Competition in Portfolio Management: Theory and Experiment

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Abstract

We develop a new theory of delegated investment whereby managers compete in terms of composition of the portfolios they promise to acquire. We study the resulting asset pricing in the inter-manager market. Managers are paid a fixed fraction of fund size. In equilibrium, investors choose managers who offer portfolios that mimic Arrow-Debreu (state) securities. Prices in the inter-manager market are predicted to satisfy a weak version of the CAPM: state-price probability ratios implicit in prices of traded assets decrease in aggregate wealth across states. An experiment involving about one hundred participants over six weeks broadly supports the theoretical predictions. Pricing quality declines, however, when fund concentration increases because funds flow towards managers who offer portfolios closer to Arrow-Debreu securities (as in the theory) and who had better recent performance (an observation unrelated to the theory).

1 Introduction

Participation in investment markets has steadily shifted away from individuals toward institutions. With 93% of US equity directly held by individuals in 1950, this figure was estimated to be as low as 25% by 2009.\footnote{See [13]} At the same time, households’ direct and indirect (through retirement plans) holdings of funds has risen from a mere 3% in 2000 to 23% of their total financial assets in 2010, with mutual funds consequently owning 27% of all US equity, 45% of all commercial paper, and 11% of US treasury securities.\footnote{Except when specified, all data reported in this introduction are taken from the 2011 Investment Company Fact Book, [17].} While there is no doubt that the participation of delegated portfolio managers who invest for others is one of the key characteristics of both modern equity and debt
markets, it remains an open question, and a subject of a growing literature, how delegation impacts asset prices and market efficiency.

In this paper we develop a theory about competition for portfolio management and its implications for asset prices and we offer experimental evidence about this theory. In our setup, investors hold securities but cannot trade those directly. Instead, they need to engage managers who will trade for their account. Managers each offer a portfolio contract to the investors and stand ready to deliver any number of units of it in exchange for a fixed fee per unit sold. Most importantly, there is free entry in the portfolio management industry and investors can hold contracts from several managers. We are interested in what contracts are offered by managers, what portfolios of contracts are chosen by investors, and in the asset prices that emerge in the inter-manager market.

In order to isolate the effect of competition for fund flows on asset pricing in a simple setting, we consider an environment where skill and information are homogeneous across managers. For the same reason, we assume a fee structure completely stripped from complications that would generate further realism. We also abstract from the question of why investors delegate their investment and “force” them to do so, thus limiting their decision to that of how to distribute their wealth among managers. Of course, delegation may introduce a shift in managers’ objectives relative to those of the investors precisely because of the skill and informational advantage that some of those managers possess and because of the exact nature of the fee arrangement. Such shifts bear consequences for both asset prices and allocations. However, even with added realism, the question we pose would still be about the component in prices due to competition, and the layers of complexity would contribute little to answering that question.

Much of the existing literature has focused on the resolution of the conflicts of interest between investors and their heterogeneously skilled or informed portfolio managers via contract design [5, 21, 16, 18, 11, 15]. Another strand of the literature has taken the unavoidable conflicts of interest and their imperfect resolution as given, and studies the behavior of the resulting markets where traders necessarily have diverse objective functions and engage in different strategies, e.g., benchmark chasing [10, 12, 14]. Our approach differs significantly from that of the extant literature. Ours is not an application of contract theory, in which one principal and one agent agree to an optimal contract. We explicitly model that principals (investors) have a choice of agents (managers). Also, the relationship between investors and managers is not exclusive: investors can engage several managers. Managers, on the other hand, compete to be chosen by the investors and to have more funds contributed towards their portfolios.

Berk & Green [4] consider competition between funds (managers) but in their model price competition offsets possible talent differences between managers. Closer to our analysis, Agranov, Bisin
& Schotter [1] investigate strategic portfolio choices by two managers when they Bertrand-compete for a single investor’s money. Ours is a general-equilibrium competitive analysis rather than a purely strategic one, in which managers may enter freely, and investors may engage different managers simultaneously. In order to stress investors’ access and use of several managers simultaneously, we do not model direct fee competition. Instead, competition occurs via the quality and indirect cost of the contracts that managers offer their clients.\footnote{Manager competition is empirically motivated by the existence of over 16000 investment companies in 2010. Fees charged cover a wide range, maximum fees have not decreased in the last decade, and the correlation between fund fees and performance – a proxy of talent – is yet to be established (see, e.g., [6]). Investors (who largely converge to low-fee managers) tend to invest in several funds: by the end of 2010, the median fund investor held shares in four different funds.} Moreover, unlike [1], we study pricing in the inter-manager market.

Our theoretical setting is one where, in the absence of delegation, the predictions of the capital asset pricing model (CAPM) would be attained. In equilibrium, agents would hold the market portfolio, which would be mean-variance efficient, and the two-fund separation theorem would hold. We introduce the layer of delegation where fund managers compete for clients and are paid according to a nominal fee proportional to the value of assets they are entrusted to manage, and investors can hold contracts offered by several managers. There is a large multiplicity of equilibria in this setting. By adding a clause of limited liability on the fees that investors owe, we induce managers to compete on reducing the effective fees paid by investors. This setup brings about sharp equilibrium predictions: managers must offer Arrow-Debreu (AD) securities. The equilibrium in the inter-manager asset market has all properties of the corresponding economy without delegation in all respects but for a lower risk premium.

Investors in our theory (and experiment) are endowed with assets that they allocate to different managers. An investor pays fees and obtains a share in a manager’s fund in accordance to the value of the allocated assets. The basic version of our theory determines value using security prices. This version bears a one-to-one relationship with the more familiar situation where investors allocate wealth to managers, for which they receive shares in the overall managers’ portfolios and pay fees that are proportional to the contributed wealth. Implementation in the laboratory, however, requires one to know the prices of the assets in the inter-manager market before these prices are determined by subjects’ participation in asset markets. Therefore, we consider a variation of the theory where the share in a fund portfolio is determined by contributed securities’ expected final payoffs. As a consequence, we lose efficiency of the market portfolio but holdings and prices still satisfy a weak version of the CAPM. Namely, state prices are ranked in accordance to wealth in every state. Our experiment implements this version.

Our theory is essentially an application of the analysis of competition in contracts to delegated
portfolio management. Rothschild & Stiglitz [20] were the first to carry out this analysis, and Asparouhova [2] has recently extended it and put it to experimental testing. Our application goes one step further: it not only analyzes what contracts will emerge in equilibrium, but also whether pricing in the securities markets is affected by delegation. Our equilibrium notion is thus closely related to that of Zame [22].

There are a number of reasons why we follow the tradition of competitive analysis of contracts that started with [20]. First, it provides a manageable notion of competitive equilibrium. Second, in the field, there is plenty of casual support for its main predictions.⁴ Third, these and other predictions have recently been confirmed in experimental testing [2].

We complete our analysis with a large scale experiment that provides an empirical test of the equilibrium predictions. With the exception that no subject could trade for his or her own account, our experimental design closely follows the design of earlier experiments that have reliably generated (weak) CAPM pricing [7, 8, 3]. The investor-subjects (investors) allocated their initial endowment to manager-subjects (managers). The managers could then trade the assets for cash in an anonymous web-based continuous open-book market within a pre-specified time period. When trading concluded, each investor was paid his share of managers’ final portfolio values net of fees. Managers received a payment proportional to the expected payoffs on the initial contributions to their fund, while investors’ obligation to pay the fee was limited by the final value of a manager’s portfolio.⁵ Basic performance indices were then reported, investors were given new batches of endowments and invited to allocate them to the managers. This process was repeated six times in the experiment.

Consistent with the theory, we found strong evidence that investors preferred managers whose portfolios were closer to generating the payoffs of Arrow-Debreu securities. However, manager choices were also partly determined by performance in the immediate past. Both elements contributed to increased concentration of holdings across managers. Consistent with our theory, price dynamics appeared to be driven by weak CAPM, but pricing quality was negatively affected by fund concentration.

Our contribution to the literature on asset prices in the presence of delegation is as follows. First, we formalize and show the experimental validity of the observation that even in the absence of

⁴In the context of lending under adverse selection, for instance, this theory anticipated the inevitability of zero-deductible insurance contracts or zero-downpayment “subprime” loan contracts, and it explained why banks could at times be casual about creditworthiness checks (because risks could easily be inferred from contract choice).

⁵To ensure that they were adequately incentivized, however, managers were always paid their fees, i.e., they always received the nominal fee they were promised. In other words, we (the experimenters) paid out managers whose final portfolios did not cover their fees. Obviously, such “bailouts” do not change investors’ preference for managers who offer portfolios that mimic AD securities. The situation is not unlike that for hedge funds in the field, where managers are compensated based on performance, but never receive less than zero. This induces managers to take risks, like emulating payoffs of AD securities.
conflicts of interest, a market populated by managers need not look identical to a market populated by their clients. Still, prices conform to the predictions of the basic asset pricing model, and while managers hold extreme positions, investors end up with well diversified portfolios. Second, since investors ultimately care for post-fee payoffs, fees and contracts are shown to have an influence on trading behavior even in the absence of skill and information asymmetries. Third, we show that in the controlled laboratory experiment, investors react to the fee structure and, consequently, managers who allow for a better post-fee distribution of payoffs are prized.

2 Theory

In our model investors face idiosyncratic and aggregate risk in a one-period investment setting. Today investors can acquire contracts offered by investment funds or managers so that their payoff tomorrow has a desirable distribution of risk, not necessarily equal to their original idiosyncratic risk. Managers supply the contracts demanded by investors via trade in financial markets. They use the assets entrusted to them by investors to pay for the necessary securities to provide the promised contracts. We will assume that investors have quadratic utility over consumption tomorrow and care about the expectation of that utility today, while managers are risk neutral and only care about the payment they receive from investors. This payment is a fee proportional to the value of assets given them by investors. In a highly stylized version of our model, this latter value is endogenously determined by equilibrium prices.

To outline our modeling strategy in the most transparent way we first present this stylized baseline model. However, implementation of this baseline in an experimental setting is not straightforward. Subsequently, we present the feasible variation that we put to test in our experiment.

Baseline

The wealth process of all investors and of the economy as a whole is expressed in terms of Arrow-Debreu securities (AD or state securities) that span all states. States are indexed by \( s (= 1, \ldots, S) \) and an AD security \( s \) pays $1 in state \( s \) and $0 otherwise. Investor \( i (i = 1, \ldots, I) \) has initial endowment \( x^0_i \) of AD security \( s \), which is thus also investor \( i \)'s initial wealth, were state \( s \) to be realized. Let \( \pi_s \) denote the chance that state \( s \) occurs. Investors may have incentives to move away from this initial wealth distribution across states. They can do so only by acquiring contracts with investment managers in exchange for a management fee.

There are \( M \) managers indexed by \( m (= 1, \ldots, M) \), who can offer investors multiple units of one specific contract. This contract, gross of management fees, is either an AD security or any convex
linear combination of AD securities. That is, manager \( m \)'s contract states a fraction, \( \theta^m_s \geq 0 \) of the $1 total payout that manager \( m \) promises in state \( s \) (\( \sum_s \theta^m_s = 1 \)). If the manager chooses to offer an AD security, \( \theta^m_s = 1 \) in some state \( s \) and \( \theta^m_{s'} = 0 \) in all other states, \( s' \neq s \). To deliver these payouts, managers trade in a set of complete securities markets. They enter the markets endowed with assets given to them by investors. Markets are competitive and managers can sell short. We express their trade in terms of state securities and let \( p^d_s \) denote the price of state security \( s \) in the inter-manager markets (\( s = 1, \ldots, S \); the superscript \( d \) indicates that these are inter-manager market prices).

Managers are paid a fixed fee, \( f \) (\( 0 < f < 1 \)), to deliver the promised payoffs of their contract, to be charged as a back-end load on the final value of the contract. Managers can sell any number of units of their contract to different investors. Investor \( i \) acquires a non-negative amount \( x^i_m \) of the contract of manager \( m \) (\( 0 \leq x^i_m \)) and owes the corresponding fee of \( x^i_m f \). Manager incentives are such that they strictly prefer to sell a contract over not selling at all, whatever the fee. Investors, on the other hand, have a quadratic utility over final wealth after fee payment.

We assume that investors are not liable for the full fee if the final value of the contract is insufficient. This limited liability for fees implies that investor \( i \)'s state-\( s \) total payoff from his shares in all managers’ contracts equals:

\[
\sum_m x^i_m \max \{ \theta^m_s - f, 0 \}.
\]

Therefore, investor \( i \)'s expected utility, given his holdings of each manager’s contract, is given by the following expression:

\[
u^i \left( \{ x^i_m \} \right) = \sum_s \pi_s \left\{ \sum_m x^i_m \max \{ \theta^m_s - f, 0 \} - \frac{b}{2} \left[ \sum_m x^i_m \max \{ \theta^m_s - f, 0 \} \right]^2 \right\}.
\]

Investors choose their holdings of managers’ contracts to maximize their utility subject to a budget constraint. When acquiring \( x^i_m \) units of manager \( m \)'s contract, investor \( i \) commits to hand over the necessary holdings of state securities so that manager \( m \) can trade to positions that allow her to deliver the contractually-specified payoffs. This means that \( i \) hands over state securities with a total value equal to \( \sum_s p^d_s x^i_m \theta^m_s \). Summing over managers, investor \( i \) will need to have satisfied his budget constraint,

\[
\sum_s p^d_s \sum_m x^i_m \theta^m_s = \sum_s p^d_s x^i_s 0.
\]

In equilibrium, state prices \( p^d_s \) should be such that demands match supplies in the inter-manager market. Moreover, contracts offered (and taken) should be such that a new entrant in the manager market cannot offer a contract that makes at least one investor strictly better off. This closely resembles the equilibrium notion defined in [22], and follows the tradition established in [20].
As we shall see later, provided that S AD contracts are already offered, the limited liability gives rise to an intuitive outcome: all newly entering managers should offer AD contracts rather than any (strict) convex linear combination of AD contracts. That is, for manager \( m \), \( \theta^m_s = 1 \) in some state \( s \) and \( \theta^m_{s'} = 0 \) in all other states \( s' \). In those states, managers receive no fee from investors. For now, we assume that managers only offer AD securities, and focus on pricing in the inter-manager market.

If managers only offer AD securities, we can assume that there are \( S \) representative managers, indexed by \( s = 1, \ldots, S \), the AD security they offer. Investor \( i \) acquires \( x^i_s \) units in manager \( s \). Since \( 0 < f < 1 \), the payoff for investor \( i \) in state \( s \) becomes:

\[
x^i_s \max\{1 - f, 0\} = x^i_s(1 - f).
\]

Absent delegation in portfolio management, CAPM pricing would obtain because of the quadratic utilities our investors hold. To understand the sequel, we remind the reader of two important features of competitive equilibrium with quadratic utility:

1. Individual demands satisfy portfolio separation. In terms of AD securities, this means that the optimal demand of investor \( i \) for state \( s \), \( z^i_s \), can be written as the sum of a constant term (constant across all states) \( z^i_c \) and a state-dependent quantity \( z^i_{e,s} \):

\[
z^i_s = z^i_c + z^i_{e,s}.
\]

We refer to \( z^i_{e,s} \) as the “risk-exposed” component of demands. The constant in the demands, \( z^i_c \), can be accommodated by purchasing riskfree securities. The vector of risk-exposed components, \( \{z^i_{e,s}\}_{i=1}^S \), is the same for all investors, up to a constant of proportionality that depends on individual risk tolerance. Mathematically, for any investors \( i \) and \( j \), and states \( s \) and \( s' \)

\[
\frac{z^i_{e,s}}{z^i_{e,s'}} = \frac{z^j_{e,s}}{z^j_{e,s'}}.
\]

2. The market portfolio is mean-variance optimal, which means that the aggregate portfolio of all AD securities available in the marketplace is optimal for some agent with quadratic utility. This means that its Sharpe ratio (expected excess return divided by return standard deviation) is maximal, or equivalently, that the well-known “beta” pricing relationship holds (expected excess returns on all risky assets are strictly proportional to their “betas;” see [19]).

Let us now spell out the argument for why CAPM pricing continues to obtain despite delegated portfolio management.

To start, it is important to realize that demands for (AD) securities in the inter-manager financial
market place are derived from investor demands for those securities. Net of fees, an investor's demand is optimal for some quadratic utility; gross of fees, managers need to acquire that many more AD securities as to cover their fee, but only in one state. That is, the fee itself is contingent: if the AD security pays in state $s$, then the fee is due only in state $s$. Thus, $i$'s demand for AD security $s$, net of fees, will be $z^i_s = x^i_s(1 - f)$. The quantity $z^i_s$ is ultimately what matters for the investor; $x^i_s$ is what the investor wants the manager to acquire.

As such, the derived demands $x^i_s$, $s = 1, \ldots, S$, equal the original net-of-fee demands times a state-independent factor. But this implies that the derived demands continue to be composed of two additive elements: a constant, independent of the state, and a risk-exposed demand:

$$x^i_s = z^i_s/(1 - f) = z^i_c/(1 - f) + z^i_{c,s}/(1 - f) = x^i_c + x^i_{c,s}.$$  

The risk-exposed demand is the same as that of any other investor with quadratic utility, up to a constant of proportionality. In particular, across states, the ratio of the risk-exposed demands is the same as that of the original (net-of-fees) demands:

$$\frac{x^i_{c,s}}{x^i_{c,s'}} = \frac{z^i_{c,s}}{z^i_{c,s'}}.$$  

Consequently, the derived demands are optimal for (some) quadratic utility as well. Since derived demands in the inter-manager market continue to be optimal, CAPM pricing continues to obtain.\(^6\)

Fig. 1 illustrates this in a two-state world. Given prices, an investor desires an allocation $a$ on the line of optimal allocations. We only show the risk-exposed allocation. Including fees, the allocation can be obtained as a fraction $\xi$ of one unit of AD security 1 and $1 - \xi$ of one unit of AD security 2. (Any optimal allocation can be obtained by scaling up from 1 unit to some number of units of both AD securities). The derived demand of the managers in the inter-manager market is obtained by adding the fees to the desired allocation in the states that they are due. Because fees are due only in states when the AD securities have strictly positive payoff, the allocation the managers need to acquire still lies on the line of optimal portfolios, and hence, is optimal under quadratic utility.

\(^6\)The mathematical proof of the above is very simple and can be obtained from the authors together with numerical examples. We also show that the equity premium will be lower than in the original economy without portfolio delegation. That is, the market portfolio will be optimal for quadratic utility with risk aversion that is less than the (harmonic) mean of the individual risk aversion parameters. The intuition is simple: the maximum difference in aggregate wealth between states is lower in the economy with delegation. For example, if aggregate wealth with three states equals $\{1, 2, 3\}$, the maximum difference in wealth levels is 2 ($= 3 - 1$). If fees are 40%, then net of fees, the maximum difference in wealth is $2(1 - f) = 1.2$. It is this maximum difference (in wealth levels) that determines the risk premium, as quadratic utility displays constant absolute risk aversion. As a result, the risk premium required is lower under delegation.
Investors’ Choices of Managers

We now turn to the question as to why investors prefer managers who offer AD securities rather than the more diversified (strict) convex linear combinations of AD securities. The elements of our model that are key to the argument that follows are the possibility that investors diversify on their own by acquiring contracts from several different managers, and the assumption that they have limited liability for manager fees when manager payoffs cannot cover their own fees.

We borrow from a classical model of a competitive insurance or banking industry (see [20, 2]) and envisage a portfolio management industry with free entry. Managers each offer a single contract. They enter only if they get a strictly positive fee income, which means that they need to be able to sell at least a positive number of their contract; otherwise they stay out. We assume that managers diligently acquire the portfolio of AD securities that their contract with the investors stipulates. We claim that reputation will ensure that this is the case.

With reference to Fig. 2, imagine that an investor would like to acquire a position which, including fees, brings him to position $a_{\text{post-fee}}$. This can be accomplished by acquiring $\xi$ units of the contract of a manager who offers the AD security 1 (which pays $1$ in state 1) and $1 - \xi$ units of 2. The position gross of fees is denoted $a'$. The total fee that our investor pays equals $\xi f$ in the 1 state, and $(1 - \xi)f$ in state 2. The AD securities expire worthless in complementary states, and hence, the investor does not owe any fee for those states, which explains the fee obligations across states.

Can a manager enter and make the investor better off when offering a contract which is a strict convex linear combination of AD securities, like E in Fig. 2? Our investor could now generate $a'$ with $\gamma$ units of AD contract 1 and $1 - \gamma$ units of contract E. (Investors can only acquire positive fractions of manager contracts, so our investor cannot obtain $a'$ by combining E with AD security 2.) Necessarily, $\gamma < \xi$, but because of this, the fee cost to our investor is strictly higher in both states: it is $\gamma f + (1 - \gamma)f = f$ ($> \xi f$) in state 1 and $(1 - \gamma)f$ ($> (1 - \xi)f$) in state 2. So, net of fees, our investor will receive strictly less in both states than required for his desired allocation $a$. The investor will not be enticed by the manager with contract E; he will stay with the one with contract 2. The argument is not specific to two states.

Hence, according to our definition of equilibrium, the presence of enough managers who offer a complete set of AD securities that satisfy the demands of all investors, is an equilibrium. There can be other equilibria where a complete set of contracts different from AD securities is offered. However, such equilibria are not intuitive and not every complete set of securities survives the entry of a single manager offering an AD security. Moreover, the equilibrium that we have just spelled out has the property that it survives the joint entry of any number of managers simultaneously, while no other equilibrium would satisfy this property.
On Weak And Strong CAPM

We showed above that managers that offer a complete set of AD securities and investors that optimally diversify among these managers constitute an equilibrium of our model. In such an equilibrium, CAPM continues to hold, meaning that the market portfolio is mean-variance optimal and portfolios of all investors are perfectly correlated. CAPM requires quadratic utility (or strict distributional assumptions such as normally distributed payoffs). Under milder assumptions - namely, complete markets and decreasing marginal utility - one still obtains rank correlation of investor portfolios. This implies perfect negative rank correlation of the ratios of state prices over probabilities, i.e., state-price probability ratios, with aggregate wealth across states. We call this result weak CAPM.

Of course strong CAPM implies weak CAPM. This obtains because state-price probability ratios are increasing in aggregate wealth. If \( p_s^d \) denotes the price of state security \( s \) under portfolio delegation, and \( x_s^0 \) the (per-capita) aggregate wealth in this state, then:

\[
\frac{\partial p_s^d}{\pi_s} \frac{\partial x_s^o}{\partial x_s^o} < 0.
\]

The result is obtained by differentiating the equilibrium pricing equations.\(^7\)

A Feasible Variation for Experimental Implementation

In the baseline theory, a manager promises payoffs that are those of a convex linear combination of AD securities. Each investor must hand over to the chosen managers assets with a worth (valued at inter-manager market prices) necessary for the managers to trade to the contractual combinations of AD securities.

This situation is equivalent to one where as above investors hand over the necessary assets and in return, receive a share in the manager payoff pro rata of their contribution (in value) to manager assets. The fee then can be expressed as a fixed percentage \( \phi \) of the value of the share an investor holds in each manager’s fund.

There is a practical problem with the baseline scenario in that one needs to know the values of the assets contributed to a manager’s fund. These values depend on prices in the inter-manager market, which necessarily occurs only after managers receive funds from investors.

In the field, this problem is routinely encountered in the case of ongoing mutual funds. Consider a new investor who contributes cash to an existing fund. Shares are traditionally assigned based on the value of all assets (both already in the fund and contributed) at the close of the market after contribution. Consider the fraction of the manager’s portfolio that the contributed cash will

\(^7\)These pricing equations are contained in the mathematical material that can be obtained from the authors.
buy if stock prices increase the subsequent day when the manager aims for a particular portfolio composition, say, an equally weighted index of two types of common shares. This fraction will be lower than the share allocated to the new investor, which was based on the lower stock prices at the close the previous day. Conversely, the cash contribution of the new investor will buy a higher fraction of the manager’s portfolio if stock prices decline, compared to the share he is being allocated. In terms of relative value in the eventual portfolio, the share generated by the assets which the new investor contributes does not correspond to the share he is being assigned.

When a new fund is launched, and to the extent contributions are solely in the form of one asset, say cash, the problem is nonexistent. Shares can then be based on relative expected payoffs (which, for cash, is always $1 plus interest for each dollar contributed). That is, an investor who contributes $2 in expected payoff will receive twice as many shares as another investor who contributes $1 in expected payoff. Since both investors use the same asset, the contribution from the first investor will buy twice as many funds as that of the second investor, independent of intended fund composition.

Here we consider the implications for the theory if shares are computed based on the expected payoff on the assets contributed instead of their value. This exploits the advantage of an experiment where the experimenter controls and announces the probabilities of different states and hence the expected payoff is readily computed. Unlike in most instances in the field, we allow investors to contribute in the form of multiple assets: investors hand over fractions of their wealth to the managers, and each manager determines investors' shares based on the relative expected payoff of the contribution. Management fees are a percentage $\phi$ of the expected payoff of contributed assets.

**Implications for Investors’ Choice of Manager**

Does this change the desire of the investors to solely choose managers who intend to trade to AD securities, and hence, to whom they owe the management fee in only one state for each AD security? No, and here is why. The investor allocates fractions of his initial endowment (wealth) to each of the managers, so the fees he pays in each state will only be a *fraction of the fraction* of the expected value of his wealth he allocates to the managers who invest for the corresponding state. Imagine that an investor’s initial endowment is expected to generate $10 in payoffs. The investor allocates $1/3$ of his wealth to managers who intend to buy AD securities for each of three possible states. If the fee rate is $\phi = 0.2$, then he owes managers $0.6 (=10*0.3*0.2)$ in each state. If instead the investor decides to assign his entire wealth to a manager who intends to buy to a portfolio that pays in each state, he

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8If he contributed $5 in cash and the existing fund was worth $10 at the close, he is assigned a share of $1/3. If prices subsequently increase 20%, the existing fund will be worth $12, but the cash contribution still only allows the manager to buy $5 in stock, and hence, the contributed assets buy only $5/17 (< 1/3) of the shares of the fund as intended by the manager.
owes the manager $2 (=10\times0.2)$ in all states (provided the manager always generates enough payoff). The latter is obviously larger, so the investor should prefer to allocate his wealth over managers who commit to trading to AD securities.

**Implications for Pricing**

To facilitate the discussion, let us translate contributions and fee payments back into a setting where $S$ (representative) managers offer AD contracts (each manager offers the AD security for a different state) in return for a fixed dollar fee per unit. Below we demonstrate that the fee expressed as a percentage of expected payoff translates into a state-dependent fixed dollar fee per unit. Thus, let $f_s$ denote the dollar fee owed in state $s$, $s = 1, ..., S$.

Investor $i$ would like manager $s$ to acquire $x_i^s$ units of AD security $s$ gross of fees (i.e., the payout of this investment should cover both investor $i$’s desired consumption in state $s$ and the manager fee). To accomplish this, investor $i$ needs to hand over assets worth $p^d_i x_i^s$. This corresponds to a fraction $\alpha^i_s$ of the value of investor $i$’s wealth, i.e.,

$$p^d_i x_i^s = \alpha^i_s \sum_{s'} p^d_{i,s'} x_{i,s'}^0.$$  \hspace{1cm} (2)

This way, the manager can implement the desired investment. To balance investor $i$’s budget constraint, we need: $\sum_s \alpha_i^s = 1$.

On the other hand, investor $i$ pays a fraction of the manager’s fee that is identical to his share in this manager’s fund. Shares, in turn, are computed *pro rata* of the expected payoff of the contributed assets. That is, suppose investor $i$ gives a proportion $\psi_i^s$ of the expected payoff of her initial endowment to manager $s$, he will pay a part of the manager’s total fee that equals

$$f^i_s x_i^s = \phi \psi_i^s \sum_{s'} \pi_{s,s'} x_{i,s'}^0.$$  \hspace{1cm} (3)

We assume $\psi_i^s = \alpha_i^s$ for all managers and investors. In our experiment this assumption is enforced by having investors choose a fraction of their holdings to assign to each manager, which is equally applied to their holdings of every asset.\(^9\) With $\psi_i^s = \alpha_i^s$ we obtain

$$f^i_s x_i^s = \alpha_i^s \sum_{s'} \pi_{s,s'} x_{i,s'}^0.$$  \hspace{1cm} (4)

\(^9\)E.g., if investor $i$ wants to give 25% of his wealth to manager $s$, in our experiment this can only be done if he gives 25% of his holdings of each of the assets in his initial portfolio.
implying a per unit fee

$$f_s^i = p_s^i \frac{\sum_{s'} \pi_{s'} x_{s'}^{i,0}}{\sum_{s'} p_s^{i,s'} x_{s'}^{i,0}} \sim p_s^i \phi.$$  

That is, \( f_s^i \) is proportional to \( p_s^i \phi \), where the constant of proportionality depends on the ratio of expected payoff to the value of the initial endowment. With the additional assumption that all investors have the same expected payoff on their initial endowment, we obtain \( f_s^i = f_s \) for all investors.\(^\text{10}\)

The important point here is that, when translated into dollar fee per contract unit, the new setting leads to a state-dependent fee, and as a result, CAPM pricing need no longer hold in the inter-manager market. Managers’ demands, derived from investor demands after the addition of (state-dependent) fees, may no longer be optimal for quadratic utility. In terms of Fig. 1, the derived risk-exposed demands no longer locate on the line of optimal portfolios. Net of fees, demands continue to be optimal for quadratic utility, and corresponding equilibrium state prices continue to satisfy CAPM, but these prices are not the ones we will observe in the inter-manager market.

Nevertheless, weak CAPM will hold, which means that the state-price probability ratios will be ranked inversely to the aggregate payouts in the states. Aggregate payouts have to be corrected: fee payments have to be subtracted from aggregate liquidation values before ranking states.

The derivation of this result is tedious, but the argument runs as follows. Strong CAPM continues to hold after properly adjusting state prices in the inter-manager market for the per-contract fee \( f_s \), and when measuring aggregate (state) payoffs net of fees. Strong CAPM implies weak CAPM. So, weak CAPM applies to the adjusted state prices. This means that a marginal increase in the aggregate state wealth (net of fee payments) decreases the adjusted state price. We show that the unadjusted state prices also decrease in state aggregate wealth, even after accounting for the impact of the marginal increase in the fee \( f_s \) and its ensuing effect on the state price adjustment.

Since strong CAPM continues to hold for properly adjusted state prices, it is worth examining to what extent unadjusted state prices would deviate from strong CAPM. We investigate this question numerically. With the parametrization of the experiment, the difference in state prices is estimated to be uniformly less than 0.005, and hence, strict CAPM misprices securities only marginally.

\(^{10}\)If in equilibrium the risk premia of different assets are not very different from each other, the ratio \( \frac{\sum_{s'} \pi_{s'} x_{s'}^{i,0}}{\sum_{s'} p_s^{i,s'} x_{s'}^{i,0}} \) is approximately equal across investors even if they have different initial endowments. In our experiment we make sure that the initial endowment of all investors have approximately the same expected payoff (see experimental details below).
3 Details of the Experimental Setup

Timeline

The experiment consisted of a multi-period main session followed by a one-period end session which we shall refer to as “the terminal session.” The purpose of the end session was to eliminate possible unraveling of manager incentives. In what follows, we outline the design of the main session.

The main session was conducted in a series of six periods, one week long each. Table 1 gives a schematic overview of a single weekly period. One can discern three stages, called Asset Allocation (first stage), Trading (second stage), and Information Disclosure (third stage). Individuals who participated in the trading functioned as managers. The 32 managers remained the same individuals during all periods. A separate group of participants, functioning as investors, were the initial owners of assets and cash. The investors needed not be the same individuals each period and their number did change slightly, but equalled 70 on average.

Investors received their endowments in the beginning of each period. Their endowments consisted of units of two risky assets, called A and B, and some cash. The assets were risky because their end-of-period liquidating dividends (in US dollars) depended on the realization of a random state that could take three values, called X, Y, and Z. Investors could not buy, sell or store assets directly, and we asked them to assign all of their initial resources to one or more of the managers, who would subsequently trade on their behalf and collect the (liquidating) dividends.

The allocations from different investors to one manager constituted this manager’s initial portfolio for the period. An investor’s share in the manager’s fund was determined by the expected dividend on his contribution to the total expected dividend of the manager’s initial portfolio. Management fees were determined as a fixed fraction (40%) of the expected dividend of the contributed assets. The fee was back-end loaded, which means that it came out of the liquidating dividend of the manager’s final portfolio. If this dividend was insufficient, investors did not owe the fee; instead, we (the experimenters) paid out the managers.

The average fee per manager per period equaled $30. The dispersion of fees among managers varied across periods, and was most extreme in period 5, when a single manager collected over $200 in fees. As we shall see later, this is because of changes in concentration of allocation of funds to managers in response to the type of portfolios they historically had invested in, and to the performance record in the prior round.

\footnote{The original design aimed for eight periods in the main session followed by the terminal period. While we did implement the eight periods, we report here the results only of the first six periods. An accounting error in the seventh and eighth periods caused inaccurate reporting in the information disclosure stage. As such, periods 7 and 8 effectively became periods in the terminal session, which absorbed possible end-of-game effects in manager behavior.}
The allocation of assets from investors to managers constituted the first stage of a period. Subsequently, managers participated in the Trading Stage, which lasted exactly thirty minutes. During that stage, managers could trade through a web-based, electronic, anonymous, continuous open-book limit order system called jMarkets. A snapshot of the trading screen is provided on the experiment web site, http://clef.caltech.edu/exp/dp. To facilitate borrowing, in addition to trading securities A and B, managers could trade a risk-free security called a “Bond.” The Bond paid an end-of-period dividend of $1 in all states of the world (X, Y, Z) and was in zero net supply.

The final portfolio of a manager generated a liquidating dividend according to the random realization of a state variable that became known only after the conclusion of trading. The dividend, with the management fee subtracted from it, was distributed to the investors according to their shares in the fund. If this residual dividend was negative, the distribution to the investors was equal to $0, which means that investors had limited liability with respect to management fees. The average payoff to investors was $26 ($3.50 when the realization of the state variable was X, $42 when Y, and $23.50 when Z).

The third and final stage of a period was the information disclosure stage. In this stage a series of performance indicators for each manager were published on the experimental webpage as well as the weekly university newspaper. To preserve the privacy of the participants in the experiment, all managers were assigned pseudonyms used for all announcements (the list of pseudonyms can be found on the experimental webpage).

We reported four performance indicators for all managers as well as for an index called the Dow-tech Index, composed of one unit of asset A, one unit of asset B, and $1 cash. The following performance indicators were used: Portfolio Return, Market Share (called “Volume” in the published reports), Residual, and Risky Share. Portfolio return is the (pre-fee) final value (based on liquidating dividends) of a manager’s portfolio as a percentage of the value of the initial portfolio, where the latter value is based on average transaction prices in the period. Market Share is the ratio of the expected liquidating value of manager j’s initial portfolio and the expected liquidating value of the portfolio comprised of all assets and cash available to all investors. The Residual is the liquidating value of a manager’s portfolio (based on paid dividends) minus the fees paid to the manager (up to the liquidating portfolio value). The risky share provides an indication of the amount of risk the portfolio manager was taking. It takes the final portfolio and computes, at average transaction prices, the value of risky securities (A and B) as a fraction of the total value of the portfolio.

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12 This open-source trading platform was developed at Caltech and is freely available under the GNU license. See http://jmarkets.ssel.caltech.edu/. The trading interface is simple and intuitive. It avoids jargon such as “book,” “bid,” “ask,” etc. The entire trading process is point-and-click. That is, subjects do not enter numbers (quantities, prices); instead, they merely point and click to submit orders, to trade, or to cancel orders.

13 The experimental webpage, http://clef.caltech.edu/exp/dp/, contains further details, including experimental instruc-
Trading: Assets and Dividends

Table 2 summarizes the liquidating dividends of assets A, B, and the Bond, expressed in cents (all accounting in the experiment was done in U.S currency). The three payoff-relevant states, X, Y, and Z, were equally likely and this was known to both managers and investors.

In the tradition of CAPM experiments [7], we split investors in two groups ("types") and gave them different endowments. An investor of type A held 100 units of asset A, and $6 of cash, while an investor of type B held 70 units of asset B, and $9 of cash. Nobody started out with bonds. The expected payoff (sum of total dividends per state weighted by state probabilities) was the same to within $0.50 ($34.3 and $34.7, respectively). As mentioned in the theory section of this paper, equal expected payoffs are important for weak CAPM pricing to obtain.

There are good reasons why in our setup investors are endowed with securities instead of cash. First, investors who have different initial endowments face idiosyncratic risk beyond a mere difference in risk aversion. Idiosyncratic risk is an important element of our model and is generally an important reason for trade, both in the experimental setup, as well as in practice. Second, this is the only way for pricing to be endogenous. If investors were endowed with cash instead, then the assets would have to be supplied exogenously, e.g., by a market maker. The incentives of such exogenous supplier of assets would be different from the incentives of other market participants, and the analysis would then no longer be one of general equilibrium but one of partial equilibrium.

Managers did not have initial endowments. They could trade only with what they received from investors.

The market portfolio is the aggregate endowment of assets A and B. Because the total number of investors as well as the fraction of investors of each type varied from one period to the next, the composition of the (per capita) market portfolio also varied. Table 3 provides period-by-period details on the distribution of investor types and the corresponding market portfolio composition as well as the resulting aggregate wealth (dividend payments) across states. As one can discern, the aggregate wealth was always highest in state Z and lowest in state X.

In the asset allocation process, an investor could choose the number of units of his risky asset (A or B, depending on the investor’s type) to allocate to each manager. If a manager was allocated a fraction of an investor’s risky portfolio, the same fraction of the investor’s cash was also allocated...
to that manager. Investors distributed holdings to managers by filling out a form over the Internet. Before trading started, each manager knew her initial portfolio but not the portfolios of the other managers.

During the Trading Stage, managers could also trade a risk-free security called Bond, alongside securities A and B. Because of the presence of cash, the bond was a redundant security. However, managers were allowed to short sell the Bond if they wished. Short sales of the Bond corresponded to borrowing. Managers could thus exploit short sales to lever their purchases of assets A or B. Managers were also allowed to short sell the risky securities. To avoid bankruptcy (and in accordance with classical general equilibrium theory), our trading software constantly checked subjects’ budget constraints. In particular, a bankruptcy rule was used to prevent managers from committing to trades (submitting limit orders) that would imply negative cash holdings at the end of the period.\footnote{Whenever a manager attempted to submit an order, her cash holdings after dividends were computed for all states, given current asset holdings and outstanding orders that were likely to trade (an order was considered likely to trade if its price was within 20\% of the last transaction price), including the order she was submitting. If these hypothetical cash holdings turned out to be negative for some state, the order was automatically canceled and the trader was informed that her order did not go through because of risk of bankruptcy. However, in trading sessions when prices changed a lot, it was possible that orders that originally passed the bankruptcy check (because they were not considered likely to trade) eventually were executed and led to negative value of final holdings. Thus, despite the best attempts to avoid bankruptcy, rates of return at times were below -100\%.}

4 Results

We shall focus on the two main theoretical predictions, regarding (i) investor preferences for managers who offer portfolios that mimic AD securities, and (ii) weak-CAPM pricing.

Before we turn to the relevant results, we offer here some descriptive statistics, namely, asset turnover and state realizations. Table 4 suggests that asset turnover was high (often more than 50\% of the outstanding securities changed hands in the inter-manager market). Across the six periods, each state was realized at least once, with the worst state (lowest aggregate dividend; state X) being drawn in periods 3 and 6.

**Investors’ Choice of Managers**

According to our theory, in equilibrium, investors allocate funds across managers who offer portfolios that mimic AD securities. We shall determine to what extent this preference is revealed in investor choices by studying manager market shares and correlating those with a variable that measures how far manager portfolios were from AD securities.

First consider individual manager market shares, i.e., percentage of total value of all assets received by a manager, where value is determined by expected liquidating dividends. In the first round,
investors allocated their initial endowment to managers very much randomly, so that each manager received an approximately equal share (1/32, or about 3%). This reflects absence of knowledge of the investment plans of the managers. Only in period 2 could investors gauge the intentions of managers, by observing past performance (return in period 1), as well as the share of risky assets in managers’ final portfolios in period 1, as reported on the experiment web site as well as in the university newspaper. Already in period 2, investors preferentially allocated their wealth to a few managers. This pattern continued over time, and led to relatively high concentration of investor wealth among a few portfolio managers. Indeed, Table 5 confirms that both the market share of the largest manager (in terms of value of assets under management) and the Gini index of concentration increased in period 2 and remained high (double-digit market share of the largest managers, and Gini Indices around 0.50) throughout the remainder of the main session.

Of course, the main issue is whether these preference patterns reflect the theoretical prediction that investors prefer to go with managers who invest in portfolios with payoffs that mimic those of AD securities. That is, were managers who attracted a disproportionate share of the flow of funds also the ones that invested in portfolios with AD-like payoffs? To verify this, we ran a number of regressions with, as dependent variable, manager market share, and as explanatory variables: (i) a measure of distance of final manager portfolio holdings from an AD security in the prior period, (ii) prior-period portfolio return. The former, called LagDistanceAD, was measured as follows. Let \( \varsigma \) denote the state where the final manager portfolio paid the highest dividend, and let \( \hat{\varsigma} \) denote the collection of remaining states. LagDistanceAD was computed as the ratio of the (state-price-weighted) average total dividend in all states in \( \hat{\varsigma} \) over the total dividend (times the state price) in state \( \varsigma \). This measure is minimal (0) when the manager perfectly mimics the payoff on AD security \( \varsigma \). It increases as managers buy portfolios that pay more total dividend in states in \( \varsigma \).

Many variations of this base regression were tried, slightly varying the definition of the distance of a manager’s final portfolio payoff pattern from an AD security, adding potentially confounding factors such as payoff variance (across states) of the manager’s final portfolio in the previous period, the expected payoff (across states) of the manager’s final portfolio, and period dummies. While addition of confounding factors at times had an effect on the significance of the effects of the two main explanatory variables [(i) and (ii) above], the signs remained robust.

Table 6 displays the regression results. Consistent with the theory, the distance of a manager’s

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\[ G = \frac{(I + 1 - 2 \times r)}{I} \]

...
portfolio from mimicking an AD security (“LagDistanceAD”) has a highly significant ($p < 0.001$ univariate and $p = 0.003$ multivariate) negative effect on a manager’s market share, suggesting that investors preferred to allocate their wealth across managers who offered portfolios that looked more like AD securities, and hence, exploited investors’ limited liability for the manager fee. The regression results are based on 31 (number of managers minus 1) times 5 (number of periods minus 1), i.e., 155, observations.

Table 6 also shows that the realized return in the previous period (“LagReturn”) has a significantly positive effect ($p < 0.001$) on market share. There are a number of interpretations of this result.

First, lagged return could be a way for investors to resolve indifference between two managers who offered portfolios with the same distance from an AD security. There is no a priori reason to believe in differential managerial skill in our experimental setting, but investors may have taken lagged return as an indicator that there was, and may have allocated funds accordingly. This phenomenon would be consistent with the theory of (and evidence of real-world) fund flows in [4].

Second, although less likely, absent transparent information about the precise composition of a manager’s portfolio (we only report the weight on risky securities), lagged return could be another, albeit noisy, indicator of how far a manager’s portfolio was from mimicking an AD security. Indeed, the correlation between our measure of distance (“LagDistanceAD”) and the lagged return, is a significant -0.22 ($p = 0.003$). In our experiment this correlation is explained by the realization of the richest state (state $Y$) in three out of five lagged periods, leading to a strong positive correlation between being an AD security and obtaining a large return. As such, “LagReturn” provided information about “LagDistanceAD.”

**Investor and Manager Holdings**

Our model predicts that fee-adjusted prices and endowments in the presence of delegated investment still satisfy CAPM properties. We therefore expect the state-dependent (risky) part of investors’ portfolio to be proportional to the market portfolio. One of the main points illustrated by our model is that because investors can hold diversified portfolios of contracts offered by managers, investor holdings satisfy CAPM properties while manager holdings need not. With limited liability of investors, we actually expect managers to hold AD securities, which is very different from holding the market. Figure 3 illustrates the extent to which this important result is verified in the experiment. It displays the histogram of the ratio of wealth in state $Y$ to the total wealth across all states for both investors and managers, given investors’ shares in the managers’ funds and given these funds’ post-fee payoffs in every state. For both groups of subjects, the wealth in every state takes into account
limited liability and truncates at zero in states where post-fee payoff would have been negative.\textsuperscript{17}

As is clear from Figure 3, not all investors hold the market portfolio, but the histogram of relative final wealth is unimodal, centered around the (post-fee) market portfolio. This replicates the findings in past asset pricing experiments where investors trade assets directly (e.g., [7, 9]). The coexistence of CAPM prices and a unimodal distribution of investor holdings centered at the market portfolio is theoretically founded by a variant of the CAPM that relaxes the assumption of quadratic utility, allowing for individual deviations that wash out in aggregate (CAPM+$\varepsilon$, [9]). Figure 3 also shows, importantly, that this unimodal distribution of investor final wealth, which is consistent with CAPM, is maintained to a large extent across all periods of the experiment. A significant deviation from a unimodal distribution centered at the market portfolio is observed only in period 5, coinciding with the observed violation of weak CAPM in price predictions (this is in agreement with the theory of CAPM+$\varepsilon$ mentioned before).

On the other hand, the distribution of final relative post-fee wealth for managers changes over the course of the experiment. Already in the second period a mode at 1 emerges. This means some managers quickly start offering contracts that mimic state-\textit{Y} AD securities. As periods pass, the histogram becomes more dispersed, closer to bimodal (second mode at 0). This is a very sharp illustration of the intuition lying at the base of our theory: investors achieve their portfolio goals by mixing different managers, whom they choose based on their ability to handle costs (fees). In our setup, managers’ correct reply to investors’ objectives is to become polarized (mimic AD securities), which they do, while prices still satisfy weak CAPM.

At the same time as managers become more polarized, investors reduce the number of managers they include in their portfolio. The median investor holds 18.5 managers in the first period – basically, investors randomly allocate their assets, 8 in the third period, and 6 in the last period. Our theory indicates that three managers would suffice to obtain any desired portfolio. There are several reasons to not expect this number to arise in our experimental implementation, not least of all, that managers – unlike in the theory – do not announce the contracts they will offer but rather that investors deduce them from past performance.

**Pricing**

According to the theory, prices in the inter-manager market should satisfy the weak version of the CAPM, which predicts that state prices, normalized by state probabilities, should be inversely ranked to aggregate wealth in the states. As such, state-price probability ratios should be highest in state

\textsuperscript{17}As previously stated, we are interested only in the risky part of these portfolios, which in the case of investors is expected to be proportional to the market portfolio. We cannot determine this part precisely, but for normal levels of risk aversion, the total portfolio plotted in Figure 3 serves as a good proxy.
$X$, where aggregate wealth is lowest (see Table 3), and lowest in state $Y$, where aggregate wealth is highest. Aggregate wealth is computed from the dividends on the outstanding assets ($A, B,$ and Bonds), added to the total cash outstanding.

Figure 4 provides time series plots of transaction prices, per period. Prices of risky securities ($A$ and $B$) are close to, and with few exceptions, below, expected payoffs (indicated with solid horizontal lines). The fact that prices are different from expectations suggests risk aversion. With risk aversion, weak CAPM predicts that when prices of traded securities are converted to prices of AD securities, their ranking (adjusted for state probabilities) should be inverse to aggregate wealth across states.

Figure 5 provides evidence. Plotted are the time series of state-price probability ratios for the three states, implied from trade prices. While the evidence looks mixed at first, late in periods 2, 3, 5 and 6, state-price probability ratios are ranked exactly as predicted by weak CAPM: the ratio for state $X$ is highest, and that for state $Y$ is lowest.

While not immediate from Figure 5, there is a statistically significant tendency for state-price probabilities to revert back to the right ranking (according to weak CAPM) when the ranking is incorrect. This is determined as follows. Following [7], we construct a statistic, called $\tau$, that measures the frequency with which state-price probability ratios move in the right direction to restore ranking according to weak CAPM when ranking is incorrect. Specifically, $\tau$ is the average across transactions of a variable that assigns a value of 1 to a change in state prices in the direction of weak CAPM, and 0 otherwise. For each possible ranking of the state-price probabilities after transaction $t$, Table 7 presents the required subsequent changes (at transaction $t + 1$) for the indicator variable determining $\tau$ to take the value of 1. To avoid bias, we eliminate observations where prices did not change (these would otherwise automatically be assigned a value of 1).

To determine correct rejection levels for $\tau$ at the 5\% and 1\% significance levels in each trading period under the assumption that state-price probabilities change randomly, we bootstrapped the original time series of transaction prices (period by period for assets $A, B$ and bond) after subtracting price drifts (so price series become martingales), inverted the resulting bootstrapped prices for state-price probability ratios, and constructed 200 series of levels and subsequent changes of state-price probability ratios with the same length as the original (period) series. For each of these 200 series, we computed the $\tau$ statistics and then determined the critical level of $\tau$ so that 5\% (1\%) of the outcomes were above this level, thus obtaining cut-off levels at $p = 0.05$ (and $p = 0.01$ respectively) for each of the six trading periods.

Table 8 displays the results. Shown are $\tau$ statistics for all periods, and corresponding critical values at the 5\% and 1\% levels. The null of no drift is rejected at the 1\% in all but one (5th) period, where $\tau$ does not even reach the 5\% cut-off level. Overall, Table 8 provides evidence that is difficult
to discern from Figure 5, namely, that prices tend to change in the direction of weak CAPM in the instances where they do not conform to this theory.

One of the reasons why weak CAPM does not always obtain is because there is substantial concentration in fund wealth after the first period. Table 5 showed that the Gini index of concentration could be as high as 0.55, and the market share of the largest manager as high as 20%. As such, idiosyncracies (relative to CAPM demands) of a few managers may have substantial effects on prices. This is further exacerbated here because successful managers do not buy diversified portfolios, but instead trade to positions that are closer to AD securities (Table 6). For CAPM pricing to obtain robustly, it is known that individual idiosyncracies need to average out, and this can only occur if there are a sufficient number of participants all of whom only hold a small fraction of wealth [8].

Consistent with the hypothesis that concentration has a detrimental effect on pricing, we discovered that the correlation between concentration measures and the $\tau$ statistic is negative, albeit only marginally significant. The correlation across periods with the Gini concentration index is -0.51 (standard error 0.30); that with the market share of the largest manager is -0.42 (standard error 0.40). When computing $\tau$ based on all observations, including those where prices do not change, these correlations become significant: -0.54 (standard error 0.26) and -0.58 (standard error 0.23), respectively. As such, some of the poor pricing should be attributed to market concentration, which itself is the consequence of investors’ preferences for managers who (i) acquire undiversified, AD security-like portfolios, (ii) had high returns in the prior period.\textsuperscript{18}

Because fund shares could not be assigned according to the value of assets contributed by individual investors, only weak CAPM obtained theoretically. Numerical analysis based on the parametrization of the experiment demonstrates, however, that mispricing relative to the original (strong) CAPM should be minor.

The strong CAPM predicts that the market portfolio should be mean-variance optimal. An effective way to gauge the distance of the market portfolio from optimality in mean-variance space is to compute the difference between the Sharpe ratio of the market portfolio and that of the maximal (mean-variance optimal) portfolio (see [7] for an early application of this metric in the context of price data from experimental markets). One can re-compute this Sharpe ratio difference after each transaction and take the average across all transactions within a period.

If we then compare the average Sharpe ratio difference with $\tau$ (which measures the extent to

\textsuperscript{18}Some direct evidence that large managers influence prices in an adverse way comes from the following. Asset B’s price was significantly larger in periods 4 and 5 than in all other periods (up to $0.39$ in period 4 and $0.38$ in period 5). A detailed look at trade in the last 10 minutes reveals that in period 4 the two largest managers, jointly holding approx. 35% market share, bought asset B (approx. 400 units), with only one large manager on the sell side (approx. 8% market share). In period 5 the largest manager, holding about 21% market share again bought (450 units) of asset B, with no large managers on the sell side. A speculator manager, who played that role in every period, somewhat palliated asset B overpricing by accumulating stocks early on for resale to the largest managers.
which prices move in the direction of weak CAPM), we observe a correlation of 0.34 (standard error 0.51). When computing $\tau$ based on all observations, including those where prices do not change, this correlation becomes significant: 0.55 (standard error 0.25). This illustrates that our conclusions about pricing in the delegated portfolio management experiment would have been qualitatively the same whether we had used the original (strong) version of the CAPM (focusing on mean-variance optimality of the market portfolio) or its weak counterpart (focusing on rankings of state-price probability ratios).

5 Conclusion

In a break with tradition in the analysis of delegated portfolio management, we here developed a theory that focused on competition in compositions of portfolios offered by managers to investors rather than on the structure of management fees. We assumed that investors have limited liability for manager fees, which were charged on initial funds under management, but paid out of final portfolio value. This aligned managers’ incentives with those of investors: managers (themselves subject to limited liability) will take risks by offering portfolios that mimic AD securities, and investors prefer this. In our setup, because investors could hold portfolios of contracts with different managers, manager polarization was more convenient to investors than having a set of perfectly-diversified managers matching each investor’s preferences.

We designed and ran a large-scale experiment involving approximately 100 subjects over multiple weeks and studied manager portfolio choices, investor fund allocation decisions, and pricing in the inter-manager market. The two main predictions of the theory were confirmed, namely: (i) investors preferred managers that offered portfolios that came closer to mimicking the payoff on AD securities; (ii) prices tended to levels that were in accordance with weak CAPM, which predicts that state-price probability ratios are ranked inversely with aggregate wealth across states.

We did observe an effect of realized returns on subsequent fund flows, as in field data [4]. The resulting concentration of funds led to a deterioration of price quality; all else equal, prices reflected weak CAPM to a lesser extent than when funds were distributed more evenly across managers.

Our experiment should be considered a proof of concept: it shows that it is possible to run meaningful experiments on delegated portfolio management in the laboratory. Part of our set-up was unrealistic, but this was done deliberately in order to obtain precise tests of the theory. Further experiments should relax some of our assumptions. In particular, we assumed a very simple management fee structure, focusing on competition in portfolio offerings. One can imagine theory and tests about joint determination of fee and portfolio structure. We leave this for future work.
References


Tables and Figures

Table 1: Timeline For One Week-Long Period

<table>
<thead>
<tr>
<th>Wed</th>
<th>Fri</th>
<th>Sat</th>
<th>Mon</th>
<th>Tue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investors and managers informed of payoffs</td>
<td>Performance indices published on website</td>
<td>Sign-up announcement for investors</td>
<td>Performance indices published in Tech</td>
<td>6pm Close of investor allocation stage</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Only First Week) Sign up for managers</td>
<td>Investors receive access to allocation software</td>
<td>10pm Managers see allocations and trading starts</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10:30pm Trading ends</td>
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Table 2: State-Dependent Liquidating Dividends (in US Cents Per Unit).

<table>
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<th>Y</th>
<th>Z</th>
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<tr>
<td>Asset A</td>
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<tr>
<td>Asset B</td>
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<tr>
<td>Bond</td>
<td>100</td>
<td>100</td>
<td>100</td>
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Table 3: Number of Participants by Type and Corresponding Per Capita Market Portfolio and Aggregate Wealth (Dividend) Per State (in dollars).

<table>
<thead>
<tr>
<th>Period</th>
<th>Number of Participants</th>
<th>Per Capita Market Portfolio</th>
<th>Aggregate Wealth Per State</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Type A</td>
<td>Type B</td>
<td>A</td>
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<tr>
<td>1 (061017)</td>
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<td>6 (061121)</td>
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</tr>
</tbody>
</table>

Table 4: Asset Turnover And State Realization (Per Period)

<table>
<thead>
<tr>
<th>Period</th>
<th>Turnover: Asset A</th>
<th>Turnover: Asset B</th>
<th>State Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (061017)</td>
<td>0.37</td>
<td>0.40</td>
<td>Z</td>
</tr>
<tr>
<td>2 (061024)</td>
<td>0.53</td>
<td>0.35</td>
<td>Y</td>
</tr>
<tr>
<td>3 (061030)</td>
<td>0.45</td>
<td>0.53</td>
<td>X</td>
</tr>
<tr>
<td>4 (061107)</td>
<td>0.47</td>
<td>0.56</td>
<td>Y</td>
</tr>
<tr>
<td>5 (061114)</td>
<td>0.48</td>
<td>0.69</td>
<td>Y</td>
</tr>
<tr>
<td>6 (061121)</td>
<td>0.57</td>
<td>0.78</td>
<td>X</td>
</tr>
</tbody>
</table>

*aAsset turnover is calculated by dividing the trading volume by the total number of units outstanding.

Table 5: Market Concentration (Gini Index) and Market Share of the Largest Manager

<table>
<thead>
<tr>
<th>Period</th>
<th>Gini Index</th>
<th>Largest Manager Market Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (061017)</td>
<td>0.1334</td>
<td>0.0524</td>
</tr>
<tr>
<td>2 (061024)</td>
<td>0.5039</td>
<td>0.1236</td>
</tr>
<tr>
<td>3 (061030)</td>
<td>0.3434</td>
<td>0.1032</td>
</tr>
<tr>
<td>4 (061107)</td>
<td>0.4905</td>
<td>0.1995</td>
</tr>
<tr>
<td>5 (061114)</td>
<td>0.4978</td>
<td>0.2034</td>
</tr>
<tr>
<td>6 (061121)</td>
<td>0.5491</td>
<td>0.1342</td>
</tr>
</tbody>
</table>

Table 6: Manager Market Share Regressed On Lagged Distance Of Offered Portfolio From AD Security (Top) And On This Lagged Distance Plus Lagged Return (Bottom).

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t value</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.047</td>
<td>0.006</td>
<td>7.77</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>LagDistanceAD</td>
<td>-0.020</td>
<td>0.004</td>
<td>-5.33</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.023</td>
<td>0.003</td>
<td>6.91</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>LagDistanceAD</td>
<td>-0.010</td>
<td>0.003</td>
<td>-3.24</td>
<td>0.003</td>
</tr>
<tr>
<td>LagReturn</td>
<td>0.023</td>
<td>0.002</td>
<td>10.43</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>
Table 7: List Of Changes That Suggest State-Price Probability Rankings Drift Back To Satisfying Weak CAPM.

<table>
<thead>
<tr>
<th>Ranking at $t$</th>
<th>Correct Subsequent Change In State-Price Probability Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_X(t) &gt; u_Z(t) &gt; u_Y(t)$</td>
<td>Any Change</td>
</tr>
<tr>
<td>$u_X(t) &gt; u_Y(t) &gt; u_Z(t)$</td>
<td>$u_Z(t + 1) - u_Y(t + 1) \geq u_Z(t) - u_Y(t)$</td>
</tr>
<tr>
<td>$u_Z(t) &gt; u_X(t) &gt; u_Y(t)$</td>
<td>$u_X(t + 1) - u_Z(t + 1) \geq u_X(t) - u_Z(t)$</td>
</tr>
<tr>
<td>$u_Z(t) &gt; u_Y(t) &gt; u_X(t)$</td>
<td>$u_X(t + 1) - u_Y(t + 1) \geq u_X(t) - u_Y(t)$</td>
</tr>
<tr>
<td>$u_Y(t) &gt; u_X(t) &gt; u_Z(t)$</td>
<td>$u_X(t + 1) - u_Z(t + 1) \geq u_X(t) - u_Z(t)$</td>
</tr>
<tr>
<td>$u_Y(t) &gt; u_Z(t) &gt; u_X(t)$</td>
<td>$u_X(t + 1) - u_Y(t + 1) \geq u_X(t) - u_Y(t)$</td>
</tr>
</tbody>
</table>

Table 8: Tests Of Whether Transaction Prices Follow A Martingale ($H_0$) Against Drift Towards Satisfying Weak CAPM ($H_1$).

<table>
<thead>
<tr>
<th>Period</th>
<th>$\tau$</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5%</td>
</tr>
<tr>
<td>1 (061017)</td>
<td>0.73</td>
<td>0.52</td>
</tr>
<tr>
<td>2 (061024)</td>
<td>0.69</td>
<td>0.38</td>
</tr>
<tr>
<td>3 (061030)</td>
<td>0.83</td>
<td>0.60</td>
</tr>
<tr>
<td>4 (061107)</td>
<td>0.68</td>
<td>0.52</td>
</tr>
<tr>
<td>5 (061114)</td>
<td>0.72</td>
<td>0.98</td>
</tr>
<tr>
<td>6 (061121)</td>
<td>0.68</td>
<td>0.29</td>
</tr>
</tbody>
</table>
Figure 1: When acquiring the optimal allocation \( a \) through managers offering AD securities 1 and 2, the derived demand (including fees) of the managers in the inter-manager market lies on the line of all optimal portfolios (red), and hence, is itself optimal.

Figure 2: The optimal allocation \( a \) can be obtained, gross of fees, through a convex linear combination \( a' \) of managers offering AD securities 1 and 2. Blue dashed lines indicate fees charged. The gross-of-fee allocation \( a' \) can also be obtained through a convex linear combinations of managers offering 1 and E. Post-fee (i.e., net of fees), however, the latter produces a strictly inferior allocation, because the fees required in each state (green dashed lines) are strictly larger. Investors will prefer combining offerings 1 and 2.
Figure 3: Histogram of final holdings of wealth in state $Y$ as a proportion of the sum of wealth in all states for investors (after fee, left column) and for managers after fee, with a floor at zero (right column).
Figure 4: Time series of transaction prices, per period. Horizontal lines indicate expected payoffs.
Figure 5: Time series of state-price probabilities, per period. Weak CAPM predicts that the state-price probability ratios should be highest for state X (when aggregate wealth is lowest – see Table 3), lowest for state Y (where aggregate wealth is highest). State-price probability ratios of X are indicated with red arrows pointing up; those for Y are displayed with blue arrows pointing down; and those for Z with black errors pointing sideways. State prices are implied from transaction prices (see Figure 4) after each trade, based on the most recent transaction prices for each security, and on inversion using the payoff matrix (Table 2).