Endogenous Growth

Senior you should have your final
due June 4 at noon

Final for all non-seniors posted June 5
5pm
due June 9 at noon
If you want to do it early, email me
Outline

• Endogenous growth
  – The problem
  – Human capital
  – Technical progress
  – Increasing returns

• Growth as Political economy
  – Private decisions in a public context
  – Public decisions influenced by private preferences
  – Growth as a complex system problem
Traditional growth theory

• Focus on capital and savings rates
• Models are clean but run into problems when we look at them in the real world context
• So we need different ways of thinking about growth problems.
Endogenous Growth

• Maybe assuming constant drift in technical change (recall in the Solow+ technical change, labor efficiency growth at constant rate $(1+\pi)$) is not so interesting

• In fact growth is a social process
  – In some situation imitation is easy
    • That would lead to convergence
  – In others it may be hard
    • That might lead to divergence

• So we need to think about how technology changes
Possibility 1. Human capital

- Solow => \( Y = K^{\alpha}L^{1-\alpha} \)
  - \( L \) evolves at rate \( (1+\eta) \), and \( K \) depends on savings
  - \( k_{t+1} = (1-d)k_t + sy_t \)  \( \text{Steady state } (\eta+d)/s = y(k)/k \)

- Now suppress the population change part and think
  - \( y = k^{\alpha}h^{1-\alpha} \) \( \iff (Y = k^{\alpha}h^{\beta}L^{1-\alpha-\beta}) \)
    - \( k_{t+1} = k_t + sy_t \)
    - \( h_{t+1} = h_t + qy_t \)

- Model is a bit more intricate but by the same logic as before this model is going to have a steady state.
  - Because there are constant returns to scale you will settle down to a constant growth rate (rather than a level)
  - This will involve a constant physical human capital ratio \( r \) \( (h/k) \) equal to \( q/s \).
  - As a result \( k_{t+1}/k_t = 1 + sy_t/k_t \) or \( \Delta k_t = sr_t^{1-\alpha} \)
  - Steady state \( \Delta k = sr^{1-\alpha} = s^\alpha q^{1-\alpha} \)
Proof

• \( k_{t+1} = k_t + sy_t \)
  • Define \( r \) to be \( \frac{h}{k} \frac{y}{k} = \frac{k^\alpha h^{1-\alpha}}{k} = h^{1-\alpha}k^{1-\alpha} = r^{1-\alpha} \)
• Divide both sides by \( k_t \)
  • A. \( \frac{(k_{t+1})}{k_t} = 1 + s(y_t)/k_t = sr_t^{1-\alpha} + 1 \)
• Similarly
  • B. \( \frac{(h_{t+1})}{h_t} = 1 + q(y_t)/h_t = qr_t^{-\alpha} + 1 \)
– Divide B by A
  • \( \frac{h_{t+1}/h_t}{k_{t+1}/k_t} = \frac{(qr_t^{-\alpha} + 1)}{(sr_t^{1-\alpha} + 1)} \)
  • \( \frac{h_t/k_t}{k_{t+1}/h_t} \)
• OR \( r_{t+1} = r_t \frac{(qr_t^{-\alpha} + 1)}{(sr_t^{1-\alpha} + 1)} \)
  – if we multiply the top by \( 1/qr_t^{-\alpha} \) and the bottom by \( 1/sr_t^{1-\alpha} \)
  • \( r_{t+1} = q/s \frac{(1+r_t^{-\alpha}/q)}{(1+r_t^{1-\alpha}/s)} \)
  – if you multiply both sides by \( r_t \) and then divide the ratio by \( r_t^{1-\alpha} \)
  • \( r_{t+1} = r_t \frac{(q/r_t + r_t^{\alpha-1})}{(s+r_t^{\alpha-1})} \)
• Now can show if \( r_t > q/s \) => \( r_t > r_{t+1} > q/s \)
  • Part 1. if \( r_t > q/s \) then \( r_t^{\alpha}/q > r_t^{1-\alpha}/s \) => \( (1 + r_t^{\alpha}/q)/(1 + r_t^{1-\alpha}/s) > 1 \) => \( r_{t+1} > q/s \)
  • Part 2. \( q/s < r_t \) => \( q/r_t < s \) => \( (q/r_t + r_t^{\alpha-1}) < (s+r_t^{\alpha-1}) \) => \( r_t > r_{t+1} \)

In fact \( r_{t+1} = r_t = q/s \) is unique solution
Implication

• Advantage over models of Tuesday
  – 1) explains why returns to physical capital may not be higher in poor countries
  – 2) does not require countries to converge...initial differences persist, and negative shocks really matter.
  – 3) but in a way artificial because there is no technical change.
  – Still Most of what is to be explain is away from trend.
Possibility 2: Knowledge

• Let knowledge be blueprints. The more blueprints the better but you have to do the early ones to get to the more interesting ones.

• $Y_t = E_t^\gamma K_t^\alpha [uH]^{1-\alpha}$

• $\Delta E = a(1-u)H$

• $K_{t+1} - K_t = sY_t$

• This model is both really similar and quite different.
Knowledge

• Similarities
  – Growth process looks the same (accumulate two factors E and K)
  – On each factor you have DMR

• Differences
  – $Y_t = E_t^\gamma K_t^\alpha [uH]^{1-\alpha}$ this is not CRS or DMR. The sum of the exponents is $1+\gamma$ so this function has increasing returns
  – What does the solution look like? In many ways just like the Solow model with exogenous technical change
  – Except that growth rate is endogenous—and it creates increasing returns (the higher your current level the higher your growth rates)
Increasing returns

• Some of what we see in the location of economic activity is in fact consistent with increasing returns

• Some of what we see suggests that increasing returns may not be very large
  – Separation of R&D and production location
  – Development of outsourcing
  – Development of trade in semi-finished goods.
The Revenge of political economy

• Growth you might study at Caltech
• Variables in the models
  – Depreciation
  – Population growth
  – Savings rates \((s)\)
  – Investment in Physical capital \((\Delta K)\)
  – Investment in Human capital \((\Delta H)\)
  – Investment in Knowledge capital \((\Delta E)\)
• Depend both on private and public decisions
Private decision influenced by social context

- Depreciation
  - Technical variable
  - But also a choice variable
- Individual can chose to spend more today to make breakdowns less likely
  - So trade off is between costs today (investment)
  - And cost tomorrow (maintenance replacement)
- Social context
  - Property rights => fear of expropriation ↓
  - Regulation => ↑ if good ↓ if bad
  - Taxation => ↑ if tax maintenance more than investment or ↓
  - Fashion? Is it chic to maintain?
True for depreciation also true for all the others

- Population growth
  - Depends on fertility decisions
  - In turn depends on human capital of women and technology (contraception?)
  - Also depends on costs and returns (wage labor vs stay at home)

- Savings rates (s)
  - Depends on what individuals decide to do (how long they live...)
  - Also on incentives and requirements of states (tax law, mandatory contributions)
  - And decisions made by firms (distribute profits or invest them)

- Investment in Physical capital ($\Delta K$)
- Investment in Human capital ($\Delta H$)
- Investment in Knowledge capital ($\Delta E$)
  - Different set of institutions (but abundance of K and H will lower the cost)
Public decisions influenced by private concerns

• Lets choose a policy
  – building code
  – child care policy
  – education policy

• Then three issues
  – Private preferences
    • I am a home owner
    • I am a would be home owner
    • I am a builder
    • I am a banker....
  – Key trade off efficiency (what is best for society) and distribution (what is best for me).
  – Aggregation of those preferences
    • Do we have a referendum
    • Do we delegate to an assembly
      – Does the assembly delegate to an agency
      – Does the agency delegate to experts
How to solve

• Make the assumption that individuals who are making these decisions weight the private gains and private costs
  – Both in terms of what they want
  – And whether to express their opinion politically

• Then different aggregating mechanisms weight these preferences
Public decisions influenced by private concerns

• Even if we solve the policy decision issue we have to take a step back

• How do we chose the aggregation rule
  – Democracy.......authoritarian government
  – Voting.........delegation
Growth as a complex process

• In 5 slides we have traveled very far from
  – $k_{t+1} = (1-d)k_t + sy_t$
• It’s a good thing.
• If the problem was simple it would have been solved
• That its complex and important means we have something to work on.
The End!

THANK YOU ALL

Best of Luck on the Final