Collective Self-Control

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Abstract

Behavioral economics presents a “paternalistic” rationale for a benevolent government’s intervention. We consider an economy where the only “distortion” is agents’ time inconsistency. We study the desirability of various forms of collective action, ones pertaining to costly commitment and ones pertaining to the timing of consumption, when government decisions respond to voters’ preferences via the political process. Three messages emerge. First, welfare is highest under either full centralization or laissez faire. Second, introducing collective action only on consumption decisions yields no commitment. Last, individuals’ relative preferences for commitment may reverse depending on whether future consumption decisions are centralized or not.

Keywords: Behavioral Political Economy, Time Inconsistency, Hyperbolic Discounting.

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1 Introduction

Traditional public economics provides efficiency rationales for government intervention that are commonly founded in payoff or information externalities. In particular, none of these rationales justify government policy in areas in which agents make private decisions that do not have impacts on other agents. The behavioral economics literature has introduced a novel justification for government intervention arising from “paternalistic attitudes.”\(^1\) This approach is controversial, partly because it drastically departs from standard normative economics.\(^2\) It has, however, proven influential in the policy realm. For instance, in the U.S., some discussion of social security takes an explicitly paternalistic approach by viewing it as a necessary program to correct for many individuals’ inability to properly save for retirement.\(^3\) In the U.K., the Behavioural Insights Team, also known as the Nudge Unit, which is partially owned by the government, was put in place with the intention of applying insights from behavioral economics and psychology to improve government policy and services. On its website, as part of its stated objectives are “improving outcomes by introducing a more realistic model of human behaviour to policy; and whenever possible, enabling people to ‘make better choices for themselves.’”\(^4\)

Just as for textbook public policy analysis, it is useful to consider what happens when we abandon the idea of a benevolent planner and instead explicitly model the fact that the political process determines the design of policy. Will politicians seeking election exploit/indulge voters’ behavioral distortions? Are behavioral distortions amenable to aggregation into collective action? What are the implications for the constitutional scope of government activity? The goal of this paper is to develop a tractable model of the potential political economy constraints on the implementation of paternalistic policies in one special, but important, setting.

There are of course many types of behavioral distortions, and each of them may lead to its own collective action environment. We focus here on self-control problems: agents have preferences that display present bias or quasi-hyperbolic discounting a-la Phelps and Polak (1968) and Laibson (1997). These self-control problems can lead to phenomena such


\(^2\)See, for instance, the essays in Caplin and Schotter (2008).


\(^4\)See www.behaviouralinsights.co.uk/about-us/
as procrastination, insufficient savings for retirement,\textsuperscript{5} and harmful obesity and addictions.\textsuperscript{6} Furthermore, self-control problems can generate a demand for commitment (rehab clinics, illiquid assets with costly withdrawals, and so on). In principle, a benevolent government could offer commitment instruments that would help the electorate overcome some of the harmful symptoms of time inconsistency.

Once we depart from the world of hypothetical social planners, however, the set of feasible outcomes is constrained by the political incentives faced by politicians and it is important to understand how these incentives are affected by the set of available policies. Time inconsistency offers a simple case study to illustrate how political forces shape allocations in a world where there is scope for “paternalistic policies.”\textsuperscript{7}

In our model, an electorate composed of heterogeneous time-inconsistent individuals must make two decisions: how much to invest in commitment instruments, and how to allocate consumption over time. We study the outcomes that emerge from several political processes that impact either or both decisions. Namely, we consider systems in which investment in commitment (say, in the form of 401K accounts that penalize early withdrawals) is mandated collectively or ones in which government intervenes at the time of consumption (say, by using government transfers). While our discussion focuses on electoral settings, our analysis applies immediately to general settings of committee decision making.

Specifically, we study a simple Wicksellian tree-cutting problem, under the standard specification that the tree is growing in value over time. In our baseline setting, agents have the option of cutting a tree at period 2, which generates a value of $v_2$, or at period 3, which generates a value of $v_3$, where $v_3 > v_2$. A tension arises since agents exhibit present bias. At any period, all future periods are discounted with a factor of $\beta \leq 1$, which is distributed in an arbitrary (but continuous) way in the population. Thus, from the perspective of period 1, all agents prefer to wait until period 3 to cut the tree. But when period 2 arrives, agents compare an immediate value of $v_2$ with a discounted value of $\beta v_3$ and some could potentially prefer to cut the tree early. This problem has been studied by O’Donoghue and Rabin (1999), who show that time-inconsistent agents tend to consume (cut the tree) inefficiently early.

\textsuperscript{5}See, for instance, O’Donoghue and Rabin 1999 and Laibson, Repetto, and Tobacman 1998.

\textsuperscript{6}E.g, O’Donoghue and Rabin (2000), Gul and Pesendorfer (2007).

\textsuperscript{7}Bisin, Lizzetti, and Yariv (2015) study a model of fiscal irresponsibility and public debt in the presence of time-inconsistent voters. The model they consider captures environments where it is either impossible for government to help agents to achieve commitments or it is positively harmful for the government to do so. Their model does not quite fit into any of the scenarios that we discuss in this paper, but does highlight the potential harmful effects government intervention may have in the realm of fiscal policy when voters exhibit time inconsistencies. In fact, the paper offers a new rationale for balanced budget rules in constitutions as they restrain governments’ responses to voters’ desires.
and that these agents would find it valuable to commit to cutting the tree later (namely, in period 3).

We modify the O’Donoghue-Rabin model to allow for continuous choices and costly commitment: by investing resources in period 1, agents can make it costly for their future selves to depart from some pre-specified plan of action. The more investment there is early on in commitment, the more costly it is for future selves to cut the tree too early. Indeed, there are many examples in which individuals use costly commitment devices. For instance, as of the writing of this paper, there has been a collective investment of over $22 million in individual contracts through stickk.com. These contracts provide explicit financial punishments for not sticking to pre-specified commitments, which vary among users and include smoking cessation, exercise, work targets, etc.\footnote{See also Della Vigna and Malmendier (2006), Ayres (2010), and references therein.} There are also various ways in which governments invest in commitment instruments, most notably ones having to do with retirement savings or drug prohibition.\footnote{For a description of current public sector pension plans, see Beshears et al. (2011) and for a review of alcohol policy in the U.S., see Babor (2003). The introduction of graphic warning labels on cigarette packs has been the topic of recent controversy and is covered in http://www.cnn.com/2013/03/19/health/fda-graphic-tobacco-warnings.}

We outline three different scenarios that vary in terms of which choices (investment in commitment and/or the timing of consumption) are subject to the political process, and which ones are left to individuals. That is, we consider collective action pertaining to either or both the commitment decision and the consumption decision, in addition to considering a laissez faire economy in which all decisions are taken individually. We assume collective decisions resulting from a political process are determined by the outcome of competition between two office-seeking candidates. We believe these scenarios offer a simple taxonomy for an array of plausible environments. In addition, the characterization of equilibria under the different scenarios helps highlight the sensitivity of the resulting welfare levels to the aspects, or timing, of choices in which collective action comes into play.\footnote{We abuse terminology by referring to “welfare” as the utilitarian social surplus measured for the period 1 selves of the voters. We acknowledge that other criteria are relevant and we discuss this more explicitly in Section 6.}

The are three main insights that emerge from our analysis. First, \textit{partial interventions are dominated by either full intervention or no intervention}. This is a complementarity result between intervention in commitment instruments and intervention in consumption decisions.\footnote{Mandating the timing of consumption can be viewed as a metaphor for, e.g., government transfer payments in the form of welfare, disability insurance, food stamps, or supplemental security income. See Stephens}
the median voter, so welfare levels depend on whether the median voter’s bias is relatively small or large. In addition, when only some decisions are centralized but others are made privately by the agents, one must take into account how agents respond to the government’s interventions: we display important ways in which partial interventions can be worse than no intervention.

Second, introducing collective action only on consumption decisions yields no commitment. One would expect that collective action on the consumption decision provides commitment and is therefore a valuable alternative to costly commitment investments. In general, delegating control seems like a good way to get around one’s self-control problems. However, this delegated control undermines the incentives to invest in commitment. Thus, the collective consumption decision that is taken by the median voter is undertaken with no help from investment in commitment. This immediately implies that individuals with preferences in a neighborhood of the median voter are made worse off by government intervention.

Last, individuals’ relative preferences for commitment may reverse depending on whether future consumption decisions are centralized or not. An individual with high \( \beta \) has sufficient self-control and may not need to invest in commitment technologies when she decides on her own consumption. However, when the consumption allocation is delegated to an agent with lower \( \beta \), she prefers a larger investment in commitment, since, from her perspective, the need for commitment becomes larger. This effect is stronger the larger the individual’s \( \beta \) is relative to the median voter.

## 2 Related Literature

There are three main components of the setting we study in this paper: 1. Agents are time inconsistent; 2. Political leaders offer policies directed at addressing those individual time inconsistencies; and 3. Political leaders are not benevolent, but rather select policies that maximize their chances of being elected under majority rule.

The three components appear in Hwang and Mollerstrom (2014) who focus on a particular environment of political reforms when voters are time-inconsistent. They show that gradualism emerges in equilibrium as a consequence of time inconsistency. They also show that election of a patient agenda setter can arise in equilibrium. Without time inconsistencies (the first component of our analysis), Ortoleva and Snowberg (2015) look at the potential effects of over-confidence on electoral outcomes.

The third component, pertaining to politicians who strategize rather than optimize wel-
fare, has been considered by several authors (Benjamin and Laibson 2003, Caplan 2007, Glæser 2006, Rizzo and Whitman 2009a,b). These authors have informally made the point that when government is not run by a benevolent social planner but by politicians influenced by voting decisions, it is not clear that government intervention is beneficial. In fact, Glæser and Caplan explicitly make the case that, if voters are boundedly rational, then the case for limited government may be even stronger than in standard models.\textsuperscript{12}

The first two components of our setting have been considered by Krusell, Kurusçu, and Smith (2002, 2010). They examine government policy for agents who suffer self-control problems. Krusell, Kurusçu, and Smith (2002) consider a neoclassical growth model with quasi-hyperbolic consumers. They show that, when government is benevolent but cannot commit, decentralized allocations are Pareto superior. This is due to a general equilibrium effect of savings that exacerbates an under-saving problem. Benabou and Tirole (2006) discuss how endogenously biased beliefs that are chosen by individuals for self-motivation can generate a belief in a just (or unjust) world and ultimately affect redistributive politics.

Outside of the political context, Gottlieb (2008) studies the optimal design of non-exclusive contracts when firms compete over time-inconsistent consumers. The paper studies the asymmetry between immediate-cost goods and immediate-reward goods that are generated by non-exclusivity. Hiedhues and Koszegi (2010) study contract choice, loan-repayment patterns, and welfare in a model of a competitive credit market when consumers are time inconsistent. They show that prohibiting large penalties for delayed repayments may be welfare enhancing for unsophisticated consumers. To the extent that firms are akin to political competitors, some of the underlying forces in these papers are relevant for the study of political processes with a time-inconsistent electorate.

Time inconsistency and commitment problems faced by politicians, rather than the electorate, have been the focus of a large literature in political economy and macroeconomics, especially in the context of government debt and monetary policy (e.g., Persson and Tabellini 1990, Alesina and Tabellini 1990). In those models, voters are time consistent, but the identity of the decision maker (or decisive voter) changes over time, generating time inconsistent policies. This in turn creates an incentive for early decision makers to manipulate state variables, such as debt, in order to influence subsequent decisions.\textsuperscript{13} There is also evidence

\textsuperscript{12}Bendor, Diermeier, Siegel, and Ting (2011) present models of boundedly rational voters that are successful in matching some features of elections that are hard to explain with rational voter models. Diermeier and Li (2013) study the outcomes of dynamic majoritarian elections with ‘behavioral’ voters who exhibit some persistence in their voting and forgetfulness of past political outcomes.

\textsuperscript{13}There is also work (e.g., Lagunoff, 2008) that shows that, if one considers governments that have policy preferences and that know that they may be kicked out of office with positive probability, endogenous present
that time inconsistency may have been at the root of the historical design of pension systems.\footnote{Jacobs (2011) provides a comparative history of pension systems, where commitment problems are emphasized as an explanation for why some countries chose, or eventually turned to, a pay-as-you-go system.} In this literature time inconsistency of political choices emerges from the interaction among time consistent agents who act at different points in time. Piguillem and Riboni (2015) also consider politicians who have a present bias for spending and bargain dynamically. They show that disagreement leads to more persistent policies and attenuation of the immediate desire of bargaining proposers to over-spend. Halac and Yared (2014) consider time-inconsistent governments and illustrate that, when shocks to the value of spending exhibit persistence, the ex-ante optimal fiscal rule is history dependent, with high shocks leading to strong future fiscal discipline compared to low shocks.

Our analysis complements this work by studying the consequences of having agents with heterogeneous degrees of time inconsistency participate in the political process. For instance, as mentioned above, a public pension system is sometimes defended as a desirable solution to a potential problem of under-saving due to self-control problems. However, the design of such a system should then take into account the political constraints generated by an electorate composed of voters with these self-control problems. As it turns out, the induced constraints are quite different from those considered in the literature on time inconsistent policy driven by a sequence of time consistent agents. These constraints may affect the choice between a pay-as-you-go system and a funded system, the kind of safeguards that are designed into the system, as well as the timing and evolution of the system.

3 A Tree Cutting Model

3.1 Preferences and Consumption Possibilities

A continuum of agents decides collectively on the timing of consumption. There are three periods. In period 1 agents make “commitment” decisions (that we specify below). In periods 2 and 3 agents consume fractions of a “tree” of growing value. The tree is worth $v_2$ in period 2, and $v_3$ in period 3. We assume that $v_2 < v_3$.\footnote{Our qualitative results remain in the presence of uncertainty over future tree values.} In period 2 agents choose a fraction $x$ of the tree to consume in period 2, with $1 - x$ remaining to be consumed in period 3. We interpret period 3 as the natural moment of maturity of the tree so that there is an extra cost in cutting part of the tree in period 2. This cost is given by the function $k(x, c)$, where $c$ is a parameter that is determined in the first period.
Agents have $\beta - \delta$ preferences. That is, for any payoffs $u_2$ and $u_3$ in periods 2 and 3, respectively, the assessed utility at time $t$, denoted by $U_t$, is given by:

\[
U_1 = \beta \delta u_2 + \beta^2 \delta u_3,
U_2 = u_2 + \beta \delta u_3,
U_3 = u_3.
\]

Individuals are heterogeneous in their present-bias parameter: $\beta$ is distributed according to a continuous distribution $G[\underline{\beta}, \overline{\beta}]$ with a median parameter denoted by $\beta_M$.

An agent with parameter $\beta$ has a utility at $t = 2$ given by:

\[
U_2(x, c, \beta) = v_2 x - k(x, c) + \beta \delta v_3 (1 - x)
\]

In period 1 a parameter $c$ is chosen (potentially by a collective action process that we soon specify). This parameter raises the cost of cutting the tree early. We assume that $\frac{\partial k(x, 0)}{\partial x} = \frac{\partial k(x, 0)}{\partial c} = 0$ for all $x$, so that absent commitment, there is no marginal cost of cutting the tree early. We also assume that $\frac{\partial k(x, c)}{\partial x} \geq 0$, $\frac{\partial^2 k(x, c)}{\partial x^2} > 0$, $\frac{\partial k(x, c)}{\partial c} > 0$, and $\frac{\partial^2 k(x, c)}{\partial x \partial c} > 0$. That is, cutting costs are weakly increasing and convex in the amount of the tree that is cut $x$ and in the extent of commitment in place, as given by the commitment parameter $c$. The marginal cost of early consumption is also increasing in $c$. Thus, $c$ serves as a commitment mechanism to delay consumption to period 3. This commitment is costly in period 1: choosing $c$ costs $I(c)$. We assume $I(0) = 0, I'(0) = 0, I'(c) \geq 0$, and $I''(c) > 0$ for all $c$.\footnote{The assumption that $I(0) = 0$ is not restrictive. Indeed, assuming $I(0) > 0$ is tantamount to assuming there is a fixed cost to entering our economy.} \footnote{For the most part we will treat the investment in commitment and the associated increase in $k(x, c)$ as resource costs that should be counted in welfare calculations. However, for some applications, such as 401K plans, these are taxes whose revenues are not a deadweight loss. Such cases can be accommodated simply by reinterpreting the resource costs as the pure resources involved in administering the tax. Of course, our welfare calculations would then have to be performed differently. Nonetheless, it turns out that our main welfare results are robust to this modification.} The regularity restrictions we impose on $k(x, c)$ and $I(c)$ are sufficient for our results and simplify our presentation, but are by no means necessary (in fact, in our running example we will drop the requirement that $\frac{\partial k(x, 0)}{\partial x} = 0$).

Utility in period 1 is given by

\[
U_1(x, c, \beta) = \beta \delta (v_2 x - k(x, c)) + \beta^2 \delta v_3 (1 - x) - I(c).
\]

Agents are assumed to be sophisticated, in the sense that they are aware that they are time inconsistent. O’Donoughue and Rabin (1999) analyzed the single person decision problem

\[\text{agents have } \beta - \delta \text{ preferences. That is, for any payoffs } u_2 \text{ and } u_3 \text{ in periods 2 and 3, respectively, the assessed utility at time } t, \text{ denoted by } U_t, \text{ is given by:}
\]

\[U_1 = \beta \delta u_2 + \beta^2 \delta u_3,
U_2 = u_2 + \beta \delta u_3,
U_3 = u_3.
\]

\[\text{individuals are heterogeneous in their present-bias parameter: } \beta \text{ is distributed according to a continuous distribution } G[\underline{\beta}, \overline{\beta}] \text{ with a median parameter denoted by } \beta_M.
\]

\[\text{an agent with parameter } \beta \text{ has a utility at } t = 2 \text{ given by:}
\]

\[U_2(x, c, \beta) = v_2 x - k(x, c) + \beta \delta v_3 (1 - x)
\]

\[\text{in period 1 a parameter } c \text{ is chosen (potentially by a collective action process that we soon specify). This parameter raises the cost of cutting the tree early. we assume that } \frac{\partial k(x, 0)}{\partial x} = \frac{\partial k(x, 0)}{\partial c} = 0 \text{ for all } x, \text{ so that absent commitment, there is no marginal cost of cutting the tree early. we also assume that } \frac{\partial k(x, c)}{\partial x} \geq 0, \frac{\partial^2 k(x, c)}{\partial x^2} > 0, \frac{\partial k(x, c)}{\partial c} > 0, \text{ and } \frac{\partial^2 k(x, c)}{\partial x \partial c} > 0. \text{ that is, cutting costs are weakly increasing and convex in the amount of the tree that is cut } x \text{ and in the extent of commitment in place, as given by the commitment parameter } c. \text{ the marginal cost of early consumption is also increasing in } c. \text{ thus, } c \text{ serves as a commitment mechanism to delay consumption to period 3. this commitment is costly in period 1: choosing } c \text{ costs } I(c). \text{ we assume } I(0) = 0, I'(0) = 0, I'(c) \geq 0, \text{ and } I''(c) > 0 \text{ for all } c.\]

\[\text{utility in period 1 is given by}
\]

\[U_1(x, c, \beta) = \beta \delta (v_2 x - k(x, c)) + \beta^2 \delta v_3 (1 - x) - I(c).
\]
in this environment, by using the notion of perception perfect equilibrium. When agents are sophisticated, this boils down to preferences that are specified a-la Strotz (1955) who perform backwards induction. We assume sophistication because we want to study how the demand for commitment is mediated by the political system.\textsuperscript{18}

For notational simplicity, we assume $\delta = 1$ for the remainder of the analysis. This assumption is effectively without loss of generality (as discounting can be encoded in the $\nu$ sequence of tree values).

3.2 The Political Process

There are two candidates running for office. Candidates are office motivated, receiving some positive payoffs from each electoral victory. It will be clear that candidates’ time preferences play no role in this model.\textsuperscript{19} We assume that the electorate has no ideological attachment to the candidates.\textsuperscript{20}

We distinguish between three types of environments. These are meant to capture different collective action settings and highlight the effects of the timing of collective decisions on commitment choices.

**Centralized Commitment, Centralized Choice.** Elections occur in periods $t = 1, 2$. At $t = 1$, each candidate offers a platform consisting of a cost $c$ that determines the cost of consumption in period 2 later on. Majority voting determines which outcome, and corresponding platform, is elected (we assume that ties are broken with a toss of a coin). If the platform $c_i$ is implemented, all agents experience an immediate commitment cost of $I(c_i)$ at $t = 1$. At $t = 2$, the candidates each offer a fraction $x_j$ of the tree to be consumed in period 2 and majority rule (with random breaking of ties) determines which policy is implemented. If an amount $x$ of the tree is consumed at $t = 2$, an agent with taste parameter $\beta$ receives the value of $v_2 x_j + \beta v_3 (1 - x_j)$. All agents experience an immediate cost of $k(x_j, c_i)$.

**Dentalized Commitment, Centralized Choice.** At $t = 1$, agents choose individually the parameter $c$ that will induce their commitment-breaking costs at time $t = 2$, the cost of which is immediate and given by $I(c)$. At $t = 2$, fraction $x$ of the tree to be consumed in period 2 and majority rule (with random breaking of ties) determines

\textsuperscript{18}In Section 7 we discuss the effects of naivete in our model.
\textsuperscript{19}This is not to say that time inconsistencies cannot take place directly at the political level. As mentioned in the discussion of the related literature, there is a body of work that focuses on time consistency of government policy.
\textsuperscript{20}Allowing agents to have idiosyncratic ideological preferences (as in Lindbeck and Weibull 1987) does not change any of the results qualitatively. Details are available from the authors upon request.
which policy \( x_i \) is implemented for the entire population. An individual with taste parameter \( \beta \) who chose a commitment parameter of \( c \) at \( t = 1 \) receives a net value of \( v_2 x_i + \beta v_3 (1 - x_i) \) and experiences an immediate cost of \( k(x_i, c) \).

**Centralized Commitment, Decentralized Choice.** Elections occur only in period \( t = 1 \), when each candidate offers a platform consisting of a commitment parameter \( c_i \) involving an immediate commitment cost of \( I(c_i) \). Majority voting determines which outcome, and corresponding platform is elected (again, ties are broken randomly). At \( t = 2 \), each of the individual agents decides what fraction \( x \) of the tree to consume. An individual with taste parameter \( \beta \) who chooses to consume a fraction \( x \) of the tree at \( t = 2 \) receives a net value of \( v_2 x + \beta v_3 (1 - x) \) and experiences an immediate cost of \( k(x, c_i) \).

In what follows, we analyze the outcomes of a fully decentralized economy in addition to each of the above settings in turn. All proofs are relegated to the Appendix.

### 3.3 Discussion of the Model

Our model of collective action builds on the individual decision-making problem proposed by O’Donoughue and Rabin (1999), who first considered a version of the type of tree-cutting decision problem analyzed in this paper. We extend this model in two ways and then introduce collective action. First, we allow the agents to **invest** in commitment or self-control. This is reasonable as there are many cases in which the government or individuals can take actions to restrain their future selves (401K plans, rehab programs, weight watchers, internal psychological mechanisms). Notice that if commitment were free, all agents would commit themselves to later consumption and there would effectively be no self-control problem. Consequently, decentralized decisions would lead to first-best outcomes in which all agents would efficiently and fully delay their consumption. The introduction of commitment costs introduces a non-trivial cost-benefit trade-off in decentralized economies, that we soon analyze, which creates room for potentially beneficial government intervention. A second, more technical modification of the model presented by O’Donoughue and Rabin is that they assume that agents only choose when to cut the tree and cannot cut fractions of it. This is tantamount to a special case of our model in which \( k(x, c) \) is linear. We delay the discussion of this special case until Section 7.5 as it introduces some technical complications within the analysis of the Centralized Commitment-Decentralized Choice scenario. We now discuss several aspects of our setting and their applicability.

**Preference Heterogeneity.** The only dimension of heterogeneity that we explore is the degree of present bias \( \beta \). We view this as a natural step given the questions we are studying.
since experimental and empirical studies of self-control problems suggest non-trivial degrees of heterogeneity (e.g., Augenblick, Niederle, and Sprenger 2015, Fang and Wang 2015, and references therein). We note, however, that some of our qualitative analysis would remain the same had we assumed a unique $\beta < 1$ in the population and a distribution of commitment technologies (each individual having access to a private commitment cost function $I$). Similarly, allowing for a non-trivial distribution of both present-bias preferences and commitment technologies would generate qualitatively similar results, though complicate our presentation.

Political Intervention. We have chosen to study four scenarios that differ in their degree of centralization of choices. These are natural scenarios and we show below that it is important to distinguish between government action over consumption, such as government transfer payments in the form of welfare, disability insurance, food stamps, or supplemental security income, and government action over commitment, such as 401K pension plans that penalize withdrawals before retirement age, prohibition, cigarette ad campaigns, etc.\(^21\) Furthermore, comparing laissez faire to various degrees of government centralization is an old question in economics and it is useful to understand how centralization affects outcomes in a world with no other reason for government intervention other than “behavioral” distortions. There are, of course, many other scenarios that can be studied. We discuss some of these in Section 7.

Format of Collective Action. We have modeled collective action as elections of office-motivated candidates. We have done this in part because this is the most standard way to approach political economy models, and is therefore a good starting point to explore collective self-control in this benchmark setting. However, the main forces behind our results are likely to be present in several alternative specifications of the political system. Two particular natural extensions (discussed in Section 7) are the following. First, one could think of collective action generating more targeted policies that affect individuals differentially, say in the form of commitment subsidies or consumption caps. Second, throughout our analysis we take the commitment technology itself (namely, the functions $I(c)$ and $k(x,c)$) as exogenous. While some of the implied costs may be psychological and rather non-malleable, others may stem from institutions and could be an object of political choice as well.\(^22\)

We intentionally ignore externalities among agents other than those induced by the

\(^{21}\) The model is not tailored to study addiction. However, the key force of commitment to delay undesirable temptations for immediate gratification is relevant for addiction as well.

\(^{22}\) We also note that our results carry through almost directly to a citizen-candidate model (under plurality rule) of collective action, for moderate candidacy costs. In that case, in all the scenarios we consider, as in Proposition 1 in Osborne and Slivinskti (1996), one citizen would put forward her candidacy and select the policies we characterize.
collective-choice process in order to isolate the forces that emerge from collective action through time inconsistency itself. However, studying the interactions between self-control problems and other reasons for collective action would certainly be interesting.

**Commitment Technologies.** We assume that the commitment technologies faced by individuals and by the government are identical. There is no empirical reason to make this assumption: the relative effectiveness of commitment by the government or by individuals will depend on the specific application. However, assuming identical technologies is a useful benchmark. In Section 7 we consider some aspects of different commitment technologies accessible to the government and the agents.

**Self-Control Problems in Elections.** The quasi-hyperbolic discounting model imposes a link between self-control problems and a difference between present- and future-period preferences. In that respect, what accounts for the “present” as opposed to the “future” is crucial for the scope of self-control problems and for evaluating potential policies that attempt to limit their impact. The use of this model in the context of voting merits some discussion.\(^{23}\)

In terms of evidence, experimental work eliciting time preferences almost inherently deals with short time lags between immediate and delayed rewards (usually by only a few weeks, see Frederick, Loewenstein, and O’Donoghue 2002 and Augenblick, Niederle, and Sprenger 2015 for references). However, there are a number of survey-based studies in which agents face intertemporal trade-offs at long horizons (several years). These studies report evidence in favor of hyperbolic discounting. Chesson and Viscusi (2000), for instance, document discount rates that decrease over time, as in hyperbolic discounting models, for business managers facing a choice between a payment option offering a known payoff time and one offering a gamble over the timing of the award, both delayed by several years depending on the treatment.\(^{24}\) Further evidence has elicited inter-temporal preferences with respect to non-monetary payoffs. For instance, Viscusi and Huber (2006) surveyed approximately one thousand subjects regarding their preferences over potential (costly) water-improvement policies that were to start in different years. Subjects’ responses were consistent with a quasi-hyperbolic model of time preferences, even when controlling for risk. Furthermore, survey studies of intertemporal preferences over health outcomes provide evidence in favor of hyperbolic discounting, a generalization of the quasi-hyperbolic model with very similar

\(^{23}\)See also Bisin, Lizzeri, and Yariv (2015) for further details.

\(^{24}\)See also Chapman (1996), Cairns and van der Pol (1997), and references therein.
behavioral implications.\textsuperscript{25,26}

Furthermore, we note that there are many ways in which short-term popular support may affect government behavior between elections, and therefore at higher frequencies than the electoral cycle. The simplest link between policies and “current-self” preferences is an extreme case in which a government readily reacts to contemporaneous opinion polls.\textsuperscript{27} There are also various other channels through which political pressure occurs at high frequencies, such as lobbying and fund-raising events.

More generally, the quasi-hyperbolic model is commonly interpreted as a convenient metaphor for capturing macroeconomic phenomena of great relevance in the political arena that are attributed to lack of commitment of political institutions. There are a variety of reasons why such political institutions may display, in and of themselves, lack of commitment: political turn-over that limits the time horizon a candidate can impact, underlying preferences of politicians, etc. There could also be a channel through the political process itself delivering such a lack of commitment, through institutions internalizing at least in part the self-control problems of their constituencies. Indeed, problems of political commitment are central to many discussions of fiscal stabilization (see, for instance, International Monetary Fund 2009 and Nerlich and Reuter 2013) and monetary unions (see, for instance, Giavazzi and Pagano 1988 on the European Monetary System and Ravenna 2005 on the European Monetary Union).

Last, there is an alternative way to interpret quasi-hyperbolic discounting that does not require any “psychological” tastes for immediate gratification. Namely, quasi-hyperbolic discounting can result from a decision maker aggregating preferences or experiencing uncertainty over future discount factors (that may be due to uncertainty over future income or subsequent available interest rates, unforeseen inflation rates, etc.). In particular, if we interpret our analysis as regarding political influence on government policy, then individuals could represent political pressure or interest groups whose political actions aggregate heterogeneous trade-offs between the present and the future. This is the interpretation given

\textsuperscript{25}For instance, in van der Pol and Cairns (2002), for instance, respondents are offered the opportunity for a spell of ill-health 2 or 3 years into the future to be delayed (by 2-5 years up to 10-13 years, depending on the experimental treatment) as a result of a medical intervention. Subjects are then asked to report a maximum number of days of future ill-health, after the delay, at which it would still be worthwhile to receive this treatment (see also Bleichrodt and Johannesson 2001, Chapman 1996, and van der Pol and Cairns 2001, 2011).

\textsuperscript{26}Interestingly, hyperbolic discounting is documented also in related survey studies eliciting inter-temporal social preferences; e.g., when respondents trade-off saving lives at different future times; see Cairns and van der Pol (1997) and Johanneson and Johannson (1996).

\textsuperscript{27}For a survey discussing the policy responsiveness to public opinion, see Erikson (2013).
to hyperbolic discounting in the study of social preferences over climate change policies in Goulder and Williams (2012) and more generally to the related “gamma discounting” over any over cost-benefit analyses in Weitzman (2001).

4 Decentralized Outcomes

Before inspecting the impacts of collective action on commitment decisions, we describe each agent’s individual decisions. This analysis corresponds to the case in which all decisions are made in a decentralized fashion.

Given the value of $c$ determined in the first period, in the second period the agent’s problem is given by:

$$\max_x U_2(x, c, \beta) \iff \max_x v_2 x - k(x, c) + \beta v_3 (1 - x).$$

Let $x(c, \beta)$ denote the solution to this problem.

$$x(c, \beta) = \begin{cases} 
1 & \beta < \left( v_2 - \frac{\partial k(1, c)}{\partial x} \right) / v_3 \\
(v_2 - \beta v_3) & \left( v_2 - \frac{\partial k(1, c)}{\partial x} \right) / v_3 \leq \beta < \left( v_2 - \frac{\partial k(0, c)}{\partial x} \right) / v_3 \\
0 & \beta \geq \left( v_2 - \frac{\partial k(0, c)}{\partial x} \right) / v_3.
\end{cases} \quad (1)$$

Intuitively, whenever the agent either experiences less present bias or higher marginal costs of immediate consumption, delay is more likely. At the extremes, if marginal costs of cutting the whole tree are not too high (namely, $\frac{\partial k(1, c)}{\partial x} < v_2$), very impatient agents will not delay any consumption. Virtuous agents, for whom the marginal costs of very little early consumption outweigh the benefits, will cut the entire tree in period 3. The monotonicity of the consumption function $x(c, \beta)$ is captured by the following lemma, which will be useful for our analysis of the collective-choice settings.

Lemma 1 (Consumption Monotonicity) The fraction of the tree consumed in period 2, $x(c, \beta)$, is decreasing in both $c$ and $\beta$.

\[28\]In fact, the idea that the mere aggregation of different (standard) time preferences may generate a time inconsistent representative agent (or, consequently, a time inconsistent social planner) has been floating around since Marglin (1963) and Feldstein (1964). Sozou (1998) illustrated that hyperbolic discounting may emerge from exponentially distributed hazard rates; see Jackson and Yariv (2014) for a generalization implying that non-dictatorial representative agents respecting Pareto efficiency are inherently time inconsistent.
The first period problem can be written as:

$$\max_c U_1(x, c, \beta) \iff \max_c \beta v_3 + x(c, \beta) (\beta v_2 - \beta v_3) - \beta k(x(c, \beta), c) - I(c)$$

Let $c(\beta)$ be the solution of this problem. We want to understand the dependence of the commitment parameter $c(\beta)$ on $\beta$, which will be an essential input into the collective-action problem.

In order to glean some intuition on the dependence of $c$ on $\beta$, consider the case in which $x(c, \beta)$ is interior and differentiable with respect to $c$. Notice that:

$$\frac{\partial U_1}{\partial c} = \frac{\partial x(c, \beta)}{\partial c} \beta (v_2 - v_3) - \beta \frac{\partial k(x(c, \beta), c)}{\partial c} \frac{\partial x}{\partial c} - I'(c).$$

In contrast to the standard dynamic optimization problem with geometric discounters, the envelope condition fails and the indirect effect on period 2 consumption does not disappear. Indeed, substituting the second period first-order conditions we obtain:

$$\frac{\partial U_1}{\partial c} = -\beta \left( \frac{\partial x(c, \beta)}{\partial c} ((1 - \beta) v_3) + \frac{\partial k(x(c, \beta), c)}{\partial c} \right) - I'(c). \quad (2)$$

The benefit of commitment is captured by the term $-\beta \frac{\partial x(c, \beta)}{\partial c} ((1 - \beta) v_3)$. This term is increasing in the degree of present bias and captures the fact that the period 1 self and the period 2 self disagree on the value of cutting the tree in period 2.

There are several effects of changes in $\beta$ on the optimal choice of $c$. As $\beta$ increases more weight is put on the future, pushing for more early commitment investment. Furthermore, the fraction of the tree consumed in period 2, $x(c, \beta)$, is smaller, leading to a smaller marginal cost $\frac{\partial k(x(c, \beta), c)}{\partial c}$ tomorrow. Nonetheless, as $\beta$ becomes larger, time inconsistency becomes less relevant, so the benefit of $(1 - \beta) v_3$ is smaller. When $\beta$ is close to zero or close to $\frac{v_2}{v_3}$ period 1 investment will be zero, so investment is not monotone. Intuitively, agents for whom time inconsistency is very severe foresee that reasonably priced commitments will not save them from excessive consumption in period 2 and therefore acquire limited commitment. On the other side of the spectrum, agents who are virtuous (characterized by high $\beta$), do not suffer from great temptation in period 2 and therefore do not require extreme commitment to enable them to postpone consumption. In particular, recall that when $\beta \geq \frac{v_2}{v_3}$, agents choose $x(c, \beta) = 0$ for all $c$ so their optimal investment in commitment is zero: $c(\beta) = 0$ for all $\beta \geq \frac{v_2}{v_3}$.

In general, $c(\beta)$ may achieve several local maxima between $\underline{\beta}$ and $\frac{v_2}{v_3}$. The following example illustrates a case in which $c(\beta)$ is concave in this region and only one maximum exists.
Example 1 (Quadratic Commitment Costs) Consider the case of $k(x,c) = (c + v_2) \frac{x^2}{2}$ and $I(c) = \frac{c^2}{2}$. The second period utility is then given by:

$$U_2(x,c,\beta) = \beta v_3 + x(v_2 - \beta v_3) - \frac{(c + v_2)x^2}{2}$$

and the corresponding first-order condition requires that:

$$x(c,\beta) = \begin{cases} \frac{(v_2 - \beta v_3)}{(c + v_2)} & \beta \leq \frac{v_2}{v_3} \\ 0 & \beta > \frac{v_2}{v_3} \end{cases}.$$

Notice that $x(c,\beta)$ is decreasing in $\beta$, achieving the maximal value of 1 when $\beta = 0$. This generates a second period utility of:

$$U_2(x(c,\beta),c,\beta) = \beta v_3 + \frac{(v_2 - \beta v_3)^2}{2(c + v_2)}.$$

Plugging these values into period 1’s objective function yields:

$$U_1 = \beta v_3 + \beta \frac{(v_2 - \beta v_3)}{(c + v_2)} (v_2 - v_3) - \beta \frac{(v_2 - \beta v_3)^2}{2(c + v_2)} - \frac{c^2}{2}.$$

The optimum is given by:

$$c(\beta) = \begin{cases} \frac{\alpha_1}{\alpha_2} & 0 < \beta \leq \frac{v_2}{v_3} \\ 0 & \frac{v_2}{v_3} < \beta \leq 1 \end{cases}.$$

where $\alpha_1, \alpha_2, \alpha_3$ are positive constants depending on $v_2$ and $v_3$, while $P_k(\beta)$ is a polynomial of degree $k$ in $\beta$ (with coefficients determined by $v_2$ and $v_3$).

Figure 1 illustrates the emerging result of $c(\beta)$. As highlighted by the figure, the greatest commitment constraints are chosen by individuals with moderate levels of time inconsistency.

5 Electoral Outcomes

We now turn to inspect the effects of collective action on agents’ choices. We start by analyzing the case in which only the choice of commitment levels is done through an electoral process. We then proceed to a case in which both commitment and the timing of consumption are decided upon collectively.

29 In particular, our specification of the cost function $k(x,c)$ assures that consumption is interior for $\beta \in (0, \frac{v_2}{v_3})$. 


Recall that the support of the preference distribution $G$ is $[\underline{\beta}, \bar{\beta}]$ and that $\frac{\partial k(x, \beta)}{\partial c} = 0$ for all $x$. Unless otherwise mentioned, we will assume that $\underline{\beta} > 0$ from now on. From equation (2), together with Lemma 1, it follows that for all $\beta \in [\underline{\beta}, \bar{\beta}^*),$ the optimal commitment level is positive, $c(\beta) > 0$. Denote by $c^* \equiv c(\bar{\beta})$. We will focus on what we term regular environments.

Definition (Regular Environments) An environment is regular if $\frac{\partial k(1,c)}{\partial x} \geq v_2$ for all $c \geq c^*$.

Regularity assures that preferences for commitment are single-peaked. Using our discussion above, we have the following:

Lemma 2 (Commitment Choices in Regular Environments) In regular environments, for any individual of type $\beta$, $x(c(\beta), \beta) < 1$. In particular, $c(\beta) > 0$ for all $\beta \in [\underline{\beta}, \bar{\beta}^*)$.

Furthermore, preferences over commitment levels are single-peaked.

That is, regularity assures that individuals, left to their own devices, would choose commitment levels that are effective to some extent. This simplifies our analysis substantially. We discuss the analysis of environments that are not regular in Section 7.5.
5.1 Collective Commitment with Decentralized Choice

In this setting, the commitment parameter $c$ is determined collectively. From the point of view of an agent of type $\beta$, the voting problem is determined as follows. From the analysis of the private decision problem of an agent of type $\beta$, if a commitment parameter $c$ is chosen, and subsequent choices are made optimally by the agent, period 1 utility is given by

$$U_1(x(c,\beta),c,\beta) = \beta v_3 + x(c,\beta)(\beta v_2 - \beta v_3) - \beta k(x(c,\beta),c) - I(c).$$

Thus, the agent votes for candidate 1 offering commitment $c_1$ over candidate 2, who offers commitment $c_2$, whenever

$$U_1(x(c_1,\beta),c_1,\beta) > U_1(x(c_2,\beta),c_2,\beta).$$

**Proposition 1** There is a unique pure strategy equilibrium of the collective commitment game in which both candidates offer a platform $c^{CD}$. Furthermore, when $c(\beta)$ has a unique local maximum in $\left(\frac{v_2}{v_3}\right)$, the platform $c^{CD}$ corresponds to the ideal policy for a voter of type $\beta^{CD}$, where $\beta^{CD}$ is higher than the median $\beta$, $\beta^{CD} > \beta_M$.

The quadratic case in the example above is useful in illustrating the intuition underlying Proposition 1. Consider Figure 1. If $1 - G(\frac{v_2}{v_3}) \geq 1/2$, there is a majority of agents who prefer no commitment and the equilibrium commitment parameter is naturally $c^{CD} = 0$, which coincides with that preferred by the median. Otherwise, for every $\tilde{c} > 0$, define $\beta_L(\tilde{c})$ and $\beta_H(\tilde{c})$ such that $\tilde{c}$ is their ideal point, i.e. $c(\beta_L(\tilde{c})) = c(\beta_H(\tilde{c})) = \tilde{c}$. All agents with preference parameters below $\beta_L(\tilde{c})$ and above $\beta_H(\tilde{c})$ prefer commitment parameters lower than $\tilde{c}$, while agents with preference parameters between $\beta_L(\tilde{c})$ and $\beta_H(\tilde{c})$ prefer preference parameters above $\tilde{c}$. In particular, the equilibrium commitment parameter $c^{CD}$ is chosen so that these two classes of agents are of equal proportions. That is, $G(\beta_L(c^{CD})) + (1 - G(\beta_H(c^{CD}))) = 1/2$. By construction, $\beta_M \in (\beta_L(c^{CD}),\beta_H(c^{CD}))$ and the result follows. In fact, note that in this case the equilibrium commitment level corresponds to a voter of type $\beta^{CD}$ that is strictly higher than the median, $\beta^{CD} > \beta_M$. In this case, it also corresponds to the commitment level of a voter of type $\tilde{\beta}^{CD}$ that is strictly lower than the median, $\tilde{\beta}^{CD} < \beta_M$. Importantly, when $c(\beta)$ has a unique local maximum, this construction suggests that equilibrium commitment is lower than that corresponding to the median preferences. That is, $c^{CD} \leq c(\beta_M)$.\textsuperscript{30} We stress, however, that Proposition 1 is still a sort of a median-voter result, in that half of the electorate prefers a commitment

\textsuperscript{30}The construction suggests that median preserving spreads of the distribution $G$ would lead to lower equilibrium commitment levels.
level lower than $c^{CD}$ and half of the electorate prefers a commitment level higher than $c^{CD}$. Since individual preferences for commitment are non-monotone in the preference type, the resulting equilibrium commitment need not correspond to the individual with median preference parameter $\beta_M$.

This construction of the equilibrium level of commitment can be adapted to environments in which $c(\beta)$ entails several local maxima, it is only the relation to the median agent’s preferred level of commitment that hinges on $c(\beta)$ having a unique maximum. However, the construction does rely on all agents having single-peaked preferences with respect to the commitment parameter $c$. Indeed, in this case, agents with high taste parameter $\beta$ prefer no investment in commitment, while all others prefer a positive amount of commitment. The condition on $\frac{\partial k(1,c)}{\partial x}$ assure that even individuals with very low parameters $\beta$ benefit from some level of commitment.\footnote{We note that it would suffice to assume that $\frac{\partial k(1,c)}{\partial x} \geq v_2$ only for $c > 0$, which is satisfied by our running quadratic example.}

### 5.2 Collective Commitment with Centralized Choice

We now discuss the case in which the second period choice is also taken via collective action. Two office-motivated candidates, 1 and 2, offer platforms $x_1$ and $x_2$ in the second period.

From the analysis of individual choices, recall that (1) provides the second period optimal choice $x(c, \beta)$ for any given commitment parameter $c$ selected in period 1. From Lemma 1, $x(c, \beta)$ is decreasing in $\beta$. It is then clear that for any given choice of $c$ in the first period, both candidates will choose to offer the ideal policy of the median voter $\beta_M$. Thus, the second period choice will be $x(c, \beta_M)$.

We can now step back and consider a generic voter’s first period utility in this scenario.

$$U_1(x(c, \beta_M), c, \beta) = \beta v_3 + x(c, \beta_M) (\beta v_2 - \beta v_3) - \beta k(x(c, \beta_M), c) - I(c). \quad (4)$$

Since $x(c, \beta_M)$ is fixed for all $\beta$, the choice of commitment in the first period is driven by the desire to commit of an agent of median taste parameter $\beta_M$. Denote by $c(\beta, \beta_M)$ the (constrained) optimal commitment parameter for an agent of taste $\beta$, foreseeing the second period choice being determined according to the taste of the median parameter $\beta_M$. As it turns out, $c(\beta, \beta_M)$ is monotonic in $\beta$, with individuals who care more about future consumption preferring greater investment in commitment, as illustrated in the following lemma.
Lemma 3 (Constrained Commitment Monotonicity) The optimal constrained commitment \( c(\beta, \beta_M) \) is increasing in \( \beta \).

Note that the monotonicity in \( \beta \) of desired commitment is in contrast with the analysis of both the fully decentralized scenario as well as of the centralized commitment with decentralized choice scenario. The logic for this is the following. The value of investment in commitment is now in reducing incentives for the median agent to cut the tree early. This is particularly valuable for the high-\( \beta \) agents.

Lemma 3 implies that it is median preferences that determine first period choices as well. This is captured in the following proposition.

Proposition 2 When both commitment and consumption are chosen collectively, equilibrium outcomes coincide with those chosen optimally by agents with the median taste parameter \( \beta_M \).

It also interesting to highlight how optimal constrained commitment \( c(\beta, \beta_M) \) changes as \( \beta_M \) changes. Indeed, the marginal benefit of commitment \( (1 - \beta_M) v_3 \) is higher when \( \beta_M \) is lower and so for the case of interior solutions it follows that:

Remark The optimal constrained commitment \( c(\beta, \beta_M) \) is decreasing in \( \beta_M \).

The following example illustrates how the optimal constrained commitment and the equilibrium outcome work for the case of quadratic consumption costs.

Example 2 (Quadratic Costs – Fully Centralized Solutions) Consider the setting of Example 1. Assume first \( \beta_M < \frac{v_2}{v_3} \). Plug in \( x(c, \beta_M) \) into \( U_2 \) to get

\[
U_2(x(c, \beta_M), c, \beta) = \beta v_3 + \frac{v_2 - \beta_M v_3}{c + v_2} (v_2 - \beta v_3) - \frac{(v_2 - \beta_M v_3)^2}{2(c + v_2)}.
\]

Moving back to period 1 we obtain:

\[
U_1(x(c, \beta_M), c, \beta) = \beta v_3 + \beta \frac{v_2 - \beta_M v_3}{c + v_2} (v_2 - v_3) - \beta \frac{(v_2 - \beta_M v_3)^2}{2(c + v_2)} - \frac{c^2}{2}.
\]

The optimal choice of commitment is given by:

\[
c(\beta, \beta_M) = \frac{\tilde{\alpha}_1}{\sqrt{3} P_1(\beta) + \sqrt{3} P_2(\beta)} + \sqrt{3} P_1(\beta) + \sqrt{3} P_2(\beta) - \tilde{\alpha}_2,
\]

\[20\]
where the positive constants $\tilde{\alpha}_1, \tilde{\alpha}_2$, as well as the coefficients of the polynomials $P_1(\beta)$ and $P_2(\beta)$ (of degrees 1 and 2, respectively) are functions of $v_1, v_2$, and $\beta_M$.

Figure 2 depicts $c(\beta, \beta_M)$ for different values of $\beta_M$. As Figure 2 illustrates, the optimal desired amount of commitment $c(\beta, \beta_M)$ is increasing in $\beta$ and decreasing in $\beta_M$. Notice, however, that the equilibrium level of commitment is given by $c(\beta_M, \beta_M) \equiv c(\beta_M)$, which is not monotonic.

We can now compare the level of commitment in the two collective-action scenarios. When $c(\beta)$ has a unique maximum, Proposition 1 assures that the platform $c^{CD}$ chosen in equilibrium corresponds to the ideal policy for a voter of type $\beta^{CD}$, where $\beta^{CD}$ is higher than the median $\beta$, $\beta^{CD} \geq \beta_M$. Furthermore, the construction of the proof of Proposition 1 (extending that appearing for the quadratic case in Example 1) illustrates that $c(\beta^{CD}) \leq c(\beta_M)$ when $c(\beta)$ has a unique local maximum. Therefore, we have the following proposition.

**Proposition 3** Assume that $c(\beta)$ has a unique local maximum in $\left(\beta_M, \frac{v_2}{v_3}\right)$. Then the equilibrium choice of commitment is higher under full centralization than in the decentralized choice scenario.

Note that this proposition shows that the optimal amount of commitment is higher in the fully centralized economy although the decisive voter is an agent with a lower $\beta$. This is,
of course, due to the non monotonicity of $c(\beta)$ and illustrates the fact that delegating the commitment choice to a more virtuous agent may not lead to higher commitment.

When $c(\beta)$ has multiple local maxima, the comparison between the equilibrium commitment levels generated by full centralization and decentralized choice is inconclusive and, in principle, can go either way.

5.3 Decentralized Commitment with Centralized Choice

We now consider the case in which individuals privately invest in commitment, but in period 2 there is an election that determines the time for consumption for all individuals.

**Proposition 4** There is a unique equilibrium of the decentralized commitment, centralized choice case in which all voters choose $c = 0$.

The intuition for this result is that there is free riding in commitment investment. Investment in commitment is only useful if it affects the choice in period 2. But, this choice is made collectively, and the probability that an agent is pivotal in period 2 is vanishingly small when there are many agents so the incentive to invest in commitment also disappears.\(^{32}\)

This result suggests the following observation. Suppose that in the decentralized setting we observe a median individual making responsible choices in period 2. One may naively conclude that centralizing consumption would be beneficial because it would lead to responsible choices for the entire population, including those who were choosing irresponsibly. However, our result shows that such partial centralization would undermine the incentive to commit that in turn allowed the median person to choose responsibly in period 3. For instance, if $\beta_M < \frac{22}{23}$, then in this scenario the median choice would be to consume the entire tree in period 2. This discussion suggests that partial centralization may be harmful: centralization of consumption choices should be accompanied by centralization of commitment. We discuss this intuition more formally in the next section.

It is also interesting to note that the zero investment result in this proposition still holds in the case of a homogeneous population. This is notable since in all of the previous

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\(^{32}\)This result does rely on the continuum of voters assumption. If the population is finite, more ‘efficient’ equilibria may exist in which an exact majority invests in the private (decentralized) commitment. In that case, any individual choosing $c = 0$ is best responding since she gains her ideal commitment choice in period $t = 2$. If an investing individual deviates to a lower commitment level, she becomes pivotal at $t = 2$. Therefore, choosing the optimal decentralized level of commitment is optimal for her. In a world with a finite number of voters more care would be needed. However, any amount of noise in turnout would still generate zero investment in commitment in the limit when the voting population becomes large.
scenarios, outcomes for a homogeneous population would coincide with those generated by a fully decentralized, laissez faire system. Indeed, in our other settings, the key ingredient determining outcomes is the identity of the decisive agent. However, in the setting where only consumption is centralized, the decision-making process itself undermines the incentive to commit. It inherently entails a free rider problem that leads no one to invest in commitment regardless of the distribution of preferences and, in turn, may harm the population as a whole.

5.4 Comparison of Outcomes

Our analysis so far illustrates that individuals’ relative preferences for commitment may reverse depending on whether future consumption decisions are centralized or not. We showed that, under some regularity assumptions, commitment investment is larger in the scenario with full centralization relative to the scenario where only commitment is centralized. Importantly, there is no investment in commitment in the case of decentralized commitment and centralized choice. Therefore, that scenario leads to the lowest aggregate investment in commitment.

The comparison between full decentralization and the systems involving centralized commitment choices is instead ambiguous. One obvious case in which the fully decentralized outcome leads to higher investment in commitment is when the median voter is virtuous: $\beta_M \geq \frac{v_2}{v_3}$. In this case all scenarios with some degree of centralization of commitment decisions generate no commitment investment, whereas some investment takes place in the fully decentralized scenario as long as there is a positive mass of individuals who are not virtuous. Finally, in order to construct an example in which centralized commitment leads to higher investment, consider a case where the median voter $\beta_M$ is such that $c(\beta_M)$ is maximal (i.e., $\beta_M \in \arg\max c(\beta)$). In this case the fully centralized scenario leads to the maximal investment that would be chosen by anybody in the population. All other scenarios lead to lower investment in the aggregate.

Figure 3 summarizes the resulting second-period consumption levels under the different systems. Were individuals able to freely commit, they would all consume the tree in full in period 3 and the first-best second-period consumption would be nil for all preference types. When the median agent is not virtuous (corresponding to the left panel of the figure), centralizing consumption leads to greater early consumption at least for some agents (when commitment decisions are centralized), if not all (when commitment decisions are decentralized). However, the first-best level is achieved whenever the median agent is virtuous and there is collective action on consumption decisions (see the right panel of the figure). Deci-
sions are then delegated to an agent who can resist temptations to consume early. We next analyze the welfare consequences of the different systems, which will echo these observations.

6 Welfare Consequences

We now turn to the welfare consequences of each of the political processes analyzed above. In the case of time inconsistent agents, the appropriate welfare criterion is debatable.\textsuperscript{33} We start by measuring welfare as the utility of first period agents. We later turn to consider period-zero welfare assessments.

6.1 Period-One Welfare

We denote by $\Pi^{DD}(G), \Pi^{DC}(G), \Pi^{CD}(G),$ and $\Pi^{CC}(G)$ the expected utilitarian welfare corresponding to the fully decentralized, decentralized-centralized, centralized-decentralized, and centralized-centralized systems, respectively, when the underlying preference distribution is given by $G$. We will at times abuse notation and drop the argument of the welfare function when clarity is not compromised. For presentation simplicity, we assume that $k(x, 0) = 0$ for all $x$ so that no commitment leads agents to experience no costs of early consumption.

The main idea behind our comparison in the welfare generated by the four institutions that we consider is that of delegation. Centralization effectively allows delegation of specific decisions to a particular individual. In the setting of our model without any externalities,

\textsuperscript{33}For a discussion, see Bernheim and Rangel (2009).
standard geometric discounters would have no reason to delegate and so laissez faire would dominate all other systems. For individuals with self-control problems, from the perspective of period one welfare, we must distinguish between delegation of consumption choices and delegation of investment in commitment. All individuals benefit from delegating consumption decisions to individuals who are more virtuous than them (higher $\beta$) and harmed by delegating decisions to individuals who have worse self-control (lower $\beta$). On the other hand, no period-1 self benefits from delegating the commitment decision because there is no self-control problem in period 1.

We start by showing that partial centralization, i.e., mandating only one decision (either commitment or consumption) is dominated by either full centralization or laissez-faire.

**Proposition 5** For all preference distributions, either full centralization or full decentralization are welfare maximizing. That is, $\max\{\Pi_{DD}, \Pi_{CC}\} \geq \max\{\Pi_{CD}, \Pi_{DC}\}$.

The proof of this result is quite straightforward. Note first that, under laissez faire, individuals can always emulate the decisions generated by the centralized-decentralized system: they can choose a commitment level of $c^{CD}$. Therefore, $\Pi_{DD} \geq \Pi_{CD}$. In fact, all agents but those who individually choose $c^{CD}$ strictly benefit from having all decisions decentralized. Now, let $\beta^*$ be the threshold preference parameter corresponding to agents who are just indifferent between postponing consumption or consuming immediately in period 2 given zero commitment, $v_2 = \beta^*v_3$. By Proposition 4, in the decentralized-centralized system there is zero investment in commitment. If the median voter is not sufficiently virtuous ($\beta_M < \beta^*$) this leads to $x = 1$: full consumption of the tree in period 2 because early consumption comes at no cost, $\frac{\partial k(x,0)}{\partial x} = 0$. In this case, welfare is higher under laissez faire: all agents with preference parameter $\beta < \beta^*$ can do no worse than this, and all virtuous agents with preference parameters $\beta \geq \beta^*$ manage to delay consumption to period 3 even if they do not invest in commitment. It follows that when $\beta_M < \beta^*, \Pi_{DD} \geq \Pi_{DC}$. Suppose now that the median voter is virtuous, $\beta_M \geq \beta^*$. In this case, under both the decentralized-centralized and centralized-centralized system, there is no investment in commitment and all consumption is delayed to period 3, so that, in this case, $\Pi_{CC} = \Pi_{DC}$. The result of Proposition 5 holds, in fact, for more general environments than the special one we study here (see Section 7.6 below).

The comparison between the fully centralized system and a laissez faire economy depends on the distribution of preferences. Roughly speaking, when the median $\beta$ is high, centralization is beneficial because a centralized political process allows all agents to delegate choice to a virtuous voter who commits to efficient actions at low costs. On the other hand, when
the median voter is prone to a strong present-bias (low $\beta_M$), collective decisions lead to bad outcomes: low investment in commitment and high levels of early consumption. In these cases, the decentralized system performs better because at least some of the virtuous voters do well: they are not bound by the self-control problems of a low median $\beta$. The following result provides sufficient conditions for ranking the two systems.

**Proposition 6**

1. If $G(\beta^*) > 0$, and $\beta_M \geq \beta^*$, then full centralization is best: $\Pi^{CC}(G) > \Pi^{DD}(G)$;

2. Consider a sequence of distributions $\{G_n\}_{n=1}^\infty$ with corresponding medians $\{\beta^n_M\}_{n=1}^\infty$. If there exists $\hat{\beta} > \beta$ such that $\{G_n(\hat{\beta})\}$ is uniformly bounded below 1 and $\lim_{n \to \infty} \beta^n_M = \beta$, then there exists $n^*$ such that for all $n > n^*$, laissez faire is best: $\Pi^{DD}(G_n) > \Pi^{CC}(G_n)$.

Part 1 of this result says that a sufficient condition for full centralization to be best is that the median voter is sufficiently high so that no commitment is required to ensure no early consumption. Of course, this condition is not necessary: for instance, if the median voter is characterized by a preference parameter slightly lower than $\beta^*$, a moderate amount of investment in commitment ensures almost no early consumption.

Part 2 is more involved: the condition on the sequence of distributions rules out the possibility that there is a sequence such that $\beta_M$ converges to $\beta$ but there is a vanishing mass of agents whose preference parameter $\beta$ is larger than $\beta_M$. In such cases centralization may still be best.

Proposition 6 is effectively a delegation result. When the median voter is sufficiently virtuous, the electorate benefits from delegating decisions to the median voter. Recall that the desire to delegate only regards consumption decisions: individuals do not like delegating commitment decisions.$^{34}$

The quadratic costs example is useful in visually illustrating how the different political processes fare in terms of welfare as a function of the underlying preference distribution in the electorate.

**Example 3 (Quadratic Costs – Welfare Comparisons)** Consider the settings of Examples 1 and 2 above and suppose that $G$ is a triangular distribution with a peak

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$^{34}$The idea that delegation to an agent whose preferences are misaligned with society’s might be useful appears in other types of environments as well, most notably in the macroeconomic literature condoning the use of central banks who do not share the social objective and weigh relatively heavily inflation-rate stabilization, see Rogoff (1985) and the literature that followed.
at \( d \in (0,1) \). Figure 4 depicts the welfare levels generated by the different processes as a function of the median agent’s preferences when \( v_2 = 1 \) and \( v_3 = 3/2 \), and \( I(c) = 0.0005c^2 \).\(^{35,36}\) We use the fully decentralized setting as a baseline for comparison. The figure illustrates the way that the four scenarios compare in terms of first-period welfare. Full centralization reaches the highest welfare when the median \( \beta \) is high. Full centralization and decentralized commitment-centralized choice have the same level of welfare when the median is above 2/3 because 2/3 = \( v_2/v_3 \) for our parameters and in these cases no commitment is necessary to induce zero tree-cutting in period two. However, the decentralized commitment-centralized choice scenario is a lot worse for lower values of the median \( \beta \). When the median \( \beta \) is lower, full decentralization leads to the best outcome, with centralized commitment-decentralized choice a close second. The reason why the comparison between these scenarios becomes much more favorable to full centralization (especially relative to the setting in which only

\[ \beta_M = \begin{cases} \sqrt{d}/\sqrt{2} & d \geq 1/2 \\ 1 - \sqrt{1-d}/\sqrt{2} & d < 1/2 \end{cases} \]

\(^{35}\) For a triangular distribution with a peak at \( d \), the corresponding median is given by:

\(^{36}\) Note that our utility specification implies a weight of 1 on period-1 instantaneous utility and a weight of \( \beta \) on each of period-2 and period-3 instantaneous utilities. In the figure, we normalize the utility of each agent with preference parameter \( \beta \) by \( 1 + 2\beta \).
commitment is centralized) for high values of median $\beta$ is that when the median $\beta$ is high there is no commitment in equilibrium, and, under decentralized choice, this harms the individuals with lower $\beta$. One interesting aspect of the comparison among welfare levels is that commitment and consumption, are ‘complementary’: either full centralization or full decentralization generate the greatest levels of welfare, whereas partial centralization yields inferior welfare results.

It is also instructive to compare the welfare resulting from our political processes to that generated by an economy that does not allow for commitment. Denote by $\Pi^S$ the expected first period utilitarian surplus absent any commitment instruments:\footnote{As in the example, the division by $1 + 2\beta$ is a normalization of agents’ utilities, so that weights of instantaneous utilities in period 1 always sum up to 1.}

$$\Pi^S = \int_0^{\beta^*} \frac{\beta v_2}{1 + 2\beta} dG(\beta) + \int_{\beta^*}^{1} \frac{\beta v_3}{1 + 2\beta} dG(\beta).$$

(5)

It is easy to construct example where the welfare generated by some centralized political system is worse than $\Pi^S$. For instance, if there is a substantial mass of virtuous agents and the median $\beta_M$ is very low, $\Pi^S > \Pi^{CC}$. In other words, if the constitution makes a bad delegation decision, welfare is even lower than in an economy with no possibility of investing in commitment. However, it is easy to see that $\Pi^S \leq \max\{\Pi^{DD}, \Pi^{CC}\}$. In fact, $\Pi^S \leq \Pi^{DD}$: under laissez-faire agents can always emulate the no commitment environment by choosing a commitment level of 0. Thus, whenever a positive commitment level is chosen by an individual, the induced first-period utility is higher than that absent commitment.

### 6.2 Period-Zero Welfare

It is also useful to consider the welfare comparison among the various scenarios from the point of view of period zero, before the commitment choice is made.

Period 0 utility for an agent of type $\beta$ is given by

$$U_0(x(c, \beta), c, \beta) = \beta \{v_3 + x(c, \beta)(v_2 - v_3) - k(x(c, \beta), c) - I(c)\}.$$  

(6)

Comparing expressions (6) and (3) makes it clear that there is an important difference between the period-zero and the period-one perspective: from the point of view of the period-zero self, the commitment choices themselves are now subject to self-control problems. The main consequence of this for our analysis of collective action is that, in contrast to our
previous discussion, the period-zero self may now have the incentive to delegate commitment decisions because her period-one self “under-commits.”

Despite these differences, an analogous result to Proposition 6 still holds when considering welfare in period zero: If the median voter is sufficiently virtuous, mandating decisions, or delegating them to the virtuous median voter through full centralization, is beneficial relative to a laissez faire setting; in contrast, if the median voter has high degree of present bias (low $\beta$), then laissez faire is better.

With respect to partial centralization, notice first that full centralization Pareto dominates (for all preference parameters $\beta$) the decentralized commitment, centralized consumption system. Indeed, both systems generate a uniform profile of commitment and consumption for all agents. Therefore, all period-zero selves, regardless of their preference parameter $\beta$, rank the two systems alike. However, the agent with median preference parameter $\beta_M$ is certainly better off in the fully centralized system, which in turn implies that all agents are.

The comparison with the centralized commitment, decentralized consumption system is more intricate. As mentioned, in period zero agents can no longer emulate period-one commitment decisions when considering a laissez faire economy, and for particular settings in which median preferences are not virtuous enough, mandating commitment alone may be superior to both full centralization and full decentralization.

7 Extensions

7.1 Naive Agents

In the literature, when modeling time inconsistent agents, an assumption of naivete is sometimes made in contrast to the assumption of sophistication we have assumed so far.\footnote{See, for instance, O’Donoughue and Rabin (1999).} Naive agents have $\beta - \delta$ preferences, but believe that they will have standard geometric preferences in any future period. Sometimes agents are assumed to be partially naive. This is modeled as agents having beliefs about their future selves that are intermediate between full sophistication and full naivete.

Most of our analysis would go through, with some modifications, if agents were partially naive. However, it is useful to comment on the qualitative impact of such agents in the electorate. To simplify our discussion, suppose that some agents in the population are fully naive.

In our model naive agents behave like time consistent (high $\beta$) individuals in period
they do not have any demand for commitment because they are unaware of their time inconsistency problem. Therefore, the higher the mass of naive agents in the economy, the lower the investment in commitment in equilibrium. However, once period 2 arrives, these agents are tempted by immediate consumption, lowering the effective pivotal $\beta$ in the centralized consumption scenario. Overall, the presence of these naive agents reduces welfare for the sophisticated agents. However, the naive agents make “worse” individual choices than sophisticated agents so they are more likely to benefit from centralization. If the naive agents constitute a majority and the median $\beta$ in the second period is such that $\beta < \frac{v_2}{v_3}$, then full decentralization is best: the political outcomes of any centralized decisions would be bad so decentralization would at least deliver good choices for the relatively high $\beta$, sophisticated agents.

If the naive agents are a minority, then there are opposing forces in favor and against centralization: the presence of the naive agents worsens the choices but the naive agents benefit more from centralization.\(^{39}\)

### 7.2 Commitment Subsidies

Instead of considering a centralized commitment scenario where the elected government chooses the amount of commitment in period 1, one could consider a scenario where candidates propose subsidies to commitment. If a voter receives a subsidy $s$, the choice of commitment in period 1 can be obtained by maximizing

$$U_1(x(c, \beta), c, \beta, s) = \beta v_3 + x(c, \beta)(\beta v_2 - \beta v_3) - \beta k(x(c, \beta), c) - I(c, s)$$

where $\frac{\partial I(c,s)}{\partial c}$ is decreasing in $s$. Thus, the amount of commitment chosen by each individual is increasing in $s$. However, the voting decision between two candidates who offer different levels of subsidies needs to take into account the budgetary impact of the subsidies and how the corresponding expenses are distributed in the population. The total amount of subsidies depends on the aggregate amount of commitment. Consider then a setting in which subsidies are chosen collectively, and consumption is chosen in a decentralized fashion. If the burden is shared equally across the electorate,\(^{40}\) it can be shown that the pivotal agent remains that

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\(^{39}\)Hiedhues and Koszegi (2010) suggested how commitment policies in the credit card market might be beneficial for naive consumers from a welfare perspective. In our setting, whenever choices are made collectively, there are additional forces due to externalities, which alters the calculus of political influence.

\(^{40}\)Formally, for any profile of commitment $c(\beta)$, the overall budgetary consequence of a subsidy level $s$ is given by:

$$\int_0^1 I(c(\beta), s)dG(\beta) - \int_0^1 I(c(\beta))dG(\beta),$$
with a preference parameter $\beta^{CD}$ (the pivotal agent in our baseline centralized commitment-decentralized consumption setting absent subsidies). If this agent invests relatively little in commitment, the value of subsidies for her is lower than her contribution to the collective pool covering overall subsidies in the population. In this case, the outcome of the election would generate zero subsidies. On the other hand, if this agent has a relatively high investment in commitment, so that she is a net beneficiary of the subsidies, she will support fairly high subsidies. In this case, the outcome would lead to higher investment in commitment by all agents relative to that chosen under the fully decentralized scenario. Note, however, that from the perspective of period 1, commitment subsidies generate lower welfare than a laissez-faire economy.

7.3 Consumption Caps

We now consider a scenario in which the government can impose caps on early consumption. Specifically, assume that commitment choices in period 1 are private but, in period 2, the two candidates each propose a cap $\bar{x}_i$ on the fraction of the tree that can be consumed by any individual. Each agent can then choose an amount $x \leq \bar{x}$ of the tree. It can easily be shown that in this setting there is an equilibrium with no caps and the same commitment and consumption choices as in the laissez faire scenario. Namely, at $t = 1$, all agents implement their preferred commitment (as given by $c(\beta)$), expecting there to be no cap ($\bar{x} = 1$). At $t = 2$, both candidates offer $\bar{x} = 1$ and all agents consume as in the fully decentralized setting, $x(c(\beta), \beta)$. All we need to show is that at $t = 2$, both candidates offering no cap is part of an equilibrium. Indeed, agents for whom $x(c(\beta), \beta) > 0$ would not like to tie their hands so they would vote against any binding cap and are indifferent to any looser cap. Agents for whom $x(c(\beta), \beta) = 0$ do not require a cap (in fact, they are indifferent between all levels of the cap), so they are willing to vote against a cap. Therefore, this profile constitutes an equilibrium. Note that this contrasts the outcomes of the decentralized commitment with centralized choice setting we study, in which individual consumption in the second period is homogenized by the choices of the median voter. Offering no cap at $t = 2$ essentially imposes no restrictions on individual period 2 consumption. In particular, when no binding caps are implemented, individuals foresee choosing their ideal consumption level in the second period and can therefore benefit from commitment investments early on.

which is shared equally within the population.
7.4 Supplementing Commitment

Another natural extension pertains to agents’ potential ability to supplement commitment investments that are chosen by the government.

Suppose public and private commitments are governed by the same technology. That is, for any government choice of commitment \(c_g\), each agent experiences a period 1 cost of \(I(c_g)\), while additional private commitment of \(c_p\) leads the agent to experience an overall period 1 cost of \(I(c_g + c_p)\). That is, the cost of supplementing public investment in commitment is incremental. Our equilibrium characterization changes only in the centralized commitment, decentralized consumption setting. Since commitment costs are convex, the government’s commitment technology is not inferior to the private technology, and the amount of commitment chosen by the government is given by our Proposition 1. Individuals who seek greater commitment will then supplement the collective commitment privately. From a welfare perspective, this setting still generates lower welfare levels than the fully decentralized one as agents can emulate the generated outcomes privately.

Suppose instead that public and private commitment technologies are independent, so that a choice of government commitment \(c_g\) and private commitment \(c_p\) generate a period 1 cost of \(I_g(c_g) + I_p(c_p)\), where \(I_g\) and \(I_p\) satisfy our assumptions on the underlying commitment technology that were made in Section 3. In this case, when commitment is subject to collective action agents will typically mix private and public investment. The precise formulation of the equilibrium characterization in the relevant two settings depends more intricately on the functional forms of our model. In such settings, centralizing commitment alone may be beneficial relative to full decentralization as that setting effectively provides individuals access to an aggregate commitment cost technology that is more efficient: individuals can smooth the cost of commitment by splitting their commitment investments between public and private ones.

7.5 Linear Commitment Costs and Single-Peaked Preferences

Throughout the paper, we have often assumed that \(\frac{\partial k(1,c)}{\partial x} > v_2\). In that case, individual preferences for commitment are single peaked. When preferences are not single peaked, our analysis needs to be modified, especially for the case of centralized commitment-decentralized consumption.

We will now outline what happens when preferences are not single peaked by considering the special case of linear costs (and dropping the requirement that \(\frac{\partial k(x,0)}{\partial x} = 0\)). This case is useful since its structure is particularly simple. We first emphasize that the main welfare
results still hold in this case. However, the equilibrium construction is more complex.

When consumption costs are linear, we can normalize parameters so that \( k(x, c) = cx \). Furthermore, the optimal choice in the second period is generically either \( x = 0 \) or \( x = 1 \). In case of indifference, we will assume that an agent breaks the indifference to favor her “commitment self,” i.e., she chooses \( x = 0 \).\(^{41}\)

Suppose that in period 1 a cost \( c \) was chosen, and consider the period 2 choice problem of a voter of type \( \beta \). She will consume in period 3 if and only if

\[
U_2 = v_2 - c \leq \beta v_3. \quad (7)
\]

Thus, as before, agents with \( \beta > \frac{v_2}{v_3} \) are not willing to pay for commitment: they do not find it necessary.

Commitment is perceived beneficial in period 1 if the delay in consumption due to commitment is worth its costs \( I(c) \). That is, whenever there is a commitment parameter \( c \) such that:

\[
\beta v_3 - I(c) \geq \beta v_2 \iff \beta (v_3 - v_2) \geq I(c). \quad (8)
\]

How do investment incentives now vary with \( \beta \)? It is very difficult (and costly) to make low \( \beta \) agents wait until period 3 to consume. On the other side of the spectrum, high \( \beta \) agents are virtuous and will wait till period 3 even with no commitment instruments. Therefore, investment only pays for intermediate \( \beta \)'s.

Thus, as in the case studied previously, incentives to invest are not monotonic in \( \beta \) since both low- and high-\( \beta \) agents dislike investment (for different reasons). However, unlike the previous case, utilities are not single peaked with respect to the commitment \( c \): for intermediate \( \beta \)'s payoffs are first decreasing in \( c \) because we violate condition (7) and so commitment initially affects utility only through its costs, but carries no benefits in terms of the timing of consumption, until we reach a level of commitment \( c^* \) such that condition (7) is satisfied, so that \( c = 0 \) and \( c = c^* \) are both local optima.

Consider now the case of collective commitment accompanied by decentralized choice. For all agents of preference parameter \( \beta \geq \frac{v_2}{v_3} \), there is no willingness to pay for commitment no matter what the commitment technology is. Recall that \( \beta^* = \frac{v_2}{v_3} \). If \( 1 - G(\beta^*) \geq 1/2 \), there is a majority supporting no commitment and, as before, there is a unique equilibrium.

\[^{41}\]This setting can fit a special case of Gul and Pesendorfer (2001, 2004, 2007) type of preferences. Namely, suppose that two functions govern an individual’s utility from consumption: \( u(x) \) is the direct utility of \( x \), while \( v(y) \) is the temptation cost of not having consumed \( y \) available at the time of choice. In such a setting, in order to delay consumption in period 2, \( u(v_3) - v(v_2) \geq u(v_2) \). Suppose \( u(x) = x \) and \( v(y) = \alpha y \), where \( \alpha > 0 \). Then delayed consumption in period 3 occurs when \( v_3 \geq v_2(1 + \alpha) \), which is analogous to our linear costs case when taking \( \beta = \frac{1}{1+\alpha} \).
in which both candidates offer commitment $c^{CD} = 0$. Suppose there is a substantial fraction of the population that is moderate, $1 - G(\beta^*) < 1/2$. Now note that by raising $c$ we obtain an increasing mass of $\beta$’s for which $\beta v_3 \geq v_2 - c$. Let $\beta(c) \equiv \frac{v_2 - c}{v_3}$. The mass is given by $G(\beta(c))$. Define $c_L$ such that

$$G(\beta^*) - G(\beta(c_L)) = \frac{1}{2}$$

and let $\beta_L \equiv \beta(c_L)$.

Let $\tilde{c}$ be the unique commitment level such that

$$\beta(\tilde{c})(v_3 - v_2) = I(\tilde{c}).$$

The next result characterizes the equilibria in this environment.

**Proposition 7** Assume that $k(x, c) = cx$. When only commitment decisions are centralized,

1. If $\beta_L(v_3 - v_2) \leq I(c_L)$, there exists a unique equilibrium with investment of zero in commitment instruments.

2. If $\beta_L(v_3 - v_2) > I(c_L)$, there is no pure strategy equilibrium. In this case, there is a continuum of equilibria in mixed strategies. All symmetric profiles having a two-point support $c_1 < c_2$ with equal probability on $c_1$ and $c_2$, where $c_2 \in [c_L, \tilde{c}]$, constitute part of an equilibrium.

The intuition for the non existence of positive commitment, pure strategy equilibria is the following. Assume $c > 0$ is part of an equilibrium. A deviation to a slightly lower commitment level attracts votes from two groups of voters: all agents with (low) $\beta$’s such that $c$ is not sufficient to generate delay and so a lower $c$ is preferable, and all agents with (high) $\beta$’s such that $c$ is more than enough. Thus, support for the deviating candidate is overwhelming, with the extremes “squeezing” the middle. Zero commitment is an equilibrium if the commitment technology is not “too efficient.” If, however, investment is very cheap ($I(c)$ is very low), then zero commitment cannot be an equilibrium because a “global” deviation to a large commitment would attract a majority of support. The proposition describes the mixed strategy equilibria in this case.

When only consumption choices are mandated (but commitment is chosen individually), the same analysis as in the general case holds and equilibrium is characterized by the entire electorate choosing not to invest in commitment.

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42Note that $\beta(\tilde{c})(v_3 - v_2)$ is decreasing in $\tilde{c}$. Since $\beta(0)(v_3 - v_2) > I(0) = 0$ and $0 = \beta(v_2)(v_3 - v_2) < I(v_2)$, the existence of a unique $\tilde{c} \in (0, v_2)$ satisfying the equality is guaranteed.
Consider, last, the case in which both commitment and choices are mandated. Incentives to vote for investment in the first period may be high for high-\(\beta\) individuals. The optimal commitment parameter \(c\) is either 0 or the \(c^*\) that is just sufficient to make the median-\(\beta\) individual choose consumption at period 3, i.e., the minimal level of cost that solves

\[
v_2 - c^* \geq \beta_M v_3 \text{ or } c^* = \max \{v_2 - \beta_M v_3, 0\}.
\]

In period 1, all voters such that \(\beta (v_3 - v_2) \geq I(c^*)\) or equivalently such that \(\beta \geq \frac{I(c^*)}{v_3 - v_2}\) prefer \(c^*\) to 0; all agents with lower \(\beta's\) prefer 0. Thus, there can be a broad consensus in favor of investing.

**Proposition 8** Suppose \(k(x, c) = cx\). When both commitment and consumption decisions are centralized, there exist \(\tilde{\beta}, \hat{\beta}\) such that if \(\beta_M \leq \tilde{\beta}\) or \(\beta_M \geq \hat{\beta}\), there is a unique equilibrium with \(c = 0\), and if \(\beta_M \in (\tilde{\beta}, \hat{\beta})\), there is an equilibrium with positive commitment.

Now that we have characterized equilibria in this environment, it can easily be seen that the main forces behind our welfare results from Section 6 are still in place: either full centralization or full decentralization are best, and the comparison between these two institutions depends on how virtuous the median voter is. In fact, the proof of Proposition 6 remains intact.

### 7.6 General Dynamic Decisions

Some of the insights highlighted by our model are inherent to any setting with time inconsistent voters. Indeed, suppose, quite generally, that at \(t = 1\), agents choose menus of actions for \(t = 2\) of the form \(A \subset \mathbb{R}\) from a set \(\mathcal{A}\). There is a default menu \(A_0\) and a switch to any menu \(A \neq A_0\) entails a cost \(c(A) > 0\) (and \(c(A_0) = 0\)). We assume that for any \(A_1 \neq A_2\), \(c(A_1) \neq c(A_2)\). That is, there is a unique mapping between costs and menus. At \(t = 2\), agents choose an action from the menu selected in period 1. That is, they choose an action \(a \in A\).

In order for a model of collective action to be non-trivial, the population needs to have heterogenous preferences, and we assume a continuum of agents parametrized by a preference type \(\beta \in [0, 1]\), with a continuous and positive density. An agent of type \(\beta\) chooses action \(a(\beta; A)\) at \(t = 2\) out of menu \(A\). Time inconsistency implies that, at \(t = 1\), any agent of type \(\beta\) prefers to commit to some action \(a \in A_0\) over entering the dynamic process and choosing \(a(\beta; A)\) at a cost \(c(A)\).
In this setting, we can study the effects of collective action, much as in the special case we focused on throughout the paper. At $t = 1$, collective action translates into agents submitting their desired costs (that correspond uniquely to menus) and the median cost (and menu) being implemented. At $t = 2$, collective action is defined through simple majority rule as we have assumed.

The observation that centralization at $t = 1$ combined with decentralization at $t = 2$ generates worse welfare than full decentralization follows immediately (even absent time inconsistency). Indeed, agents in the fully decentralized setting can always emulate collective decisions at $t = 1$. This generality may make one wonder why we analyze the centralized commitment, decentralized consumption setting. The reason is that we believe this setting is close in spirit to many applications and our formulation allows us to illustrate some natural comparative statics on the equilibrium that emerges in such settings. Furthermore, it allows us to identify more precisely the impacts of welfare considerations at $t = 0$, when only $t = 1$ centralization can be useful.

Suppose we further assume a minimal well-behavedness condition on preferences: at $t = 2$, preferences are single-peaked and for each $A$, $a(\beta; A)$ is monotonic in $\beta$.

Then, under decentralization at $t = 1$ and centralization at $t = 2$, all choosing no investment at $t = 1$, i.e., remaining with menu $A_0$ and allowing the median agent of type $\beta_M$ select the action $a(\beta_M; A_0)$, constitutes an equilibrium. That is, collective action at $t = 1$ undermines the median agent’s ability to commit. Suppose that left to her own devices, the median agent would select menu $A_M$ in period 1. As long as $a(\beta_M; A_M)$ (at a cost of $c(A_M)$) is preferred in terms of welfare to $a(\beta_M; A_0)$, full decentralization is superior to centralization only in period 2.

8 Conclusions

The paper considers a simple setting in which behavioral agents, who in our case suffer from present bias, are also political actors, electing the government that is charged with “solving” their behavioral biases. While commitment instruments can be beneficial to individuals left to their own devices, we show the sensitivity of collective outcomes to the precise timing in which political processes take place and the underlying distribution of biases in the population. Commitment levels are lowest when only consumption is mandated. Under some regularity assumptions, commitment is highest when only it is subject to collective action. When both commitment and consumption decisions are decided upon collectively, they reflect the preferences of the individual with median present-bias preferences.
From a welfare perspective, there is a complementarity between centralization at the commitment and consumption stages. Indeed, we show that either full centralization or laissez faire economies generate the highest welfare. Full centralization can be beneficial when there is a virtuous median voter. In that case, centralization effectively allows the population to delegate decisions to a virtuous agent.

These results are potentially relevant for many settings. One notable example is the design of pension systems in the U.S. and abroad. Indeed, a public pension system is sometimes defended as an effective solution to under-saving driven by self-control problems. Our results suggest that an analysis of the collective action of self-control is an important aspect of the design of such pension systems. A careful study of the design of a pension system for agents subject to self-control problems requires many specific details that are missing from our model. However, their design needs to take into consideration the political constraints imposed by those same individuals who are prone to self-control problems and comprise the electorate. These constraints may in principle affect the choice between a pay-as-you-go system and a funded system, the safeguards that are embedded in the system, as well as the timing and response of the system to demographic shocks. We view this as an important potential avenue for subsequent research.

Accounts of behavioral biases have generated a rich literature that, by and large, focuses on individual actions. The paper offers some first steps to studies that allow for these same biases when considering policy determination, either ones that attempt at overcoming these biases (such as in the case of retirement savings) or otherwise. In particular, it suggests the potential importance of considering different biases (e.g., overconfidence, belief distortions in general, limited memory, etc.) as well as different types of policies (e.g., debt limits, general upholding of political promises, etc.) when inspecting political processes.43

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43 As mentioned, some recent work has started moving in that direction. Ortoleva and Snowberg (2015) study the impacts of over-confidence, while Diermeier and Li (2015) consider impacts on re-election and effort put by politicians who are facing voters with limited memory and an inclination to persist with their past voting behavior.
9 Appendix – Proofs

Proof of Lemma 1. Consider the intermediate region of \( \beta \) parameters:

\[
\beta \in \left[ \left( v_2 - \frac{\partial k(1,c)}{\partial x} \right) / v_3, \left( v_2 - \frac{\partial k(0,c)}{\partial x} \right) / v_3 \right].
\]

Since \( \frac{\partial^2 k(x(c,\beta),c)}{\partial x \partial c} > 0 \), increasing \( c \) leads to an increase in the right hand side of the first order condition specified in (1). Since \( \frac{\partial^2 k(x(c,\beta),c)}{\partial x^2} > 0 \) it follows that \( x(c,\beta) \) is decreasing in \( c \). Similarly, notice that \( \frac{d(v_2 - \beta v_3)}{d\beta} < 0 \) so that the left hand side of the first order condition is decreasing in \( \beta \). Now, notice that since \( \frac{\partial^2 k(x,c)}{\partial x \partial c} > 0 \), \( \frac{\partial k(1,c)}{\partial x} \) and \( \frac{\partial k(0,c)}{\partial x} \) are decreasing in \( c \). The lemma’s claim then follows.

Proof of Proposition 1. Lemma 2 assures that, under our regularity assumption, \( c(\beta) > 0 \) for all \( \beta \in \left( \beta, \frac{v_2}{v_3} \right) \). Furthermore, since we assume that fundamentals are such that \( x(c,\beta) \) is well-behaved, \( c(\beta) \) is continuous.

If \( 1 - G(\frac{v_2}{v_3}) \geq 1/2 \), there is a majority of agents who prefer no commitment and the equilibrium commitment parameter is \( c^{CD} = 0 \), which coincides with that preferred by the median.

Suppose instead that \( 1 - G(\frac{v_2}{v_3}) < 1/2 \). Let \( B(\bar{c}) = \{ \beta \mid c(\beta) \leq \bar{c} \} \). From Lemma 2, agents have single-peaked preferences. Therefore, for any \( \bar{c} \), all agents of type \( \beta \in B(\bar{c}) \) strictly prefer commitment level \( \bar{c} \) to any commitment level \( c > \bar{c} \). For any \( 0 < c_1 < c_2 \leq \max_{\beta} c(\beta) \), \( B(c_1) \subseteq B(c_2) \). Therefore, continuity of \( c(\beta) \) assures there exists a unique \( c^{CD} \) such that \( G(B(c^{CD})) = 1/2 \). For any parameter \( c > c^{CD} \), there is a strict majority preferring lower commitment, while for any \( c < c^{CD} \), there is a strict majority preferring greater commitment.

It follows that \( c^{CD} \) defines the unique equilibrium commitment level.

When \( 1 - G(\frac{v_2}{v_3}) < 1/2 \) and \( c(\beta) \) has a unique maximum, there exist \( \beta_L, \beta_H \in \left( \beta, \frac{v_2}{v_3} \right) \) such that \( B(c^{CD}) = [\beta, \beta_L) \cup [\beta_H, \beta] \). Since \( G \) is continuous and \( G(\beta_L) + (1 - G(\beta_H)) = 1/2 \), we have that \( G(\beta_L) < 1/2 \) and \( 1 - G(\beta_H) < 1/2 \). It follows that \( \beta_M \in (\beta_L, \beta_H) \). Notice that \( c^{CD} = c(\beta_L) = c(\beta_H) \). It follows that \( \beta^{CD} = \beta_H > \beta_M \).
Proof of Lemma 3. Let us first consider the case in which \( x(c, \beta_M) \) is interior.

From the first order condition of the median voter in period 2 we know that whenever \( x(c, \beta_M) > 0 \),
\[
v_2 - \beta_M v_3 = \frac{\partial k(x(c, \beta_M), c)}{\partial x}.
\]

Consider the effect of \( c \) on an agent of taste parameter \( \beta \) who foresees that period 2 decisions will be made by the median voter.

\[
\frac{\partial U_1}{\partial c} = \frac{\partial x(c, \beta_M)}{\partial c} (\beta v_2 - \beta v_3) - \beta \frac{\partial k(x(c, \beta_M), c)}{\partial c} - \beta \frac{\partial k(x(c, \beta_M), c)}{\partial x} \frac{\partial x(c, \beta_M)}{\partial c} - I'(c).
\]

The first order condition for \( \beta_M \) implies that
\[
v_2 - v_3 = -(1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial x}
\]
and so
\[
\frac{\partial U_1}{\partial c} = -\beta \left( \frac{\partial x(c, \beta_M)}{\partial c} (1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial c} \right) - I'(c).
\]

Now, if \( \frac{\partial x(c, \beta_M)}{\partial c} (1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial c} \geq 0 \), all agents prefer \( c = 0 \) and the claim follows. Otherwise, \( \frac{\partial x(c, \beta_M)}{\partial c} (1 - \beta_M) v_3 + \frac{\partial k(x(c, \beta_M), c)}{\partial c} < 0 \) and \( \frac{\partial U_1}{\partial c} \) is increasing in \( \beta \). At a maximum, \( U_1 \) is (weakly) concave and the claim follows.

We now consider the cases in which \( x(c, \beta_M) \) may be at a corner solution. Note first that when \( \beta_M \geq \frac{v_2}{v_3} \), then \( x(c, \beta_M) = 0 \) for all \( c \). In this case, all agents prefer \( c = 0 \) in period 1 and the claim follows. More generally, we have
\[
x(c, \beta_M) = \begin{cases} 0 & c \geq c_H(\beta_M) \\
(v_2 - \beta_M v_3) = \frac{\partial k(x(c, \beta_M), c)}{\partial x} & c_L(\beta_M) < c < c_H(\beta_M) \\
1 & c \leq c_L(\beta_M)
\end{cases}.
\]

Clearly, there is no value in choosing \( c > c_H(\beta_M) \). Thus, \( c \geq c_H(\beta_M) \) and
\[
U_1(\beta, c_H(\beta_M)) = v_3 - I(c_H(\beta_M)).
\]

Comparing this to interior cases:
\[
U_1(\beta, c_H(\beta_M)) - U_1(\beta, c) = \beta v_3 - I(c_H(\beta_M)) - \beta v_3 + x(c, \beta_M) (\beta v_2 - \beta v_3) - \beta k(x(c, \beta_M), c) - I'(c) = \\
= \beta (x(c, \beta_M) (v_3 - v_2) - k(x(c, \beta_M), c)) - (I(c_H(\beta_M)) - I(c)).
\]

If \( c_H(\beta_M) \) is optimal for some \( \hat{\beta} \), it has to be the case that
\[
\hat{\beta} (x(c, \beta_M) (v_3 - v_2) - k(x(c, \beta_M), c)) > (I(c_H(\beta_M)) - I(c))
\]

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for all \( c < c_L (\beta_M) \). But then, this also holds for all \( \beta > \beta \).

It is easy to see that it must be the case that when \( c \leq c_L (\beta_M) \), then the optimal \( c \) is zero: there is no point in investing anything in commitment if it does not help. In this case, the payo\( \tilde{u} \) in the first period is \( U_1 (\beta, 0) = v_2 \). Comparing this to interior cases:

\[
U_1 (\beta, c_H (\beta_M)) - U_1 (\beta, c) = \beta v_2 - (\beta v_3 + x (c, \beta_M) (\beta v_2 - \beta v_3) - \beta k (x (c, \beta_M), c) - I (c)) \\
= -\beta ((v_3 - v_2) (1 - x (c, \beta_M)) - k (x (c, \beta_M), c)) + I (c).
\]

If a choice of zero commitment is optimal for some \( \beta \) it has to be the case that

\[
\tilde{\beta} ((v_3 - v_2) (1 - x (c, \beta_M)) - k (x (c, \beta_M), c)) < I (c)
\]

for all \( c > c_L (\beta_M) \). But then this also holds for all \( \beta < \tilde{\beta} \).

**Proof of Proposition 4.** In period 1, all agents but the foreseen pivotal voter of period 2 best respond by choosing \( c = 0 \), as their choice of commitment parameter affects only the commitment and consumption costs they experience, but not the levels of future consumption. If any agent of taste parameter \( \beta \neq \beta_M \) invests in commitment in period 1, the median preferences in period 2 would correspond to those of the median agent with preferences \( \beta_M \) and so investment by the agent of taste parameter \( \beta \) are strictly sub-optimal. Suppose the median agent invests in period 1. In that case, in period 2 her preferences no longer coincide with the median preferences and so her commitment investment does not affect ultimate choice and is thus strictly sub-optimal. The claim then follows.

**Proof of Proposition 6.**

1. Whenever \( \beta_M \geq \beta^* \), in the fully centralized system there is fully delayed consumption, \( x(c(\beta_M), \beta_M) = 0 \) even with no commitment, \( c(\beta_M) = 0 \). In particular, since \( k(x, 0) = 0 \), any agent of preference parameter \( \beta \) experiences a period 1 utility of \( \beta v_3 \). In contrast, in the fully decentralized system, while all agents with \( \beta \geq \beta^* \) experience the same period 1 utility as in the fully centralized system, agents with \( \beta \in (\underline{\beta}, \beta^*) \) choose \( c(\beta) > 0 \) at a cost of \( I(c(\beta)) > 0 \) (as well as potentially some early consumption), hence they receive a utility that is strictly lower than \( \beta v_3 \). Since the distribution \( G \) is continuous and \( G(\beta^*) > 0 \), the result follows.

2. Consider a sequence of distributions \( \{G_n\}_{n=1}^{\infty} \) such that there exists \( \tilde{\beta} > \beta \) for which \( \{G_n(\tilde{\beta})\} \) is uniformly bounded below 1 and \( \lim_{n \to \infty} \beta_M^n = \beta \). Under full centralization,
continuity implies that for any \( \varepsilon > 0 \), there exists \( \tilde{n}(\varepsilon) \) such that for any \( n > \tilde{n}(\varepsilon) \), for all \( \beta \), period 1 utility under full centralization, \( U_{1CC}(\beta; n) \) satisfies:

\[
U_{1CC}(\beta; n) < \beta \left( v_2 x \left(c(\beta), \beta\right) + v_3 \left(1 - x \left(c(\beta), \beta\right)\right) - I \left(c(\beta)\right)\right) + \varepsilon.
\]

By Lemma 2, \( c(\beta) > 0 \) for all \( \beta < \beta^* \). Hence, under full decentralization, from Lemma 1, \( x \left(c(\beta), \tilde{\beta}\right) \) is strictly decreasing in \( \beta \). Since agents can always choose \( c(\beta) \) themselves, it follows that for sufficiently small \( \varepsilon \), there exists \( \beta(\varepsilon) \in (\bar{\beta}, \tilde{\beta}) \) such that for any \( \beta \geq \beta(\varepsilon) \), and any \( n > \tilde{n}(\varepsilon) \), decentralized choices generate first-period utility \( U_1(x(c, \beta), c(\beta), \beta) \) satisfying:

\[
U_1(x(c, \beta), c(\beta), \beta) - U_{1CC}(\beta; n) > 0.
\]

Furthermore, \( \lim_{\varepsilon \to 0} \beta(\varepsilon) = \beta \) and for sufficiently small \( \varepsilon > 0 \), for any \( \beta \geq \tilde{\beta}, n > \tilde{n}(\varepsilon) \),

\[
U_1(x(c, \beta), c(\beta), \beta) - U_{1CC}(\beta; n) > \Delta > 0.
\]

Suppose then that \( G_n(\tilde{\beta}) \leq \gamma < 1 \) for all \( n \). For sufficiently small \( \varepsilon \), it follows that

\[
\int_\beta^{\beta(\varepsilon)} \frac{U_{1CC}(\beta; n)}{1 + 2\beta} dG_n(\beta) < \int_\beta^{\beta(\varepsilon)} \frac{U_1(x(c, \beta), c(\beta), \beta)}{1 + 2\beta} dG_n(\beta) + \int_{\beta}^{\tilde{\beta}} \frac{\Delta}{1 + 2\beta} dG_n(\beta).
\]

In particular, for any \( n > n^* = n(\varepsilon) \), where \( \varepsilon \) is sufficiently small, welfare satisfies:

\[
\Pi_{CC} = \int_\beta^{\beta(\varepsilon)} \frac{U_{1CC}(\beta; n)}{1 + 2\beta} dG_n(\beta) < \int_\beta^{\beta(\varepsilon)} \frac{U_1(x(c, \beta), c(\beta), \beta)}{1 + 2\beta} dG_n(\beta) + \int_{\beta}^{\tilde{\beta}} \frac{\Delta}{1 + 2\beta} dG_n(\beta) < \Pi^{DD}.
\]

\[
\Pi_{CC} = \int_\beta^{\beta(\varepsilon)} \frac{U_{1CC}(\beta; n)}{1 + 2\beta} dG_n(\beta) < \int_\beta^{\beta(\varepsilon)} \frac{U_1(x(c, \beta), c(\beta), \beta) - \varepsilon}{1 + 2\beta} dG_n(\beta) + \int_{\beta}^{\tilde{\beta}} \frac{U_{1CC}(\beta) - \varepsilon + \Delta}{1 + 2\beta} dG_n(\beta) < \Pi^{DD}.
\]

\[\blacksquare\]
Proof of Proposition 7. We first show that with linear costs there is no pure strategy equilibrium with positive commitment. Assume by way of contradiction that candidate 1 chooses $c > 0$ with probability 1. Then candidate 2 can win with probability 1 by choosing $c - \varepsilon$ for $\varepsilon$ sufficiently small. All voters with preference parameter $\beta$ such that $\beta v_3 \geq v_2 - (c - \varepsilon)$ prefer candidate 2 because they still get to consume in period 3 but the lower investment in commitment is sufficient to do so. Furthermore, all voters with $\beta$ such that $\beta v_3 < v_2 - c$ prefer candidate 2 because they consume in period 2 with both levels of commitment, so prefer the candidate who offers the lower level. The only voters who may prefer $c$ over $c - \varepsilon$ are those whose preference parameter $\beta$ is such that $\beta v_3 \geq v_2 - c$ and $\beta v_3 < v_2 - (c - \varepsilon)$. However, because the distribution $G$ is continuous, the mass of these voters can be made arbitrarily small by choosing $\varepsilon$ small enough.

If $\beta_L (v_3 - v_2) \leq I(c_L)$, then all agents with preference parameter $\beta$ such that $\beta \leq \beta_L$ prefer $c = 0$ to $c_L$. Since $I(c)$ is convex, they prefer $c = 0$ to all $c > c_L$. Furthermore, any $0 < c < c_L$ is also worse than $c = 0$ for these agents because $\beta v_3 < v_2 - c$ by the definition of $c_L$ and $\beta_L$. Since $(1 - G(\beta^*)) + G(\beta_L) = \frac{1}{2}$, there is a majority in favor of $c = 0$ against all other $c$’s.

If $\beta_L (v_3 - v_2) > I(c_L)$, then all $\beta$’s between $\beta^*$ and $\beta_L$ strictly prefer $c_L + \varepsilon$ to $c = 0$. Furthermore, some $\beta$’s slightly higher than $\beta_L$ also prefer $c_L + \varepsilon$ to $c = 0$. Since there half the mass of voters is concentrated between $\beta_L$ and $\beta^*$, $c_L + \varepsilon$ defeats $c = 0$. As shown above, there is no pure strategy equilibrium with positive commitment. This establishes that when $\beta_L (v_3 - v_2) > I(c_L)$, there is no pure strategy equilibrium.

We now show that when $\beta_L (v_3 - v_2) > I(c_L)$ the mixed-strategy profiles in the statement of the proposition constitute equilibria. Note first that $c_1$ and $c_2$ as defined in the proposition tie. Consider now a policy $\hat{c} > c_2$. This policy may win against $c_1$. However, $\hat{c}$ loses against $c_2$ because all agents of preference parameter $\beta > \beta(c_2) - \delta$ (for some $\delta$) would vote for $c_2$ over $\hat{c}$. Since $G(\beta(c_1)) - G(\beta(c_2)) = \frac{1}{2}$, there is more than 50% of the voters supporting $c_2$. Thus, $\hat{c}$ wins with probability $1/2$. Consider now a policy $c_1 < \hat{c} < c_2$. Such a policy may win against $c_2$. However, against $c_1$, the only potential supporters are agents with preference parameters within $[\beta(\hat{c}), \beta(c_1)]$, which by construction entails less than 50% of the population. In particular, $c_L$ is a policy that would lose against $c_1$. Last, consider a policy $\hat{c} < c_1$. This policy may win against $c_1$. Against $c_2$, its only potential supporters are agents with preference parameters $\beta \leq \beta(c_2)$ or $\beta \geq \beta(\hat{c})$, which from the definition of the pair $(c_1, c_2)$ account for less than 50% of the voters. Thus, the candidate equilibrium strategy profile wins with probability at least $1/2$ against all possible deviations and no deviation is strictly beneficial.
References


