Collective Self-Control

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Abstract

Behavioral economics presents a “paternalistic” rationale for a benevolent government’s intervention. We consider an economy where the only “distortion” is agents’ time inconsistency. We study the desirability of various forms of collective action, ones pertaining to costly commitment and ones pertaining to the timing of consumption, when government decisions respond to voters’ preferences via the political process. Three messages emerge. First, welfare is highest under either full centralization or laissez faire. Second, introducing collective action only on consumption decisions yields no commitment. Last, individuals’ relative preferences for commitment may reverse depending on whether future consumption decisions are centralized or not.

Keywords: Behavioral Political Economy, Time Inconsistency, Hyperbolic Discounting.

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1 Introduction

The behavioral economics literature has considered many ways in which individuals violate the assumptions of homo economicus—they are prone to various belief updating distortions, they selectively expose themselves to particular sources of information, they make intertemporal decisions in dynamically inconsistent ways, etc.\(^1\) These behavioral tendencies generate an efficiency rational for government intervention. A benevolent social planner may want to “assist” individuals in making decisions that are ultimately good for them.\(^2\) In this paper, we abandon the idea of a benevolent planner and instead explicitly model the fact that the political process determines the design of policy. We develop a model in which politicians seeking election cater to voters who are prone to behavioral biases. We characterize settings in which the political process can ameliorate or exacerbate these distortions. Our characterization has natural implications for the constitutional scope of government activity.

There are of course many types of behavioral distortions, and each of them may lead to its own collective action environment. We focus here on self-control problems: agents have preferences that display present bias or quasi-hyperbolic discounting a-la Phelps and Polak (1968) and Laibson (1997). These self-control problems can lead to phenomena such as procrastination, insufficient savings for retirement,\(^3\) and harmful obesity and addictions.\(^4\) Furthermore, self-control problems can generate a demand for commitment (rehab clinics, illiquid assets with costly withdrawals, and so on). In principle, a benevolent government could offer commitment instruments that would help the electorate overcome some of the harmful symptoms of time inconsistency. Our goal is to understand the potential political economy constraints on the implementation of such policies.

Specifically, we study a simple Wicksellian tree-cutting problem, under the standard specification that the tree is growing in value over time. In our baseline setting, agents have the option of cutting a tree at period 2, which generates a value of \(v_2\), or at period 3, which generates a value of \(v_3\), where \(v_3 > v_2\). A tension arises since agents exhibit present bias. At


\(^2\)This approach has infiltrated policy in recent years. For instance, in the U.S., some discussion of social security takes an explicitly paternalistic approach. In the U.K., the Behavioural Insights Team, also known as the Nudge Unit, which is partially owned by the government, was put in place with the intention of applying insights from behavioral economics and “enabling people to make better choices for themselves” (see www.behaviouralinsights.co.uk/about-us/).

\(^3\)See, for instance, O’Donoghue and Rabin (1999) and Laibson, Repetto, and Tobacman (1998).

any period, all future periods are discounted with a factor of $\beta \leq 1$, which is distributed in an arbitrary (but continuous) way in the population. Thus, from the perspective of period 1, all agents prefer to wait until period 3 to cut the tree. But when period 2 arrives, agents compare an immediate value of $v_2$ with a discounted value of $\beta v_3$ and some could potentially prefer to cut the tree early. This problem has been studied by O’Donoghue and Rabin (1999), who show that time-inconsistent agents tend to consume (cut the tree) inefficiently early, and that these agents would find it valuable to commit to cutting the tree later (namely, in period 3).

We modify the O’Donoghue-Rabin model to allow for continuous choices and costly commitment: by investing resources in period 1, agents can make it costly for their future selves to depart from some pre-specified plan of action. The more investment there is early on in commitment, the more costly it is for future selves to cut the tree too early. Indeed, there are many examples in which individuals use costly commitment devices. For instance, as of the writing of this paper, there has been a collective investment of over $22$ million in individual contracts through stickk.com. These contracts provide explicit financial punishments for not sticking to pre-specified commitments, which vary among users and include smoking cessation, exercise, work targets, etc. There are also various ways in which governments invest in commitment instruments, most notably ones having to do with retirement savings or drug prohibition.

We outline three different scenarios that vary in terms of which choices (investment in commitment and/or the timing of consumption) are subject to the political process, and which ones are left to individuals. That is, we consider collective action pertaining to either or both the commitment decision and the consumption decision, in addition to considering a laissez faire economy in which all decisions are taken individually. We assume collective decisions resulting from a political process are determined by the outcome of competition between two office-seeking candidates. We believe these scenarios offer a simple taxonomy for an array of plausible environments. In addition, the characterization of equilibria under the different scenarios helps highlight the sensitivity of the resulting welfare levels to the aspects, or timing, of choices in which collective action comes into play.

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5See also Della Vigna and Malmendier (2006), Ayres (2010), and references therein.
6For a description of current public sector pension plans, see Beshears et al. (2011) and for a review of alcohol policy in the U.S., see Babor (2003). The introduction of graphic warning labels on cigarette packs has been the topic of recent controversy and is covered in http://www.cnn.com/2013/03/19/health/fda-graphic-tobacco-warnings
7We abuse terminology by referring to “welfare” as the utilitarian social surplus measured for the period 1 selves of the voters. We acknowledge that other criteria are relevant and we discuss this more explicitly in Section 6.
The are three main insights that emerge from our analysis. First, *partial interventions are dominated by either full intervention or no intervention*. This is a complementarity result between intervention in commitment instruments and intervention in consumption decisions.\(^8\) When decision-making is fully centralized, policies reflect the preferences of the median voter, so welfare levels depend on whether the median voter’s bias is relatively small or large. In addition, when only some decisions are centralized but others are made privately by the agents, one must take into account how agents respond to the government’s interventions: we display important ways in which partial interventions can be worse than no intervention.

Second, *introducing collective action only on consumption decisions yields no commitment*. One would expect that collective action on the consumption decision provides commitment and is therefore a valuable alternative to costly commitment investments. In general, delegating control seems like a good way to get around one’s self-control problems. However, this delegated control undermines the incentives to invest in commitment. Thus, the collective consumption decision that is taken by the median voter is undertaken with no help from investment in commitment. This immediately implies that individuals with preferences in a neighborhood of the median voter are made worse off by government intervention.

Last, *individuals’ relative preferences for commitment may reverse depending on whether future consumption decisions are centralized or not*. An individual with high \(\beta\) has sufficient self-control and may not need to invest in commitment technologies when she decides on her own consumption. However, when the consumption allocation is delegated to an agent with lower \(\beta\), she prefers a larger investment in commitment, since, from her perspective, the need for commitment becomes larger. This effect is stronger the larger the individual’s \(\beta\) is relative to the median voter.

## 2 Related Literature

There are three main components of the setting we study in this paper: 1. Agents are time inconsistent; 2. Political leaders offer policies directed at addressing those individual time inconsistencies; and 3. Political leaders are not benevolent, but rather select policies that maximize their chances of being elected under majority rule.

The three components appear in Hwang and Mollerstrom (2014) who focus on a par-

\(^8\) Mandating the timing of consumption can be viewed as a metaphor for, e.g., government transfer payments in the form of welfare, disability insurance, food stamps, or supplemental security income. See Stephens (2003), Shapiro (2005), Dobkin and Puller (2007), and references therein.
ticular environment of political reforms when voters are time-inconsistent. They show that gradualism emerges in equilibrium as a consequence of time inconsistency. They also show that election of a patient agenda setter can arise in equilibrium. Without time inconsistencies (the first component of our analysis), Ortoleva and Snowberg (2015) look at the potential effects of over-confidence on electoral outcomes.

The third component, pertaining to politicians who strategize rather than optimize welfare, has been considered by several authors (Benjamin and Laibson 2003, Caplan 2007, Glaeser 2006, Rizzo and Whitman 2009a,b). These authors have informally made the point that when government is not run by a benevolent social planner but by politicians influenced by voting decisions, it is not clear that government intervention is beneficial. In fact, Glaeser and Caplan explicitly make the case that, if voters are boundedly rational, then the case for limited government may be even stronger than in standard models.9

The first two components of our setting have been considered by Krusell, Kurusçu, and Smith (2002, 2010). They examine government policy for agents who suffer self-control problems. Krusell, Kurusçu, and Smith (2002) consider a neoclassical growth model with quasi-hyperbolic consumers. They show that, when government is benevolent but cannot commit, decentralized allocations are Pareto superior. This is due to a general equilibrium effect of savings that exacerbates an under-saving problem. Benabou and Tirole (2006) discuss how endogenously biased beliefs that are chosen by individuals for self-motivation can generate a belief in a just (or unjust) world and ultimately affect redistributive politics.

Time inconsistency and commitment problems faced by politicians, rather than the electorate, have been the focus of a large literature in political economy and macroeconomics, especially in the context of government debt and monetary policy (e.g., Persson and Tabellini 1990, Alesina and Tabellini 1990). In those models, voters are time consistent, but the identity of the decision maker (or decisive voter) changes over time, generating time inconsistent policies.10 There is some evidence that this sort of time inconsistency may have been at the root of the historical design of pension systems.11 See also Piguillem and Riboni (2015) and

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9Bendor, Diermeier, Siegel, and Ting (2011) present models of boundedly rational voters that are successful in matching some features of elections that are hard to explain with rational voter models. Diermeier and Li (2013) study the outcomes of dynamic majoritarian elections with ‘behavioral’ voters who exhibit some persistence in their voting and forgetfulness of past political outcomes.

10There is also work (e.g., Lagunoff, 2008) that shows that, if one considers governments that have policy preferences and that know that they may be kicked out of office with positive probability, endogenous present bias may emerge. Related, Jackson and Yariv (2015) illustrate that non-dictatorial representative agents respecting Pareto efficiency are inherently time inconsistent.

11Jacobs (2011) provides a comparative history of pension systems, where commitment problems are emphasized as an explanation for why some countries chose, or eventually turned to, a pay-as-you-go system.
Halac and Yared (2014) for the effects of politicians’ time inconsistencies on fiscal policy.\footnote{Our analysis complements this literature by studying the consequences of having agents with heterogeneous degrees of time inconsistency participate in the political process. For instance, as mentioned above, a public pension system is sometimes defended as a desirable solution to a potential problem of under-saving due to self-control problems. However, the design of such a system should then take into account the political constraints generated by an electorate composed of voters with these self-control problems. As we show, these constraints are quite different from those that have been considered in this literature. Such constraints may affect the choice between a pay-as-you-go system and a funded system, the kind of safeguards that are designed into the system, as well as the timing and evolution of the system.}

Outside of the political context, there are several papers studying the interactions between firms and time-inconsistent consumers (e.g., Gottlieb, 2008 and Heidhues and Koszegi, 2010). To the extent that firms are akin to political competitors, some of the underlying forces in these papers are also relevant for the study of political processes with a time-inconsistent electorate.

\section{A Tree Cutting Model}

\subsection{Preferences and Consumption Possibilities}

A continuum of agents decides collectively on the timing of consumption. There are three periods. In period 1 agents make “commitment” decisions (that we specify below). In periods 2 and 3 agents consume fractions of a “tree” of growing value. The tree is worth $v_2$ in period 2, and $v_3$ in period 3. We assume that $v_3 > v_2 > 0$.\footnote{Our qualitative results remain in the presence of uncertainty over future tree values.} In period 2 agents choose a fraction $x$ of the tree to consume in period 2, with $1 - x$ remaining to be consumed in period 3. We interpret period 3 as the natural moment of maturity of the tree so that there is an extra cost in cutting part of the tree in period 2. This cost is given by the function $k(x, c)$, where $c$ is a parameter that is determined in the first period.

Agents have $\beta - \delta$ preferences. For notational simplicity, we assume there is no standard discounting: in the standard notation of $\beta - \delta$ preferences, $\delta = 1$. This assumption is effectively without loss of generality for our analysis (as discounting can be encoded in the $v$ sequence of tree values). That is, for any payoffs $u_2$ and $u_3$ in periods 2 and 3, respectively, the assessed utility at time $t$, denoted by $U_t$, is given by:

\begin{align*}
U_1 &= \beta u_2 + \beta u_3, \\
U_2 &= u_2 + \beta u_3, \\
U_3 &= u_3.
\end{align*}
Individuals are heterogeneous in their present-bias parameter: $\beta$ is distributed according to a continuous distribution $G[\underline{\beta}, \overline{\beta}]$ with a median parameter denoted by $\beta_M$.

An agent with parameter $\beta$ has a utility at $t = 2$ given by:

$$U_2(x, c, \beta) = v_2 x - k(x, c) + \beta v_3 (1 - x)$$

In period 1 a parameter $c$ is chosen (potentially by a collective action process that we soon specify). This parameter raises the cost of cutting the tree early. We assume that $\frac{\partial k(x, 0)}{\partial x} = \frac{\partial k(x, 0)}{\partial c} = 0$ for all $x$, so that absent commitment, there is no marginal cost of cutting the tree early. We also assume that $\frac{\partial k(x, c)}{\partial x} \geq 0$, $\frac{\partial^2 k(x, c)}{\partial x^2} > 0$, $\frac{\partial k(x, c)}{\partial c} > 0$, and $\frac{\partial^2 k(x, c)}{\partial x \partial c} > 0$. That is, cutting costs are weakly increasing and convex in the amount of the tree that is cut $x$ and in the extent of commitment in place, as given by the commitment parameter $c$. The marginal cost of early consumption is also increasing in $c$. Thus, $c$ serves as a commitment mechanism to delay consumption to period 3. This commitment is costly in period 1: choosing $c$ costs $I(c)$. We assume $I(0) = 0$, $I'(0) = 0$, $I'(c) \geq 0$, and $I''(c) > 0$ for all $c$. The regularity restrictions we impose on $k(x, c)$ and $I(c)$ are sufficient for our results and simplify our presentation, but are by no means necessary (in fact, in our running example we will drop the requirement that $\frac{\partial k(x, 0)}{\partial x} = 0$).

Utility in period 1 is given by

$$U_1(x, c, \beta) = \beta (v_2 x - k(x, c)) + \beta v_3 (1 - x) - I(c).$$

Agents are assumed to be sophisticated, in the sense that they are aware that they are time inconsistent. O’Donoughue and Rabin (1999) analyzed the single person decision problem in this environment, by using the notion of perception perfect equilibrium. When agents are sophisticated, they correctly foresee their preferences that are specified a-la Strotz (1955) and use backwards induction to determine their optimal behavior at the outset of the process.

We assume sophistication because we want to study how the demand for commitment is mediated by the political system.\footnote{The assumption that $I(0) = 0$ is not restrictive. Indeed, assuming $I(0) > 0$ is tantamount to assuming there is a fixed cost to entering our economy.}

\footnote{For the most part we will treat the investment in commitment and the associated increase in $k(x, c)$ as resource costs that should be counted in welfare calculations. However, for some applications, such as 401K plans, these are taxes whose revenues are not a deadweight loss. Such cases can be accommodated simply by reinterpreting the resource costs as the pure resources involved in administering the tax. Of course, our welfare calculations would then have to be performed differently. Nonetheless, it turns out that our main welfare results are robust to this modification.}

\footnote{In the Online Appendix we discuss the effects of naivete in our model.}
3.2 The Political Process

There are two candidates running for office. Candidates are office motivated, receiving some positive payoffs from each electoral victory. It will be clear that candidates’ time preferences play no role in this model.\(^{17}\) We assume that the electorate has no ideological attachment to the candidates.\(^{18}\)

We distinguish between three types of environments. These are meant to capture different collective action settings and highlight the effects of the timing of collective decisions on commitment choices.

**Centralized Commitment, Centralized Choice.** Elections occur in periods \(t = 1, 2\). At \(t = 1\), each candidate offers a platform consisting of a cost \(c\) that determines the cost of consumption in period 2 later on. Majority voting determines which outcome, and corresponding platform, is elected (we assume that ties are broken with a toss of a coin). If the platform \(c_i\) is implemented, all agents experience an immediate commitment cost of \(I(c_i)\) at \(t = 1\). At \(t = 2\), the candidates each offer a fraction \(x_j\) of the tree to be consumed in period 2 and majority rule (with random breaking of ties) determines which policy is implemented. If an amount \(x\) of the tree is consumed at \(t = 2\), an agent with taste parameter \(\beta\) receives the value of \(v_2x_j + \beta v_3(1 - x_j)\). All agents experience an immediate cost of \(k(x_j, c_i)\).

**Decentralized Commitment, Centralized Choice.** At \(t = 1\), agents choose individually the parameter \(c\) that will induce their commitment-breaking costs at time \(t = 2\), the cost of which is immediate and given by \(I(c)\). At \(t = 2\), fraction \(x\) of the tree to be consumed in period 2 and majority rule (with random breaking of ties) determines which policy \(x_i\) is implemented for the entire population. An individual with taste parameter \(\beta\) who chose a commitment parameter of \(c\) at \(t = 1\) receives a net value of \(v_2x_i + \beta v_3(1 - x_i)\) and experiences an immediate cost of \(k(x_i, c)\).

**Centralized Commitment, Decentralized Choice.** Elections occur only in period \(t = 1\), when each candidate offers a platform consisting of a commitment parameter \(c_i\) involving an immediate commitment cost of \(I(c_i)\). Majority voting determines which outcome, and corresponding platform is elected (again, ties are broken randomly). At

\(^{17}\)This is not to say that time inconsistencies cannot take place directly at the political level. As mentioned in the discussion of the related literature, there is a body of work that focuses on time consistency of government policy.

\(^{18}\)Allowing agents to have idiosyncratic ideological preferences (as in Lindbeck and Weibull 1987) does not change any of the results qualitatively. Details are available from the authors upon request.
$t = 2$, each of the individual agents decides what fraction $x$ of the tree to consume. An individual with taste parameter $\beta$ who chooses to consume a fraction $x$ of the tree $t = 2$ receives a net value of $v_2 x + \beta v_3 (1 - x)$ and experiences an immediate cost of $k(x, c_i)$. 

In what follows, we analyze the outcomes of a fully decentralized economy in addition to each of the above settings in turn. Throughout, we often use the shorthand of DD, CC, DC, and CD for the fully decentralized, fully centralized, decentralized commitment and centralized choice, and centralized commitment and decentralized choice systems, respectively. 

For each political process, we consider voters' (pure) strategies that are measurable with respect to the Lebegue-Stieltjes measure induced by $G$. An equilibrium is then a profile of such strategies in which each individual best responds to population play. We note that this requires fairly limited knowledge on the part of individuals regarding overall population play—when decisions are decided collectively, individuals need only know the median choice in order to best respond.\footnote{We do not model how this knowledge comes about. While a standard assumption in equilibrium analysis, it may not be a trivial assumption from an applied perspective. We make this assumption for two reasons. First, the paper can be interpreted as showing that even when voters are as sophisticated as possible, certain institutions can still exploit their behavioral biases. Second, were we to relax this assumption, we would need to take a stand on the specification of “naive” beliefs held by voters, which would inherently be somewhat ad-hoc, as the empirical literature is still not rich with results identifying such beliefs.}

All proofs are relegated to the Appendix.

3.3 Discussion of the Model

Our model of collective action builds on the individual decision-making problem proposed by O’Donoughue and Rabin (1999), who first considered a version of the type of tree-cutting decision problem analyzed in this paper. We extend this model in two ways and then introduce collective action. First, we allow the agents to invest in commitment or self-control. This is reasonable as there are many cases in which the government or individuals can take actions to restrain their future selves (401K plans, rehab programs, weight watchers, internal psychological mechanisms). If commitment were free, all agents would commit themselves to later consumption and there would effectively be no self-control problem. Consequently, decentralized decisions would lead to first-best outcomes in which all agents would efficiently and fully delay their consumption. We therefore introduce commitment costs to allow for a non-trivial cost-benefit trade-off that also creates room for potentially beneficial government intervention. A second, more technical modification of the model presented by O’Donoughue
and Rabin is that they assume that agents only choose when to cut the tree but cannot cut fractions of it. This is tantamount to a special case of our model in which $k(x, c)$ is linear.\footnote{This special case introduces some important technical complications within the analysis of the Centralized Commitment-Decentralized Choice scenario as preferences are no longer single peaked. This case is discussed in the online appendix.}

We now discuss some features of our model and their limitations.

**Political Intervention.** We have chosen to study four scenarios that differ in their degree of centralization of choices. These are natural scenarios and we show below that it is important to distinguish between government action over consumption, such as government transfer payments in the form of welfare, disability insurance, food stamps, or supplemental security income, and government action over commitment, such as 401K pension plans that penalize withdrawals before retirement age, prohibition, cigarette ad campaigns, etc.\footnote{The model is not tailored to study addiction. However, the key force of commitment to delay undesirable temptations for immediate gratification is relevant for addiction as well.} Furthermore, comparing laissez faire to various degrees of government centralization is an old question in economics and it is useful to understand how centralization affects outcomes in a world with no other reason for government intervention other than “behavioral” distortions. There are, of course, many other scenarios that can be studied. We discuss some of these in the Online Appendix.\footnote{In the Online Appendix, we also consider endogenous turnout.}

**Format of Collective Action.** We have modeled collective action as elections of office-motivated candidates. We have done this in part because this is the most standard way to approach political economy models, and is therefore a good starting point to explore collective self-control. However, the main forces behind our results are likely to be present in several alternative specifications of the political system. Two particular natural extensions (discussed in the Online Appendix) are the following. First, one could think of collective action generating more targeted policies that affect individuals differentially, say in the form of commitment subsidies or consumption caps. Second, throughout our analysis we take the commitment technology itself (namely, the functions $I(c)$ and $k(x, c)$) as exogenous. While some of the implied costs may be psychological and rather non-malleable, others may stem from institutions and could be an object of political choice as well.\footnote{We also note that our results carry through almost directly to a citizen-candidate model (under plurality rule) of collective action, for moderate candidacy costs. In that case, in all the scenarios we consider, as in Proposition 1 in Osborne and Slivinski (1996), one citizen would put forward her candidacy and select the policies we characterize.}

We intentionally ignore externalities among agents other than those induced by the collective-choice process in order to isolate the forces that emerge from collective action.
through time inconsistency itself. However, studying the interactions between self-control problems and other reasons for collective action would certainly be interesting.

**Commitment Technologies.** We assume that the commitment technologies faced by individuals and by the government are identical. There is no empirical reason to make this assumption: the relative effectiveness of commitment by the government or by individuals will depend on the specific application. However, assuming identical technologies is a useful benchmark. In the Online Appendix we consider some aspects of different commitment technologies accessible to the government and the agents.

**Self-Control Problems in Elections.** The quasi-hyperbolic discounting model imposes a link between self-control problems and a difference between present- and future-period preferences. In that respect, what accounts for the “present” as opposed to the “future” is crucial for the scope of self-control problems and for evaluating potential policies that attempt to limit their impact. For a detailed discussion and description of empirical research supporting the relevance of self-control problems for the voting context, see Bisin, Lizzeri, and Yariv (2015).

4 Decentralized Outcomes

Before inspecting the impacts of collective action on commitment decisions, we describe each agent’s individual decisions. This analysis corresponds to the case in which all decisions are made in a decentralized fashion.

Given the value of $c$ determined in the first period, in the second period the agent’s problem is given by:

$$\max_x U_2(x, c, \beta) \leftrightarrow \max_x v_2 x - k(x, c) + \beta v_3 (1 - x).$$

Let $x(c, \beta)$ denote the solution to this problem.

$$x(c, \beta) = \begin{cases} 1 & \beta < \left( v_2 - \frac{\partial k(1, c)}{\partial x} \right) / v_3 \\ (v_2 - \beta v_3) & \left( v_2 - \frac{\partial k(1, c)}{\partial x} \right) / v_3 \leq \beta \leq \left( v_2 - \frac{\partial k(0, c)}{\partial x} \right) / v_3 \\ 0 & \beta \geq \left( v_2 - \frac{\partial k(0, c)}{\partial x} \right) / v_3 \end{cases}. \quad (1)$$

Intuitively, whenever the agent either experiences less present bias or higher marginal costs of immediate consumption, delay is more likely. At the extremes, if marginal costs of cutting the whole tree are not too high (namely, $\frac{\partial k(1, c)}{\partial x} < v_2$), very impatient agents will not
delay any consumption. Virtuous agents, for whom the marginal costs of very little early consumption outweigh the benefits, will cut the entire tree in period 3. The monotonicity of the consumption function $x(c, \beta)$ is captured by the following lemma, which will be useful for our analysis of the collective-choice settings.

**Lemma 1 (Consumption Monotonicity)** The fraction of the tree consumed in period 2, $x(c, \beta)$, is decreasing in both $c$ and $\beta$.

The first period problem can be written as:

$$\max_c U_1(x, c, \beta) \iff \max_c \beta v_3 + x(c, \beta) (\beta v_2 - \beta v_3) - \beta k(x(c, \beta), c) - I(c)$$

Let $c(\beta)$ be the solution of this problem. We want to understand the dependence of the commitment parameter $c(\beta)$ on $\beta$, which will be an essential input into the collective-action problem.

In order to glean some intuition on the dependence of $c$ on $\beta$, consider the case in which $x(c, \beta)$ is interior and differentiable with respect to $c$. Notice that:

$$\frac{\partial U_1}{\partial c} = \beta \left( (v_2 - v_3) - \frac{\partial k(x(c, \beta), c)}{\partial x} \right) - \beta \frac{\partial k(x(c, \beta), c)}{\partial c} - I'(c).$$

In contrast to the standard dynamic optimization problem with geometric discounters, the envelope condition fails and the indirect effect on period 2 consumption does not disappear. Indeed, substituting the second period first-order conditions we obtain:

$$\frac{\partial U_1}{\partial c} = -\beta \left( \frac{\partial x(c, \beta)}{\partial c} ((1 - \beta) v_3) + \frac{\partial k(x(c, \beta), c)}{\partial c} \right) - I'(c). \quad (2)$$

The benefit of commitment is captured by the term $-\beta \frac{\partial x(c, \beta)}{\partial c} ((1 - \beta) v_3)$. This term is increasing in the degree of present bias and captures the fact that the period 1 self and the period 2 self disagree on the value of cutting the tree in period 2.

There are several effects of changes in $\beta$ on the optimal choice of $c$. As $\beta$ increases more weight is put on the future, pushing for more early commitment investment. Furthermore, the fraction of the tree consumed in period 2, $x(c, \beta)$, is smaller, leading to a smaller marginal cost $\frac{\partial k(x(c, \beta), c)}{\partial c}$ tomorrow. Nonetheless, as $\beta$ becomes larger, time inconsistency becomes less relevant, so the benefit of $(1 - \beta) v_3$ is smaller. When $\beta$ is close to zero or close to $\frac{v_2}{v_3}$ period 1 investment will be zero, so investment is not monotone. Intuitively, agents for whom time inconsistency is very severe foresee that reasonably priced commitments will not save them from excessive consumption in period 2 and therefore acquire limited commitment. On the other side of the spectrum, agents who are virtuous (characterized by high $\beta$), do not suffer
from great temptation in period 2 and therefore do not require extreme commitment to enable them to postpone consumption. In particular, recall that when \( \beta \geq v_2/v_3 \), agents choose \( x(c, \beta) = 0 \) for all \( c \) so their optimal investment in commitment is zero: \( c(\beta) = 0 \) for all \( \beta \geq v_2/v_3 \).

Figure 1 illustrates a case in which \( c(\beta) \) is concave between \( \bar{\beta} \) and \( v_2/v_3 \) in this region and only one maximum exists. This holds, for instance, when the cost functions are quadratic.\(^\text{24}\) As highlighted by the figure, the greatest commitment constraints are chosen by individuals with moderate levels of time inconsistency.

5 Electoral Outcomes

We now turn to inspect the effects of collective action on agents’ choices. We start by analyzing the case in which only the choice of commitment levels is done through an electoral process. We then proceed to a case in which both commitment and the timing of consumption are decided upon collectively.

Recall that the support of the preference distribution \( G \) is \( [\underline{\beta}, \bar{\beta}] \) and that \( \frac{\partial k(x, \beta)}{\partial c} = 0 \) for all \( x \). Unless otherwise mentioned, we will assume that \( \bar{\beta} > 0 \) from now on. From equation (2), together with Lemma 1, it follows that for all \( \beta \in [\underline{\beta}, \beta^*] \), the optimal commitment

\(^{24}\)In general, however, \( c(\beta) \) may achieve several local maxima between \( \bar{\beta} \) and \( v_2/v_3 \).
level is positive, \(c(\beta) > 0\). Denote by \(c^* \equiv c(\bar{\beta})\). We will focus on what we term regular environments.

**Definition (Regular Environments)** An environment is regular if \(\frac{\partial k(1,c)}{\partial x} \geq v_2\) for all \(c \geq c^*\).

Regularity assures that preferences for commitment are single-peaked. Using our discussion above, we have the following:

**Lemma 2 (Commitment Choices in Regular Environments)** In regular environments, for any individual of type \(\beta\), \(x(c(\beta), \beta) < 1\). In particular, \(c(\beta) > 0\) for all \(\beta \in [\underline{\beta}, \beta^*]\). Furthermore, preferences over commitment levels are single-peaked.

That is, regularity assures that individuals, left to their own devices, would choose commitment levels that are effective to some extent. This simplifies our analysis substantially. We discuss the analysis of environments that are not regular in the Online Appendix.

### 5.1 Collective Commitment with Decentralized Choice

In this setting, the commitment parameter \(c\) is determined collectively. From the point of view of an agent of type \(\beta\), the voting problem is determined as follows. From the analysis of the private decision problem of an agent of type \(\beta\), if a commitment parameter \(c\) is chosen, and subsequent choices are made optimally by the agent, period 1 utility is given by

\[
U_1(x(c, \beta), c, \beta) = \beta v_3 + x(c, \beta)(\beta v_2 - \beta v_3) - \beta k(x(c, \beta), c) - I(c).
\]  

Thus, the agent votes for candidate 1 offering commitment \(c_1\) over candidate 2, who offers commitment \(c_2\), whenever

\[
U_1(x(c_1, \beta), c_1, \beta) > U_1(x(c_2, \beta), c_2, \beta).
\]

**Proposition 1** There is a unique pure strategy equilibrium of the collective commitment game in which both candidates offer a platform \(c^{CD}\). Furthermore, when \(c(\beta)\) has a unique local maximum in \((\underline{\beta}, v_3)\), the platform \(c^{CD}\) corresponds to the ideal policy for a voter of type \(\beta^{CD}\), where \(\beta^{CD}\) is higher than the median \(\beta^{CD} \geq \beta_M\).

The quadratic case in the example above is useful in illustrating the intuition underlying Proposition 1. Consider Figure 1. If \(1 - G(v_3) \geq 1/2\), there is a majority of agents who prefer no commitment and the equilibrium commitment parameter is naturally \(c^{CD} = 0\),
which coincides with that preferred by the median. Otherwise, for every $c > 0$, define $\beta_L(\tilde{c})$ and $\beta_H(\tilde{c})$ such that $\tilde{c}$ is their ideal point, i.e. $c(\beta_L(\tilde{c})) = c(\beta_H(\tilde{c})) = \tilde{c}$. All agents with preference parameters below $\beta_L(\tilde{c})$ and above $\beta_H(\tilde{c})$ prefer commitment parameters lower than $\tilde{c}$, while agents with preference parameters between $\beta_L(\tilde{c})$ and $\beta_H(\tilde{c})$ prefer commitment parameters above $\tilde{c}$. In particular, the equilibrium commitment parameter $c^{CD}$ is chosen so that these two classes of agents are of equal proportions. That is, $G(\beta_L(c^{CD})) + (1 - G(\beta_H(c^{CD}))) = 1/2$. By construction, $\beta_M \in (\beta_L(c^{CD}), \beta_H(c^{CD}))$ and the result follows. In fact, note that in this case the equilibrium commitment level corresponds to a voter of type $\beta^{CD}$ that is strictly higher than the median, $\beta^{CD} > \beta_M$. In this case, it also corresponds to the commitment level of a voter of type $\tilde{\beta}^{CD}$ that is strictly lower than the median, $\tilde{\beta}^{CD} < \beta_M$. Importantly, when $c(\beta)$ has a unique local maximum, this construction implies that equilibrium commitment is lower than the ideal point of the median $\beta$. That is, $c^{CD} \leq c(\beta_M)$. We stress, however, that Proposition 1 is still a sort of a median-voter result, in that half of the electorate prefers a commitment level lower than $c^{CD}$ and half of the electorate prefers a commitment level higher than $c^{CD}$. Since individual preferences for commitment are non-monotone in the preference type, the resulting equilibrium commitment need not correspond to the individual with median preference parameter $\beta_M$.

This construction of the equilibrium level of commitment can be adapted to environments in which $c(\beta)$ entails several local maxima, it is only the relation to the median agent’s preferred level of commitment that hinges on $c(\beta)$ having a unique maximum. However, the construction does rely on all agents having single-peaked preferences with respect to the commitment parameter $c$. Indeed, in this case, agents with high taste parameter $\beta$ prefer no investment in commitment, while all others prefer a positive amount of commitment. The condition on $\frac{\partial k(1,c)}{\partial x}$ assure that even individuals with very low parameters $\beta$ benefit from some level of commitment.\(^{26}\)

### 5.2 Collective Commitment with Centralized Choice

We now discuss the case in which the second period choice is also taken via collective action. Two office-motivated candidates, 1 and 2, offer platforms $x_1$ and $x_2$ in the second period.

From the analysis of individual choices, recall that (1) provides the second period optimal choice $x(c, \beta)$ for any given commitment parameter $c$ selected in period 1. From Lemma 1,\(^{25}\) This also implies that median preserving spreads of the distribution $G$ would lead to lower equilibrium commitment levels.

\(^{26}\)We note that it would suffice to assume that $\frac{\partial k(1,c)}{\partial x} \geq v_2$ only for $c > 0$, which is satisfied by our running quadratic example.
\( x(c, \beta) \) is decreasing in \( \beta \). It is then clear that for any given choice of \( c \) in the first period, both candidates will choose to offer the ideal policy of the median voter \( \beta_M \). Thus, the second period choice will be \( x(c, \beta_M) \).

We can now step back and consider a generic voter’s first period utility in this scenario.

\[
U_1(x(c, \beta_M), c, \beta) = \beta v_3 + x(c, \beta_M)(\beta v_2 - \beta v_3) - \beta k(x(c, \beta_M), c) - I(c). \tag{4}
\]

Since \( x(c, \beta_M) \) is fixed for all \( \beta \), the choice of commitment in the first period is driven by the desire to commit of an agent of median taste parameter \( \beta_M \). Denote by \( c(\beta, \beta_M) \) the (constrained) optimal commitment parameter for an agent of taste \( \beta \), foreseeing the second period choice being determined according to the taste of the median parameter \( \beta_M \).

As it turns out, \( c(\beta, \beta_M) \) is monotonic in \( \beta \), with individuals who care more about future consumption preferring greater investment in commitment, as illustrated in the following lemma.

**Lemma 3 (Constrained Commitment Monotonicity)**: The optimal constrained commitment \( c(\beta, \beta_M) \) is increasing in \( \beta \).

Note that the monotonicity in \( \beta \) of desired commitment is in contrast with the analysis of both the fully decentralized scenario as well as of the centralized commitment with decentralized choice scenario. The logic for this is the following. The value of investment in commitment is now in reducing incentives for the median agent to cut the tree early. This is particularly valuable for the high-\( \beta \) agents.

Lemma 3 implies that it is median preferences that determine first period choices as well. This is captured in the following proposition.

**Proposition 2**: When both commitment and consumption are chosen collectively, equilibrium outcomes coincide with those chosen optimally by agents with the median taste parameter \( \beta_M \).

It also interesting to highlight how optimal constrained commitment \( c(\beta, \beta_M) \) changes as \( \beta_M \) changes. Indeed, the marginal benefit of commitment \( (1 - \beta_M) v_3 \) is higher when \( \beta_M \) is lower and so for the case of interior solutions it follows that:

**Remark**: The optimal constrained commitment \( c(\beta, \beta_M) \) is decreasing in \( \beta_M \).

Figure 2 depicts \( c(\beta, \beta_M) \) for different values of \( \beta_M \in \left( \frac{\beta_1 v_2}{v_3}, \frac{\beta_2 v_1}{v_3} \right) \).
Figure 2: Constrained Commitment for Different Median Preferences

As Figure 2 illustrates, the optimal desired amount of commitment $c(\beta, \beta_M)$ is increasing in $\beta$ and decreasing in $\beta_M$. Notice, however, that the equilibrium level of commitment is given by $c(\beta_M, \beta_M) \equiv c(\beta_M)$, which is not monotonic.

We can now compare the level of commitment in the two collective-action scenarios. When $c(\beta)$ has a unique maximum, Proposition 1 assures that the platform $c^{CD}$ chosen in equilibrium corresponds to the ideal policy for a voter of type $\beta^{CD}$, where $\beta^{CD}$ is higher than the median $\beta$, $\beta^{CD} \geq \beta_M$. Furthermore, the construction of the proof of Proposition 1 (extending that appearing for the quadratic case in Example 1) illustrates that $c(\beta^{CD}) \leq c(\beta_M)$ when $c(\beta)$ has a unique local maximum. This discussion can be summarized by the following proposition:

**Proposition 3** Assume that $c(\beta)$ has a unique local maximum in $\left(\beta_1, \frac{\beta_4}{\beta_3}\right)$. Then the equilibrium choice of commitment is higher under full centralization than in the decentralized choice scenario.

Note that this proposition shows that the optimal amount of commitment is higher in the fully centralized economy although the decisive voter is an agent with a lower $\beta$. This is, of course, due to the non monotonicity of $c(\beta)$ and illustrates the fact that delegating the commitment choice to a more virtuous agent may not lead to higher commitment.
When $c(\beta)$ has multiple local maxima, the comparison between the equilibrium commitment levels generated by full centralization and decentralized choice is inconclusive and, in principle, can go either way.

5.3 Decentralized Commitment with Centralized Choice

We now consider the case in which individuals privately invest in commitment, but in period 2 there is an election that determines the time for consumption for all individuals.

Proposition 4 In an equilibrium of the decentralized commitment, centralized choice scenario, all voters choose $c = 0$.

The intuition for this result is the following. Each agent is powerless to affect choices in the second period. Investment in commitment is only useful if it affects the choice in period 2. But, this choice is made collectively, and the probability that an agent is pivotal in period 2 is zero when there is an infinite number of agents so the incentive to invest in commitment also disappears.

This result suggests the following observation. Suppose that in the decentralized setting we observe a median individual making responsible choices in period 2. One may naively conclude that centralizing consumption would be beneficial because it would lead to responsible choices for the entire population, including those who were choosing irresponsibly. However, our result shows that such partial centralization would undermine the incentive to commit that in turn allowed the median person to choose responsibly in period 3. For instance, if $\beta_M < \frac{\mu_2}{\sigma_2}$, then in this scenario the median choice would be to consume the entire tree in period 2. This discussion suggests that partial centralization may be harmful: centralization of consumption choices should be accompanied by centralization of commitment. We discuss this intuition more formally in the next section.

It is also interesting to note that the zero investment result in this proposition still holds in the case of a homogeneous population. This is notable since in all of the previous

\[27\] With a finite voting population, the no commitment equilibrium still exists, but equilibria with positive investment may exist. In those, an exact majority invests in the private (decentralized) commitment. In that case, any individual choosing $c = 0$ is best responding since she gains her ideal commitment choice in period $t = 2$: If an investing individual deviates to a lower commitment level, she becomes pivotal at $t = 2$: Therefore, choosing the optimal decentralized level of commitment is optimal for her. However, the zero investment equilibrium is the only one that is robust in the following sense. Consider a finite population of voters drawn from a distribution $G(\beta, \epsilon)$ so that the identity of the median voter is uncertain. In this variation of the model, as the number of voters grows large, the probability that any given voter is pivotal in the second period becomes negligible. Hence, the benefit from investing in commitment converges to zero.
scenarios, outcomes for a homogeneous population would coincide with those generated by a fully decentralized, laissez faire system. Indeed, in our other settings, the key ingredient determining outcomes is the identity of the decisive agent. However, in the setting where only consumption is centralized, the decision-making process itself undermines the incentive to commit. It leads to no one investing in commitment regardless of the distribution of preferences and, in turn, may harm the population as a whole.

5.4 Comparison of Outcomes

Our analysis so far illustrates that individuals’ relative preferences for commitment may reverse depending on whether future consumption decisions are centralized or not. We showed that, under some regularity assumptions, commitment investment is larger in the scenario with full centralization relative to the scenario where only commitment is centralized. Importantly, there is no investment in commitment in the case of decentralized commitment and centralized choice. Therefore, that scenario leads to the lowest aggregate investment in commitment.

The comparison between full decentralization and the systems involving centralized commitment choices is instead ambiguous. One obvious case in which the fully decentralized outcome leads to higher investment in commitment is when the median voter is virtuous: $\beta_M \geq \frac{2}{3}$. In this case all scenarios with some degree of centralization of commitment decisions generate no commitment investment, whereas some investment takes place in the fully decentralized scenario as long as there is a positive mass of individuals who are not virtuous. Finally, in order to construct an example in which centralized commitment leads to higher investment, consider a case where the median voter $\beta_M$ is such that $c(\beta_M)$ is maximal (i.e., $\beta_M \in \arg\max c(\beta)$). In this case the fully centralized scenario leads to the maximal investment that would be chosen by anybody in the population. All other scenarios lead to lower investment in the aggregate.

Figure 3 summarizes the resulting second-period consumption levels under the different systems. Were individuals able to freely commit, they would all consume the tree in full in period 3 and the first-best second-period consumption would be nil for all preference types. When the median agent is not virtuous (corresponding to the left panel of the figure), centralizing consumption leads to greater early consumption at least for some agents (when commitment decisions are centralized), if not all (when commitment decisions are decentralized). However, the first-best level is achieved whenever the median agent is virtuous and there is collective action on consumption decisions (see the right panel of the figure). Decisions are then delegated to an agent who can resist temptations to consume early. We next
analyze the welfare consequences of the different systems, which will echo these observations.

6 Welfare Consequences

We now turn to the welfare consequences of each of the political processes analyzed above. In the case of time inconsistent agents, the appropriate welfare criterion is debatable.\textsuperscript{28} We start by measuring welfare as the utility of first period agents. We later turn to consider period-zero welfare assessments.

6.1 Period-One Welfare

We denote by $\Pi^{DD}(G), \Pi^{DC}(G), \Pi^{CD}(G),$ and $\Pi^{CC}(G)$ the expected utilitarian welfare corresponding to the fully decentralized, decentralized-centralized, centralized-decentralized, and centralized-centralized systems, respectively, when the underlying preference distribution is given by $G$. We will at times abuse notation and drop the argument of the welfare function when clarity is not compromised. For presentation simplicity, we assume that $k(x, 0) = 0$ for all $x$ so that no commitment leads agents to experience no costs of early consumption.

The main idea behind our comparison in the welfare generated by the four institutions that we consider is that of delegation. Centralization effectively allows delegation of specific decisions to a particular individual. In the setting of our model without any externalities, standard geometric discounters would have no reason to delegate and so laissez faire would

\textsuperscript{28}For a discussion, see Bernheim and Rangel (2009).
dominate all other systems. For individuals with self-control problems, from the perspective of period one welfare, we must distinguish between delegation of consumption choices and delegation of investment in commitment. All individuals benefit from delegating consumption decisions to individuals who are more virtuous than them (higher $\beta$) and harmed by delegating decisions to individuals who have worse self-control (lower $\beta$). On the other hand, no period-1 self-benefits from delegating the commitment decision because there is no self-control problem in period 1.

We start by showing that partial centralization, i.e., mandating only one decision (either commitment or consumption) is dominated by either full centralization or laissez-faire.

**Proposition 5** For all preference distributions, either full centralization or full decentralization are welfare maximizing. That is, $\max \{\Pi^{DD}, \Pi^{CC}\} \geq \max \{\Pi^{CD}, \Pi^{DC}\}$.

The proof of this result is straightforward. Note first that, under laissez faire, individuals can always emulate the decisions generated by the centralized-decentralized system: they can choose a commitment level of $c^{CD}$. Therefore, $\Pi^{DD} \geq \Pi^{CP}$. In fact, all agents but those who individually choose $c^{CD}$ strictly benefit from having all decisions decentralized. Now, let $\beta^*$ be the threshold preference parameter corresponding to agents who are just indifferent between postponing consumption or consuming immediately in period 2 given zero commitment, $v_2 = \beta^* v_3$. By Proposition 4, in the decentralized-centralized system there is zero investment in commitment. If the median voter is not sufficiently virtuous ($\beta_M < \beta^*$) this leads to $x = 1$: full consumption of the tree in period 2 because early consumption comes at no cost, $\frac{\partial k(x,0)}{\partial x} = 0$. In this case, welfare is higher under laissez faire: all agents with preference parameter $\beta < \beta^*$ can do no worse than this, and all virtuous agents with preference parameters $\beta \geq \beta^*$ manage to delay consumption to period 3 even if they do not invest in commitment. It follows that when $\beta_M < \beta^*$, $\Pi^{DD} \geq \Pi^{DC}$. Suppose now that the median voter is virtuous, $\beta_M \geq \beta^*$. In this case, under both the decentralized-centralized and centralized-centralized system, there is no investment in commitment and all consumption is delayed to period 3, so that, in this case, $\Pi^{CC} = \Pi^{DC}$. The result of Proposition 5 holds, in fact, for more general environments than the special one we study here (see Section 7 below).

The comparison between the fully centralized system and a laissez faire economy depends on the distribution of preferences. Roughly speaking, when the median $\beta$ is high, centralization is beneficial because a centralized political process allows all agents to delegate choice to a virtuous voter who commits to efficient actions at low costs. On the other hand, when the median voter is prone to a strong present-bias (low $\beta_M$), collective decisions lead to bad
outcomes: low investment in commitment and high levels of early consumption. In these
cases, the decentralized system performs better because at least some of the virtuous voters
do well: they are not bound by the self-control problems of a low median $\beta$. The following
result provides sufficient conditions for ranking the two systems.

**Proposition 6**

1. If $G(\beta^*) > 0$, and $\beta_M \geq \beta^*$, then full centralization is best: $\Pi^{CC}(G) > $ $\Pi^{DD}(G)$;

2. Consider a sequence of distributions $\{G_n\}_{n=1}^{\infty}$ with corresponding medians $\{\beta^n_M\}_{n=1}^{\infty}$. If there exists $\bar{\beta} > \beta$ such that $\{G_n(\bar{\beta})\}$ is uniformly bounded below 1 and $\lim_{n \to \infty} \beta^n_M = \beta$, then there exists $n^*$ such that for all $n > n^*$, laissez faire is best: $\Pi^{DD}(G_n) > $ $\Pi^{CC}(G_n)$.

Part 1 of this result says that a sufficient condition for full centralization to be best is
that the median voter is sufficiently high so that no commitment is required to ensure no
early consumption. Of course, this condition is not necessary: for instance, if the median
voter is characterized by a preference parameter slightly lower than $\beta^*$, a moderate amount
of investment in commitment ensures almost no early consumption.

Part 2 is more involved: the condition on the sequence of distributions rules out the
possibility that there is a sequence such that $\beta_M$ converges to $\bar{\beta}$ but there is a vanishing
mass of agents whose preference parameter $\beta$ is larger than $\beta_M$. In such cases centralization
may still be best.

Proposition 6 is effectively a delegation result. When the median voter is sufficiently
virtuous, the electorate benefits from delegating decisions to the median voter. Recall that
the desire to delegate only regards consumption decisions: individuals do not like delegating
commitment decisions.\textsuperscript{29}

An example is useful in visually illustrating how the different political processes fare in
terms of welfare as a function of the underlying preference distribution in the electorate.

**Example 3 (Quadratic Costs – Welfare Comparisons)** Consider the case of $k(x, c) =
(c + v_2) \frac{c^2}{2}$ and $I(c) = \frac{c^2}{2}$ and suppose that $G$ is a triangular distribution with a peak
at $d \in (0, 1)$. Figure 4 depicts the welfare levels generated by the different processes
as a function of the median agent’s preferences when $v_2 = 1$ and $v_3 = 3/2$, and

\textsuperscript{29}The idea that delegation to an agent whose preferences are misaligned with society’s might be useful
appears in other types of environments as well, most notably in the macroeconomic literature condoning
the use of central banks who do not share the social objective and weigh relatively heavily inflation-rate
stabilization, see Rogoff (1985) and the literature that followed.
Figure 4: Welfare Comparison for Different Median Preferences

\[ I(c) = 0.0005c^2. \] \(^{30,31}\)

We use the fully decentralized setting as a baseline for comparison. The figure illustrates the way that the four scenarios compare in terms of first-period welfare. Full centralization reaches the highest welfare when the median \( \beta \) is high. Full centralization and decentralized commitment-centralized choice have the same level of welfare when the median is above \( 2/3 \) because \( 2/3 = v_2/v_3 \) for our parameters and in these cases no commitment is necessary to induce zero tree-cutting in period two. However, the decentralized commitment-centralized choice scenario is a lot worse for lower values of the median \( \beta \). When the median \( \beta \) is lower, full decentralization leads to the best outcome, with centralized commitment-decentralized choice a close second. The reason why the comparison between these scenarios becomes much more favorable to full centralization (especially relative to the setting in which only commitment is centralized) for high values of median \( \beta \) is that when the median \( \beta \) is high there is no commitment in equilibrium, and, under decentralized choice, this harms the individuals

\[^{30}\text{For a triangular distribution with a peak at } d, \text{ the corresponding median is given by:}\]

\[ \beta_M = \begin{cases} \sqrt{d}/\sqrt{2} & d \geq 1/2 \\ 1 - \sqrt{1-d}/\sqrt{2} & d < 1/2 \end{cases}. \]

\[^{31}\text{Note that our utility specification implies a weight of 1 on period-1 instantaneous utility and a weight of } \beta \text{ on each of period-2 and period-3 instantaneous utilities. In the figure, we normalize the utility of each agent with preference parameter } \beta \text{ by } 1 + 2\beta.\]
with lower $\beta$. One interesting aspect of the comparison among welfare levels is that commitment and consumption, are ‘complementary’: either full centralization or full decentralization generate the greatest levels of welfare, whereas partial centralization yields inferior welfare results.

It is also instructive to compare the welfare resulting from our political processes to that generated by an economy that does not allow for commitment. Denote by $\Pi^S$ the expected first period utilitarian surplus absent any commitment instruments:

$$\Pi^S = \int_0^{\beta^*} \frac{\beta v_2}{1 + 2\beta} dG(\beta) + \int_{\beta^*}^1 \frac{\beta v_3}{1 + 2\beta} dG(\beta).$$

(5)

It is easy to construct example where the welfare generated by some centralized political system is worse than $\Pi^S$. For instance, if there is a substantial mass of virtuous agents and the median $\beta_M$ is very low, $\Pi^S > \Pi^{CC}$. In other words, if the constitution makes a bad delegation decision, welfare is even lower than in an economy with no possibility of investing in commitment. However, it is easy to see that $\Pi^S \leq \max \{\Pi^{DD}, \Pi^{CC}\}$. In fact, $\Pi^S \leq \Pi^{DD}$: under laissez-faire agents can always emulate the no commitment environment by choosing a commitment level of 0. Thus, whenever a positive commitment level is chosen by an individual, the induced first-period utility is higher than that absent commitment.

### 6.2 Period-Zero Welfare

It is also useful to consider the welfare comparison among the various scenarios from the point of view of period zero, before the commitment choice is made.

Period 0 utility for an agent of type $\beta$ is given by

$$U_0(x(c, \beta), c, \beta) = \beta \{v_3 + x(c, \beta) (v_2 - v_3) - k(x(c, \beta), c) - I(c)\}.$$  

(6)

Comparing expressions (6) and (3) makes it clear that there is an important difference between the period-zero and the period-one perspective: from the point of view of the period-zero self, the commitment choices themselves are now subject to self-control problems. The main consequence of this for our analysis of collective action is that, in contrast to our previous discussion, the period-zero self may now have the incentive to delegate commitment decisions because her period-one self “under-commits.”

\[32\text{As in the example, the division by } 1 + 2\beta \text{ is a normalization of agents' utilities, so that weights of instantaneous utilities in period 1 always sum up to 1.}\]
Despite these differences, an analogous result to Proposition 6 still holds when considering welfare in period zero: If the median voter is sufficiently virtuous, mandating decisions, or delegating them to the virtuous median voter through full centralization, is beneficial relative to a laissez faire setting; in contrast, if the median voter has high degree of present bias (low $\beta$), then laissez faire is better.

With respect to partial centralization, notice first that full centralization Pareto dominates (for all preference parameters $\beta$) the decentralized commitment, centralized consumption system. Indeed, both systems generate a uniform profile of commitment and consumption for all agents. Therefore, all period-zero selves, regardless of their preference parameter $\beta$, rank the two systems alike. However, the agent with median preference parameter $\beta_M$ is certainly better off in the fully centralized system, which in turn implies that all agents are.

The comparison with the centralized commitment, decentralized consumption system is more intricate. As mentioned above, in period zero agents can no longer emulate period-one commitment decisions when considering a laissez faire economy, and for particular settings in which median preferences are not virtuous enough, mandating commitment alone may be superior to both full centralization and full decentralization. For example, suppose there are three groups of agents: ones that are rather impatient, ones that are moderately impatient, and ones that experience no self-control issues. Suppose further that the group of moderately impatient agents forms a majority, so whenever commitment is mandated, they dictate the level of commitment. Full centralization may be sub-optimal if the very patient individuals are sufficiently prominent: they would end up paying a significant cost for commitment and experience early cutting of the tree that they would not have had they been able to decide for themselves. Full decentralization may not be optimal since, now, the very impatient individuals may strictly benefit from the moderately patient agents forcing them to commit more substantially. Indeed, from period-zero perspective, the costs of commitment are effectively reduced by a factor of $\beta$. If they are sufficiently prominent as well, mandating commitment alone can be optimal.

Having said this, it is difficult to characterize general conditions in which the centralized commitment, decentralized consumption setting is optimal. For instance, for the quadratic example described in the paper, from period-zero perspective, full centralization is always optimal:

**Example 4 (Quadratic Costs – Period-Zero Welfare)** Consider the environment of Example 3. Figure 5 is the analogue of Figure 4—it depicts the period-zero welfare levels generated by the different processes as a function of the median agent’s preferences.\(^{33}\)

\(^{33}\)In analogy to Example 3, period-zero utility implies a weight of $\beta$ on each period’s instantaneous utility.
As can be seen, from period-zero perspective, full centralization is always optimal. Nonetheless, mandating commitment alone can generate greater period-zero welfare than the full decentralization when there are sufficiently many agents who are very impatient.

7 General Dynamic Decisions

Some of the insights highlighted by our model are shared by more general settings with time inconsistent voters. Indeed, suppose, quite generally, that at \( t = 1 \), agents choose menus of actions for \( t = 2 \) of the form \( A \subset \mathbb{R} \) from a set \( \mathcal{A} \). There is a default menu \( A_0 \) and a switch to any menu \( A \neq A_0 \) entails a cost \( c(A) > 0 \) (and \( c(A_0) = 0 \)). We assume that for any \( A_1 \neq A_2 \), \( c(A_1) \neq c(A_2) \), i.e., there is a unique mapping between costs and menus. At \( t = 2 \), agents choose an action from the menu selected in period 1. That is, they choose an action \( a \in A \).

For instance, in our tree-cutting context, we could consider \( A_0 = [0, 1] \) and a commitment technology that generates a cap on consumption. That is, for a cost \( c(\bar{x}) \), consumption in

We therefore normalize utilities by a factor of \( 3\beta \). We note, however, that even absent normalization, the qualitative message remains the same—mandating only commitment, or only consumption, is suboptimal.
period 1 would be constrained to the menu $A = [0, \bar{x}]$.

In order for a model of collective action to be non-trivial, the population needs to have heterogenous preferences, and we assume a continuum of agents parametrized by a preference type $\beta \in [0, 1]$, with a continuous and positive density. An agent of type $\beta$ chooses action $a(\beta; A)$ at $t = 2$ out of menu $A$. Time inconsistency implies that, at $t = 1$, any agent of type $\beta$ prefers to commit to some action $a \in A_0$ rather than entering the dynamic process and choosing $a(\beta; A)$ at a cost $c(A)$.

In this setting, we can study the effects of collective action, much as in the special case we focused on throughout the paper. At $t = 1$, each candidate $i$ offers a menu $A_i$, and the menu achieving majority support prevails. At $t = 2$, collective action is defined through simple majority rule as we have assumed throughout, though actions are confined to belong to the menu that was selected in period 1.

The observation that centralization at $t = 1$ combined with decentralization at $t = 2$ generates lower welfare than full decentralization follows immediately (even absent time inconsistency). Indeed, agents in the fully decentralized setting can always emulate collective decisions at $t = 1$. This generality may make one wonder why we analyze the centralized commitment, decentralized consumption setting. The reason is that we believe this setting is close in spirit to many applications and our formulation allows us to illustrate some natural comparative statics on the equilibrium that emerges in such settings. Furthermore, it allows us to identify more precisely the impacts of welfare considerations at $t = 0$, when only $t = 1$ centralization can be useful.

Suppose we further assume a minimal well-behavedness condition on preferences: at $t = 2$, preferences are single-peaked and for each $A$, $a(\beta; A)$ is monotonic in $\beta$.

Many of the observations we make throughout the paper carry through to this more general setting. For instance, under decentralization at $t = 1$ and centralization at $t = 2$, all choosing no investment at $t = 1$, i.e., remaining with menu $A_0$ and allowing the median agent of type $\beta_M$ to select the action $a(\beta_M; A_0)$, constitutes an equilibrium. That is, collective action at $t = 1$ undermines the median agent’s ability to commit. Suppose that left to her own devices, the median agent would select menu $A_M$ in period 1. As long as $a(\beta_M; A_M)$ (at a cost of $c(A_M)$) is preferred in terms of welfare to $a(\beta_M; A_0)$, full decentralization is superior to centralization only in period 2.

Let’s consider the special case of consumption caps for period 2 and discuss how political competition determines the choice of such caps in period 1, when consumption choices are decentralized in period 2. Our general setting translates into the following: in period 1, candidate $i$ proposes a cap $\bar{x}_i$ on the fraction of the tree that can be consumed by any
individual. If a candidate proposing $\bar{x}$ wins, then all voters can choose to consume $x \in [0, \bar{x}]$ in period 2. We assume that consumption caps are costly, with a cost of $\bar{x}$ denoted by $c(\bar{x})$ assumed to be decreasing in $\bar{x}$ to capture the idea that a looser cap is easier to enforce. In particular, $c(1) = 0$, so the cost of a non-binding cap is nil. As mentioned, in period 2 each agent can choose to consume a (non-negative) amount $x \leq \bar{x}$. Denote by $x(\beta)$ the ideal point from the perspective of an agent of type $\beta$ in period 1. Note first that for agents with $\beta \geq \frac{v_2}{v_3}$ consumption caps are never binding since these agents choose to consume zero in period 2. Therefore, $x(\beta) = 1$ for $\beta \geq \frac{v_2}{v_3}$ and hence these agents, with mass $1 - G\left(\frac{v_2}{v_3}\right)$, always vote for the highest (least costly) consumption cap. Of course, if this mass of voters forms a majority, then there cannot be any investment in consumption caps in equilibrium. Assume, then, that $G\left(\frac{v_2}{v_3}\right) > \frac{1}{2}$. Observe that for agents with $\beta < \frac{v_2}{v_3}$, any consumption cap $\bar{x} < 1$ is binding, since, absent any commitment, these agents choose to consume the entire tree in period 2. Figure 6 depicts $\bar{x}(\beta)$. Because, from the perspective of period 1 selves, the desired period 2 consumption decreases with $\beta$, this implies that $\bar{x}(\beta)$ is decreasing in $\beta$ (tighter caps) for $\beta < \frac{v_2}{v_3}$. Let $\beta^*$ be such that $G\left(\frac{v_2}{v_3}\right) - G(\beta^*) = 1/2$. That is, half the population has preferences that are between $\beta^*$ and $\frac{v_2}{v_3}$. Type $\beta^*$ is the pivotal agent in this environment, so the equilibrium level of consumption caps is $\bar{x}(\beta^*)$. It follows, and depicted in Figure 6, that $\beta^*$ is lower than the median $\beta$ and lower than $\beta^{CD}$ (the pivotal agent in the CD scenario).
8 Conclusions

The paper considers a simple setting in which behavioral agents, who in our case suffer from present bias, are also political actors, electing the government that is charged with “solving” their behavioral biases. While commitment instruments can be beneficial to individuals left to their own devices, we show the sensitivity of collective outcomes to the precise timing in which political processes take place and the underlying distribution of biases in the population. Commitment levels are lowest when only consumption is mandated. Under some regularity assumptions, commitment is highest when it alone is subject to collective action. When both commitment and consumption decisions are decided upon collectively, they reflect the preferences of the individual with median present-bias preferences.

From a welfare perspective, there is a complementarity between centralization at the commitment and consumption stages. Indeed, we show that either full centralization or full decentralization generate the highest welfare. Full centralization can be beneficial when there is a virtuous median voter. In that case, centralization effectively allows the population to delegate decisions to a virtuous agent.

These results are potentially relevant for many settings. One notable example is the design of pension systems in the U.S. and abroad. Indeed, a public pension system is sometimes defended as an effective solution to under-saving driven by self-control problems. Our results suggest that an analysis of the collective action of self-control is an important aspect of the design of such pension systems. A careful study of the design of a pension system for agents subject to self-control problems requires many specific details that are missing from our model. However, their design needs to take into consideration the political constraints imposed by those same individuals who are prone to self-control problems and comprise the electorate. These constraints may in principle affect the choice between a pay-as-you-go system and a funded system, the safeguards that are embedded in the system, as well as the timing and response of the system to demographic shocks. We view this as an important potential avenue for future research.

There are two important assumptions that underlie our analysis. First, we assume that agents are time inconsistent across periods of political decision making. The classical $\beta - \delta$ model does not specify the length of the time period for which lowered discount factors kick in. Nonetheless, as mentioned in the body of the paper, there is evidence supporting the idea that time inconsistency might be very relevant for political contexts (see also Bisin, Lizzeri, and Yariv, 2015). Second, we take a partial equilibrium approach in that the costs of commitment we analyze do not depend on the volume of demand for commitment. A more elaborate analysis of markets for commitment is left for future research.
From a broader perspective, accounts of behavioral biases have generated a rich literature that, by and large, focuses on individual actions. The paper offers some first steps to studies that allow for these same biases when considering policy determination, either ones that attempt at overcoming these biases (such as in the case of retirement savings) or otherwise. In particular, it suggests the potential importance of considering different biases (e.g., overconfidence, belief distortions in general, limited memory, etc.) as well as different types of policies (e.g., debt limits, general upholding of political promises, etc.) when inspecting political processes.34

34 As mentioned, some recent work has started moving in that direction. For example, Ortoleva and Snowberg (2015) study the impacts of over-confidence, while Diermeier and Li (2015) consider impacts on re-election and effort put by politicians who are facing voters with limited memory and an inclination to persist with their past voting behavior.
9 Appendix – Proofs

Proof of Lemma 1. Consider the intermediate region of $\beta$ parameters:

$$\beta \in \left(\frac{v_2 - \frac{\partial k(1,c)}{\partial x}}{v_3}, \frac{v_2 - \frac{\partial k(0,c)}{\partial x}}{v_3}\right).$$

Since $\frac{\partial^2 k(x(c,\beta),c)}{\partial x \partial c} > 0$, increasing $c$ leads to an increase in the right hand side of the first order condition specified in (1). Since $\frac{\partial^2 k(x(c,\beta),c)}{\partial x^2} > 0$ it follows that $x(c,\beta)$ is decreasing in $c$. Similarly, notice that $\frac{d(v_2 - \beta v_3)}{d\beta} < 0$ so that the left hand side of the first order condition is decreasing in $\beta$. The assumption that $\frac{\partial^2 k(x(c,\beta),c)}{\partial x \partial c} > 0$ then assures that $x(c,\beta)$ is decreasing in $c$. The lemma’s claim then follows.

Proof of Proposition 1. Lemma 2 assures that, under our regularity assumption, $c(\beta) > 0$ for all $\beta \in (\frac{v_2}{v_3}, \frac{v_2}{v_3})$. Furthermore, since we assume that fundamentals are such that $x(c,\beta)$ is well-behaved, $c(\beta)$ is continuous.

If $1 - G(\frac{v_2}{v_3}) \geq 1/2$, there is a majority of agents who prefer no commitment and the equilibrium commitment parameter is $c^{CD} = 0$, which coincides with that preferred by the median.

Suppose instead that $1 - G(\frac{v_2}{v_3}) < 1/2$. Let $B(\tilde{c}) = \{\beta \mid c(\beta) \leq \tilde{c}\}$. From Lemma 2, agents have single-peaked preferences. Therefore, for any $\tilde{c}$, all agents of type $\beta \in B(\tilde{c})$ strictly prefer commitment level $\tilde{c}$ to any commitment level $c > \tilde{c}$. For any $0 < c_1 < c_2 \leq \max_{\beta} c(\beta)$, $B(c_1) \subseteq B(c_2)$. Therefore, continuity of $c(\beta)$ assures there exists a unique $c^{CD}$ such that $G(B(c^{CD})) = 1/2$. For any parameter $c > c^{CD}$, there is a strict majority preferring lower commitment, while for any $c < c^{CD}$, there is a strict majority preferring greater commitment. It follows that $c^{CD}$ defines the unique equilibrium commitment level.

When $1 - G(\frac{v_2}{v_3}) < 1/2$ and $c(\beta)$ has a unique maximum, there exist $\beta_L, \beta_H \in (\frac{v_2}{v_3}, \frac{v_2}{v_3})$ such that $B(c^{CD}) = [\beta, \beta_L] \cup [\beta_H, \beta]$. Since $G$ is continuous and $G(\beta_L) + (1 - G(\beta_L)) = 1/2$, we have that $G(\beta_L) < 1/2$ and $1 - G(\beta_H) < 1/2$. It follows that $\beta_M \in (\beta_L, \beta_H)$. Notice that $c^{CD} = c(\beta_L) = c(\beta_H)$. It follows that $\beta^{CD} = \beta_H > \beta_M$. 

\[\boxed{}\]
Proof of Lemma 3. Let us first consider the case in which \( x(c, \beta_M) \) is interior.

From the first order condition of the median voter in period 2 we know that whenever \( x(c, \beta_M) > 0 \),

\[
v_2 - \beta_M v_3 = \frac{\partial k(x(c, \beta_M), c)}{\partial x}.
\]

Consider the effect of \( c \) on an agent of taste parameter \( \beta \) who foresees that period 2 decisions will be made by the median voter.

\[
\frac{\partial U_1}{\partial c} = \frac{\partial x(c, \beta_M)}{\partial c} (\beta v_2 - \beta v_3) - \beta \frac{\partial k(x(c, \beta_M), c)}{\partial c} - \beta \frac{\partial k(x(c, \beta_M), c)}{\partial x} \frac{\partial x(c, \beta_M)}{\partial c} - I'(c).
\]

The first order condition for \( \beta_M \) implies that

\[
v_2 - v_3 = -(1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial x}
\]

and so

\[
\frac{\partial U_1}{\partial c} = -\beta \left( \frac{\partial x(c, \beta_M)}{\partial c} (1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial c} \right) - I'(c).
\]

Now, if \( \frac{\partial x(c, \beta_M)}{\partial c} (1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial c} \geq 0 \), all agents prefer \( c = 0 \) and the claim follows.
Otherwise, \( \frac{\partial x(c, \beta_M)}{\partial c} (1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial c} < 0 \) and \( \frac{\partial U_1}{\partial c} \) is increasing in \( \beta \). At a maximum, \( U_1 \) is (weakly) concave and the claim follows.

We now consider the cases in which \( x(c, \beta_M) \) may be at a corner solution. Note first that when \( \beta_M \geq \frac{v_2}{v_3} \), then \( x(c, \beta_M) = 0 \) for all \( c \). In this case, all agents prefer \( c = 0 \) in period 1 and the claim follows. More generally, we have

\[
x(c, \beta_M) = \begin{cases} 
0 & c \geq c_H(\beta_M) \\
(v_2 - \beta_M v_3) = \frac{\partial k(x(c, \beta_M), c)}{\partial x} & c_L(\beta_M) < c < c_H(\beta_M) \\
1 & c \leq c_L(\beta_M)
\end{cases}.
\]

Clearly, there is no value in choosing \( c > c_H(\beta_M) \). Thus, \( c \geq c_H(\beta_M) \) and

\[
U_1(\beta, c_H(\beta_M)) = \beta v_3 - I(c_H(\beta_M)).
\]

Comparing this to interior cases:

\[
U_1(\beta, c_H(\beta_M)) - U_1(\beta, c) = \beta v_3 - I(c_H(\beta_M)) - \beta v_3 + x(c, \beta_M) (\beta v_2 - \beta v_3) - \beta k(x(c, \beta_M), c) - I(c) = \\
= \beta (x(c, \beta_M)(v_3 - v_2) - k(x(c, \beta_M), c)) - (I(c_H(\beta_M)) - I(c)).
\]

If \( c_H(\beta_M) \) is optimal for some \( \hat{\beta} \), it has to be the case that

\[
\hat{\beta} (x(c, \beta_M)(v_3 - v_2) - k(x(c, \beta_M), c)) > (I(c_H(\beta_M)) - I(c))
\]

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for all \( c < c_H (\beta_M) \). But then, this also holds for all \( \beta > \tilde{\beta} \).

It is easy to see that it must be the case that when \( c \leq c_L (\beta_M) \), then the optimal \( c \) is zero: there is no point in investing anything in commitment if it does not help. In this case, the payoff in the first period is \( U_1 (\beta, 0) = v_2 \). Comparing this to interior cases:

\[
U_1 (\beta, c_H (\beta_M)) - U_1 (\beta, c) = \beta v_2 - (\beta v_3 + x (c, \beta_M) (\beta v_2 - \beta v_3) - \beta k (x (c, \beta_M), c) - I (c)) = -\beta ((v_3 - v_2) (1 - x (c, \beta_M)) - k (x (c, \beta_M), c)) + I (c).
\]

If a choice of zero commitment is optimal for some \( \tilde{\beta} \) it has to be the case that

\[
\tilde{\beta} ((v_3 - v_2) (1 - x (c, \beta_M)) - k (x (c, \beta_M), c)) < I (c)
\]

for all \( c > c_L (\beta_M) \). But then this also holds for all \( \beta < \tilde{\beta} \).

\[\text{Proof of Proposition 4.}\] Investment in commitment is beneficial to an individual voter only if the investment changes the consumption decision in the second period. However, no individual agent is pivotal in the second period. Hence, consumption choices in the second period are not responsive to individual investment decisions. Thus, the optimal level of investment in equilibrium is zero.

\[\text{Proof of Proposition 6.}\]

1. Whenever \( \beta_M \geq \beta^* \), in the fully centralized system there is fully delayed consumption, \( x (c (\beta_M), \beta_M) = 0 \) even with no commitment, \( c (\beta_M) = 0 \). In particular, since \( k (x, 0) = 0 \), any agent of preference parameter \( \beta \) experiences a period 1 utility of \( \beta v_3 \). In contrast, in the fully decentralized system, while all agents with \( \beta \geq \beta^* \) experience the same period 1 utility as in the fully centralized system, agents with \( \beta \in (\underline{\beta}, \beta^*) \) choose \( c (\beta) > 0 \) at a cost of \( I (c (\beta)) > 0 \) (as well as potentially some early consumption), hence they receive a utility that is strictly lower than \( \beta v_3 \). Since the distribution \( G \) is continuous and \( G (\beta^*) > 0 \), the result follows.

2. Consider a sequence of distributions \( \{G_n\}_{n=1}^{\infty} \) such that there exists \( \tilde{\beta} > \beta \) for which \( \{G_n (\tilde{\beta})\} \) is uniformly bounded below 1 and \( \lim_{n \to \infty} \beta_M^n = \underline{\beta} \). Under full centralization, continuity implies that for any \( \varepsilon > 0 \), there exists \( \tilde{n} (\varepsilon) \) such that for any \( n > \tilde{n} (\varepsilon) \), for all \( \beta \), period 1 utility under full centralization, \( U_1^{CC} (\beta; n) \) satisfies:

\[
U_1^{CC} (\beta; n) < \beta \left( v_2 x (c (\underline{\beta}, \beta) + v_3 (1 - x (c (\underline{\beta}, \beta))) - I (c (\underline{\beta})) + \varepsilon.
\]
By Lemma 2, $c(\beta) > 0$ for all $\beta < \beta^*$. Hence, under full decentralization, from Lemma 1, $x(c(\beta), \beta)$ is strictly decreasing in $\beta$. Since agents can always choose $c(\beta)$ themselves, it follows that for sufficiently small $\varepsilon$, there exists $\beta(\varepsilon) \in (\underline{\beta}, \overline{\beta})$ such that for any $\beta \geq \beta(\varepsilon)$, and any $n > \tilde{n}(\varepsilon)$, decentralized choices generate first-period utility $U_1(x(c, \beta), c(\beta), \beta)$ satisfying:

$$U_1(x(c, \beta), c(\beta), \beta) - U_1^{CC}(\beta; n) > 0.$$ 

Furthermore, $\lim_{\varepsilon \to 0} \beta(\varepsilon) = \underline{\beta}$, and for sufficiently small $\varepsilon > 0$, for any $\beta \geq \tilde{\beta}$, $n > \tilde{n}(\varepsilon)$,

$$U_1(x(c, \beta), c(\beta), \beta) - U_1^{CC}(\beta; n) > \Delta > 0.$$ 

Suppose then that $G_n(\tilde{\beta}) \leq \gamma < 1$ for all $n$. For sufficiently small $\varepsilon$, it follows that

$$\int_{\beta}^{\beta(\varepsilon)} U_1^{CC}(\beta; n) dG_n(\beta) < \int_{\beta}^{\beta(\varepsilon)} U_1(x(c, \beta), c(\beta), \beta) dG_n(\beta) + \int_{\beta}^{\tilde{\beta}} \frac{\Delta}{1 + 2\beta} dG_n(\beta).$$

In particular, for any $n > n^* = n(\varepsilon)$, where $\varepsilon$ is sufficiently small, welfare satisfies:

$$\Pi^{CC} = \int_{\beta}^{\beta(\varepsilon)} U_1^{CC}(\beta; n) dG_n(\beta) < \int_{\beta}^{\beta(\varepsilon)} U_1^{CC}(\beta; n) dG_n(\beta) + \int_{\beta}^{\beta(\varepsilon)} U_1(x(c, \beta), c(\beta), \beta) dG_n(\beta)$$

$$+ \int_{\beta}^{\tilde{\beta}} U_1(x(c, \beta), c(\beta), \beta) - \Delta \frac{dG_n(\beta)}{1 + 2\beta} < \Pi^{DD}.$$ 

$$\Pi^{CC} = \int_{\beta}^{\beta(\varepsilon)} U_1^{CC}(\beta; n) dG_n(\beta) < \int_{\beta}^{\beta(\varepsilon)} U_1^{CC}(\beta; n) - \varepsilon dG_n(\beta) + \int_{\beta(\varepsilon)}^{\beta} U_1^{CC}(\beta) - \varepsilon + \Delta \frac{dG_n(\beta)}{1 + 2\beta} < \Pi^{DD}.$$ 

\[\square\]
References


