Problem 1 (Tirole 8.6)

A monopolist faces demand curve \( q = 1 - p \) in each of two periods (A and B). Its unit cost is \( c \) in period A and \( c - \lambda q^A \) in period B, where \( q^A \) is the first-period output (the firm learns by doing). The discount factor between the periods is \( \delta = 1 \).

(i) Show that the first-period output is \( d/(2 - \gamma) \), where \( d \equiv 1 - c \).

Suppose now that the monopolist (firm 1) faces an entrant (firm 2, with unit cost \( c \)) in the second period. They play Cournot (quantity) competition, which yields profits

\[
\Pi^B_i = \frac{(1 + c^B_j - 2c^B_i)^2}{9}
\]

and outputs

\[
q^B_i = \frac{1 + c^B_j - 2c^B_i}{3}.
\]

(ii) Compute \( q^A_i \) when this quantity is not observed by the entrant before second-period competition.

(iii) Now compute \( q^A_i \) when the quantity is observed by the entrant. How does this compare to the output of the previous part? Give the economic intuition for this result.

(iv) Now assume that \( q^A_i \) is observed and that the entrant faces a fixed cost of entry. How would you expect \( q^A_i \) to compare to the result of the previous part? No computations required - just give the economic intuition.

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Problem 2 (Tirole 8.9)
Suppose that two firms, producing substitute but differentiated products, compete in prices. Assume that profit functions are concave, and that the resulting equilibrium is unique and stable. Show that a government-imposed floor on firm 1’s price will increase that firm’s profit, if the floor is close enough to the equilibrium price. Give an intuitive explanation for this result. (Hint: the effect is strategic. View the price floor as a form of commitment by firm 1 to charge a higher price.)

Problem 3 (Tirole 7.1)
Consider the linear city model described in Tirole §7.1.1. Suppose that firm 1 is located at point \( a \geq 0 \) and firm 2 is at point \( 1 - b \), where \( b \geq 0 \) and \( 1 - b \geq a \). (Firm 1 is to the left of firm 2.) Both firms have the same marginal cost of production \( c \). Transportation costs (borne by the consumer) are quadratic: a consumer at point \( x \) incurs a cost of \( t(x - a)^2 \) to go to store 1 and a cost of \( t(x - (1 - b))^2 \) to go to store 2.

(i) Show that the demand functions faced by each firm are

\[
D_1(p_1, p_2) = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)}
\]

and

\[
D_2(p_1, p_2) = 1 - D_1(p_1, p_2) = b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t(1 - a - b)}.
\]

(You may assume that prices are such that \( 0 \leq D_i \leq 1 \) for each \( D_i \). You may also assume that the gross consumer surplus \( \tilde{s} \) is high enough that every consumer purchases the good, i.e. the entire market is covered.)

(ii) Show that the equilibrium prices charged under Bertrand competition are

\[
p_1(a, b) = c + t(1 - a - b) \left( 1 + \frac{a - b}{3} \right)
\]

and

\[
p_2(a, b) = c + t(1 - a - b) \left( 1 + \frac{b - a}{3} \right).
\]
Problem 4 (Tirole 7.2)

We return to the linear city, this time choosing $a = b = 0$ (i.e. firms are maximally differentiated in location). The firms have marginal costs $c_1$ and $c_2$ (not necessarily equal, but you may assume they are close enough that both firms have positive market share in equilibrium). Now costs are linear: a consumer at $x$ incurs a cost $t|x-a|$ to go to store 1 and a cost of $t|x-(1-b)|$ to go to store 2. The firms again engage in Bertrand (price) competition.

(i) Compute the reaction functions $p_i = R_i(p_j)$.

(ii) Find the equilibrium prices and profits of each firm as a function of $c_1$ and $c_2$.

(iii) Show that

$$\frac{\partial^2 \Pi_i}{\partial c_i \partial c_j} < 0.$$  

Interpret this result: specifically, does an investment by firm 1 to lower his marginal cost make a similar investment by firm 2 more or less profitable?

Now suppose that, before competing in prices, the firms play a first-period game in which they simultaneously choose investments which determine their marginal cost. Let $\phi(c)$ be the cost of investing in marginal cost $c$, where $\phi' < 0$ (it costs less to invest in a higher marginal cost) and $\phi'' > 0$ (decreasing returns from investment).

(iv) Show that the investment game gives rise to a direct effect and a strategic effect on profits in the second-period price-competition game. (Ignore the cost of investment.) Describe in words the origin of these effects (i.e. what caused the change in profit corresponding to each effect).

Problem 5 (Tirole 6.4) Consider an $n$-firm repeated game over an infinite discrete-time horizon $t = 0, 1, ...$ with discount factor $\delta \leq 1$. The firms’ products are perfect substitutes, each firm has constant marginal cost $c$, and the firms engage in Bertrand competition. The demand at date $t$ is $q_t = \mu^t D(p_t)$, where $\mu \delta < 1$. $\mu$ may be interpreted as an expectation of future expansion or contraction of an industry.
(i) Given fixed $\mu$ and $n$, for what values of $\delta$ is full collusion at the monopoly price a sustainable equilibrium of the repeated game?

(ii) How does the ease of sustaining collusion change as the rate of expansion or contraction of the industry changes?

Problem 6 (Tirole 6.6)

Consider two firms interacting repeatedly over an infinite discrete-time horizon in two identical and independent markets. The markets differ in that in market 1 a firm’s price at time $t$ is observed at $t+1$, whereas in market 2 it is learned only at $t+2$. Thus, although each of the markets meets every period, market 2 has longer information lags.

(i) Show that in the absence of multimarket contact (i.e. no communication between markets), collusion in market 2 is sustainable iff $\delta \geq 1/\sqrt{2} \simeq 0.71$, where $\delta$ is the discount factor.

(ii) Show that under multimarket contact, collusion in both markets is sustainable iff $\delta \geq \tilde{\delta}$, where

$$\tilde{\delta} = \frac{1 + \sqrt{17}}{8} \simeq 0.64.$$ 

So collusion is more easily sustainable under multimarket contact - the decreased information lag makes punishment harsher and deviation less profitable.