Outline

1. Nonsequential vs. sequential search
2. Nonsequential search model
3. Nonsequential search
4. Sequential search model
Why are prices for the same item so different across stores?

A puzzle considering basic economic theory: review this. Consider the benchmark of *perfect competition*.
Leaving the PC world

- One important implicit assumption of PC paradigm is that consumers are aware of prices at all stores. This implies an infinitely elastic demand curve facing firms. (ie. if one firm raises prices slightly, he will lose all demand).

- Obviously, this assumption is not realistic. Here we consider what happens, if we relax just this assumption, but maintain other assumptions of PC paradigm: large #firms, perfect substitutes, etc.
Search model

- Each consumer demands one unit;
- Starts out at one store, incurs cost $c > 0$ to search at any other store.
- Consumer only knows prices at stores that she has been to, and buys from the canvassed store with the lowest price. “free recall”
- Utility $u$ from purchasing product: demand function is

\[
\begin{cases} 
  \text{purchase if } p \leq u \\
  \text{don’t purchase otherwise}
\end{cases}
\]

(1)

- What is equilibrium in this market?
Diamond paradox

- Claim: a nonzero search cost $c > 0$ leads to equilibrium price equal to $u$ ("monopoly price")
- Assume that marginal cost=0, so that under PC, $p = 0$
- $n$ firms, with $n$ large. Consumers equally distributed initially among all firms.
- Start out with all firms at PC outcome. What happens if one firm deviates, and charges some $p^1$ such that $0 < p^1 < c$?
  - Consumers at this store?
  - Consumers at other stores?
  - How will other stores respond?
  - By iterating this reasoning ....
Now start at “monopoly outcome”, where all firms are charging $u$.

- What are consumers’ purchase rules?
- Do firms want to undercut? Given consumer behavior, what do they gain?
- Role for advertising?
Diamond result quite astounding, since it suggests PC result is “knife-edge” case.

But still doesn’t explain price dispersion

Assume consumers differ in search costs

Two types of consumers: “natives” are perfectly informed about prices, but “tourists” are not.
Tourist-natives model

- Tourists and natives, in proportions $1 - \alpha$ and $\alpha$. $L$ total consumers (so $\alpha L$ natives, and $(1 - \alpha)L$ tourists).
- Tourists buy one unit as long as $p \leq u$, but natives always shop at the cheapest store.
- Each of $n$ identical firms has U-shaped AC curve
- Each firm gets equal number of tourists $\left(\frac{(1-\alpha)L}{n}\right)$; natives always go to cheapest store.
- Consider world in which all firms start by setting $p^c = \min_q AC(q)$.
- Note that deviant store always wants to price higher. Demand curve for a deviant firm is kinked (graph). Deviant firm sells exclusively to tourists.
Deviant firm will always charge $u$. Only tourists shop at this store. If charge above $u$, no demand. If below $u$, then profits increase by charging $u$.

**First case:** many informed consumers ($\alpha$ large)

- Number $q^u \equiv \frac{(1-\alpha)L}{n}$ of tourists at each store so small that $u < AC(q^u)$.
- In free-entry equilibrium, then, all firms charge $p^c$, and produce the same quantity $L/n$.
- If enough informed consumers, competitive equilibrium can obtain (not surprising)
**Second case:** few informed consumers ($\alpha$ small)

- Assume enough tourists so that $u > AC(q^u)$.
- But now: hi-price firms making positive profits, while lo-price firms making (at most) zero profits. Not stable.
- In order to have equilibrium: ensure that given a set of high-price firms (charging $u$) and low-price firms (charging $p^c$), no individual firm wants to deviate. Free entry ensures this.
- Let $\beta$ denote proportion of lo-price firms.
- Each high-price firm charges $u$ and sells an amount

$$q^u = \frac{(1 - \alpha)L(1 - \beta)}{n(1 - \beta)} = \frac{(1 - \alpha)L}{n} \quad (2)$$

- Each low-price firm charges $p^c$ and sells

$$q^c = \frac{\alpha L + (1 - \alpha)L\beta}{n\beta} \quad (3)$$
In equilibrium, enough firms of each type enter such that each firm makes zero profits. Define quantities $q^a, q^A$ such that (graph):

$$AC(q^a) = u; \quad AC(q^c) = p^c.$$  

(Quantities at which both hi- and lo-price firms make zero profits.)

With free entry, $n$ and $\beta$ must satisfy

$$q^a = q^u = \frac{(1 - \alpha)L}{n} AC(q^c); \quad q^A = q^c \frac{\alpha L + (1 - \alpha) L \beta}{n \beta} \quad (4)$$

Solving the two equations for $n$ and $\beta$ yields

$$n = \frac{(1 - \alpha)L}{q^a}; \quad \beta = \frac{\alpha q^a}{(1 - \alpha)(q^A - q^a)} \quad (5)$$

N.B: arbitrary which firms become high or low price. Doesn’t specify process whereby price dispersion develops.

As $\alpha \to 0$, then $\beta \to 0$ (Diamond result)
Consider two search models:


2. Sequential search model: consumer decides after each search whether to buy at current store, or continue searching.
Main assumptions:

- Infinite number ("continuum") of firms and consumers
- Observed price distribution $F_p$ is equilibrium mixed strategy on the part of firms, with bounds $p, \bar{p}$.
- $r$: constant per-unit cost (wholesale cost), identical across firms
- Firms sell homogeneous products
- Each consumer buys one unit of the good
- Consumer $i$ incurs cost $c_i$ to search one store; drawn independently from search cost distribution $F_c$
- First store is "free"
- $q_k$: probability that consumer searches $k$ stores before buying
Consumers in nonsequential model

- Consumer with search cost $c$ who searches $n$ stores incurs total cost

$$c \times (n - 1) + E[\min(p_1, \ldots, p_n)]$$

$$= c \times (n - 1) + \int_{\bar{p}}^{\tilde{p}} p \cdot n(1 - F_p(p))^{n-1} f_p(p) dp.$$

(6)

- This is decreasing in $c$. Search strategies characterized by cutoff-points, where consumer indifferent between $n$ and $n + 1$ must have cost

$$c_n = E[\min(p_1, \ldots, p_n)] - E[\min(p_1, \ldots, p_{n+1})].$$

and $c_1 > c_2 > c_3 > \cdots$.

- Similarly, define $\tilde{q}_n = F_c(c_{n-1}) - F_c(c_n)$ (fraction of consumers searching $n$ stores). Graph.
FIGURE 2
IDENTIFICATION SCHEME FOR SEARCH-COST DISTRIBUTION IN NONSEQUENTIAL-SEARCH MODEL

Let $\hat{F}_p$ denote the empirical distribution of the observed prices. First, we note that we can obtain estimates of these indifference points from the empirical price distribution $\hat{F}_p$, via the relation (2). Second, define $\tilde{q}_1 \equiv 1 - F_c(\Delta_1)$: the proportion of consumers with one price quote; $\tilde{q}_2 \equiv F_c(\Delta_1) - F_c(\Delta_2)$: the proportion of consumers with two price quotes; $\tilde{q}_3 \equiv F_c(\Delta_2) - F_c(\Delta_3)$: the proportion of consumers with three price quotes.

We can estimate $\tilde{q}_1, \tilde{q}_2, \ldots$ by exploiting the firms' equilibrium pricing conditions. To see this, note that a firm's profits from following the mixed pricing strategy $F_p(\cdot)$ are (see Burdett and Judd, 1983)

$$\Pi_1(p) = (p - r) \left[ \sum_{k=1}^{\infty} \tilde{q}_k (1 - F_p(p))^k - 1 \right]$$

for all $p \in [\hat{p}, \hat{p}]$. The characterization of the equilibrium price distribution starts with the mixed-strategy condition that firms be indifferent between charging the monopoly price $p$ (and selling only to people who never search but receive an initial free draw equal to $p$) and any other price $p$ in the equilibrium support $[\hat{p}, \hat{p}]$:

$$\left( p - r \right) \tilde{q}_1 = \left( p - r \right) \left[ \sum_{k=1}^{\infty} \tilde{q}_k (1 - F_p(p))^k - 1 \right].$$

The optimality equation (4) allows us to recover a nonparametric estimate of the search-cost distribution $F_c$ from $\hat{F}_p$ alone, as we now show. Let $\hat{p}$ and $\hat{p}$ denote the lowest and highest observed prices, respectively. For convenience, we index the $n$ observed prices in ascending order, so that $\hat{p} = p_1 \leq p_2 \leq \cdots \leq p_{n-1} \leq p_n = \hat{p}$.

Let $K (\leq n - 1)$ denote the maximum number of firms from which a consumer obtains price quotes in this market. Given this condition, the indifference condition (equation (4)) for each of

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Firms in nonsequential model

- Firm’s profit from charging $p$ is:

$$
\Pi(p) = (p - r) \left[ \sum_{k=1}^{\infty} \tilde{q}_k \cdot k \cdot (1 - F_p(p))^{k-1} \right], \quad \forall p \in [\underline{p}, \bar{p}]
$$

- For mixed strategy, firms must be indifferent btw all $p$:

$$
(\bar{p} - r)\tilde{q}_1 = (p - r) \left[ \sum_{k=1}^{\infty} \tilde{q}_k \cdot k \cdot (1 - F_p(p))^{k-1} \right], \quad \forall p \in [\underline{p}, \bar{p}) \quad (7)
$$
Estimating search costs

- Observe data $P_n \equiv (p_1, \ldots, p_n)$. Sorted in increasing order.
- Empirical price distribution $\hat{F}_p = \text{Freq}(p \leq \tilde{p}) = \frac{1}{n} \sum_i 1(p_i \leq \tilde{p})$.
- Take $\underline{p} = p_1$ and $\bar{p} = p_n$.
- Consumer cutpoints $c_1, c_2, \ldots$ can be estimated directly by simulating from observed prices $P_n$. This are “absissae” of search cost CDF.
- Corresponding “ordinates” recovered from firms’ indifference condition.
  Assume that consumers search at most $K (< N - 1)$ stores. Then can solve for $\tilde{q}_1, \ldots, \tilde{q}_K$ from

$$
(\bar{p} - r)\tilde{q}_1 = (p_i - r) \left[ \sum_{k=1}^{K-1} \tilde{q}_k \cdot k \cdot (1 - \hat{F}_p(p_i))^{k-1} \right], \quad \forall p_i, i = 1, \ldots, n - 1.
$$

$n - 1$ equations with $K$ unknowns.
Nonsequential search model

TABLE 2  Search-Cost Distribution Estimates for Nonsequential-Search Model

<table>
<thead>
<tr>
<th>Product</th>
<th>$K^a$</th>
<th>$M^b$</th>
<th>$\tilde{q}_1^c$</th>
<th>$\tilde{q}_2$</th>
<th>$\tilde{q}_3$</th>
<th>Selling Cost $r$</th>
<th>MEL Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stokey-Lucas</td>
<td>3</td>
<td>5</td>
<td>.480 (.170)</td>
<td>.288 (.433)</td>
<td></td>
<td>49.52 (12.45)</td>
<td>102.62</td>
</tr>
<tr>
<td>Lazear</td>
<td>4</td>
<td>5</td>
<td>.364 (.926)</td>
<td>.351 (.660)</td>
<td>.135 (.692)</td>
<td>27.76 (8.50)</td>
<td>84.70</td>
</tr>
<tr>
<td>Billingsley</td>
<td>3</td>
<td>5</td>
<td>.633 (.944)</td>
<td>.309 (.310)</td>
<td></td>
<td>69.73 (68.12)</td>
<td>199.70</td>
</tr>
<tr>
<td>Duffie</td>
<td>3</td>
<td>5</td>
<td>.627 (1.248)</td>
<td>.314 (.195)</td>
<td></td>
<td>35.48 (96.30)</td>
<td>109.13</td>
</tr>
</tbody>
</table>

Parameter estimates and standard errors: nonsequential-search model

<table>
<thead>
<tr>
<th>Product</th>
<th>$\Delta_1$</th>
<th>$F_r(\Delta_1)$</th>
<th>$\Delta_2$</th>
<th>$F_r(\Delta_2)$</th>
<th>$\Delta_3$</th>
<th>$F_r(\Delta_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stokey-Lucas</td>
<td>2.32</td>
<td>.520</td>
<td>.68</td>
<td>.232</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lazear</td>
<td>1.31</td>
<td>.636</td>
<td>.83</td>
<td>.285</td>
<td>.57</td>
<td>.150</td>
</tr>
<tr>
<td>Billingsley</td>
<td>2.90</td>
<td>.367</td>
<td>2.00</td>
<td>.058</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duffie</td>
<td>2.41</td>
<td>.373</td>
<td>1.42</td>
<td>.059</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Search-cost distribution estimates

- $^a$ Number of quantiles of search cost $F_r$ that are estimated (see equation (5)). In practice, we set $K$ and $M$ to the largest possible values for which the parameter estimates converge. All combinations of larger $K$ and/or larger $M$ resulted in estimates that either did not converge or did not move from their starting values (suggesting that the parameters were badly identified).
- $^b$ Number of moment conditions used in the empirical likelihood estimation procedure (see equation (17)).
- $^c$ For each product, only estimates for $\tilde{q}_1, \ldots, \tilde{q}_{K-1}$ are reported; $\tilde{q}_K = 1 - \sum_{k=1}^{K-1} \tilde{q}_k$.
- $^d$ Indifferent points $\Delta$ computed as $E p_{1:k} - E p_{1:k+1}$ (the expected price difference from having $k$ versus $k + 1$ price quotes), using the empirical price distribution. Including shipping and handling charges.
Sequential model

- Consumer decides after each search whether to accept lowest price to date, or continue searching.
- Optimal “reservation price” policy: accept first price which falls below some optimally chosen reservation price.
- NB: “no recall”
Consumers in sequential model

- Heterogeneity in search costs leads to heterogeneity in reservation prices
- For consumer with search cost $c_i$, let $z^*(c_i)$ denote price $z$ which satisfies the following indifference condition

$$c_i = \int_0^z (z - p)f(p)dp = \int_0^z F(p)dp.$$

Now, for consumer $i$, her reservation price is:

$$p_i^* = \min(z^*(c_i), \bar{p}).$$

- Let $G$ denote CDF of reservation prices, ie. $G(\bar{p}) = P(p^* \leq \bar{p}).$
Firms in sequential search model

- Again, firms will be indifferent between all prices.
- Let $D(p)$ denote the demand (number of people buying) from a store charging price $p$. Indifference condition is:

\[(\bar{p} - r)D(\bar{p}) = (p - r)D(p) \iff (\bar{p} - r) \times (1 - G(\bar{p})) = (p - r) \times (1 - G(p))\]

for each $p \in [\underline{p}, \bar{p})$. 
Observe prices $p_1, \ldots, p_n$ (order in increasing order, so $\bar{p} = p_n$).

Indifference conditions, evaluated at each price, are:

$$(\bar{p} - r) \cdot (1 - G(\bar{p})) = (p_i - r) \cdot (1 - G(p_i)), \quad i = 1, \ldots, n - 1$$

This gives $n - 1$ equations, but $n + 1$ unknowns: $G(p_i)$ for $i = 1, \ldots, n$ as well as $r$.

Define $\alpha = 1 - G(\bar{p})$: percentage of people who don’t search.

Assume that search distribution is Gamma distribution. (Eq. (13) in paper).

Estimate model parameters $(\delta_1, \delta_2, \alpha, r)$ by maximum likelihood (Eq. 9).
Results: sequential search model

TABLE 3 Estimates of Sequential-Search Model

<table>
<thead>
<tr>
<th>Product</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>Median$^a$</th>
<th>Selling Cost $r$</th>
<th>$\alpha^b$</th>
<th>$F_c^{-1}(1 - \alpha; \theta)$</th>
<th>Log-L Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stokey-Lucas</td>
<td>.46 (.02)</td>
<td>1.55 (.03)</td>
<td>29.40 (1.45)</td>
<td>22.90 (1.31)</td>
<td>.58</td>
<td>19.19</td>
<td>31.13</td>
</tr>
<tr>
<td>Lazear</td>
<td>.40 (.01)</td>
<td>1.15 (.01)</td>
<td>16.37 (1.00)</td>
<td>11.31 (.79)</td>
<td>.69</td>
<td>4.56</td>
<td>34.35</td>
</tr>
<tr>
<td>Billingsley</td>
<td>.25 (.01)</td>
<td>2.01 (.04)</td>
<td>9.22 (.94)</td>
<td>65.37 (.83)</td>
<td>.51</td>
<td>8.43</td>
<td>23.73</td>
</tr>
<tr>
<td>Duffie</td>
<td>.21 (.02)</td>
<td>4.57 (.29)</td>
<td>10.57 (2.01)</td>
<td>28.24 (1.63)</td>
<td>.54</td>
<td>7.00</td>
<td>18.93</td>
</tr>
</tbody>
</table>

Note: Including shipping and handling charges. Standard errors in parentheses. $\delta_1$ and $\delta_2$ are parameters of the gamma distribution; see equation (13).

$^a$ As implied by estimates of the parameters of the gamma search-cost distribution.

$^b$ Proportion of consumers with reservation price equal to $\overline{p}$, implied by estimate of $r$ (see equation (11)).
Additional empirical evidence

- Online book markets (redux)
- Gasoline markets