allocation?

**E** (4 points) What is the size of the Pigouvian per-unit-tax that restores optimality to the market?
F (7 points) What is the size of the Pigouvian ad-valorem tax that restores optimality to the market? (Hint: remember that an ad-valorem tax is expressed as a percentage of the sale price)