Risk and the market for insurance

Armen Alchian:

“To lower auto insurance rate premium we should put a stake on each steering wheel”
Outline

• Risk and Risk attitudes
• Kinds of risk
• Mitigating risk
• Insurance
• Diversification
• Mitigating risk and incentives
Risk

- Consumer max utility
- But uncertainty
  - Income
  - Prices
  - Quality
  - Preferences

Some outcomes are better than others
- How to plan?
- How to mitigate?

- Firm Max profits
- But uncertainty
  - Output price (demand)
  - Input prices
  - Technology
  - Innovation
  - Regulation
Risk attitudes

• Take a coin flip if heads $10 if tails $0.
  – Probability of heads 0.5, probability of tails 0.5
• How much are you willing to pay for the outcome of a flip
  – Less than $5, more than $5
  – If less than $5 then risk averse, if more Risk loving
• How much are you willing to pay for
  – Same coin flip if heads $100,000 if tails -$99990.
  – Expected value is $5
• Coin flip, heads 1/1000000 pays $500,000 tails 0, cost $1
  – Expected value -0.5
Risk attitudes

- Risk averse

\[ U(\text{PX}_l + (1-P)\text{PX}_h) \]

\[ PU(X_l) + (1-P)U(X_h) \]

- Risk loving

\[ U(\text{PX}_l + (1-P)\text{PX}_h) \]

\[ PU(X_l) + (1-P)U(X_h) \]
Kinds of risk

• Three kinds of risk
  • Exogenous risk (F. Knight’s risk)
    – Sets of events to which we can assign probabilities
    – And those events do not depend on the actors
  • Endogenous risk
    – Sets of events given but probability of an event depends on the actors of the game
    – E.g. Adverse selection
      • Risk depends on who takes up a contract (life insurance, subprime mortgages)
    – Moral Hazard
      • Risk depends on care one takes (do you take up smoking after buying a life insurance policy…)
  • Uncertainty
    – Things that matter but either the relevant list of events is not known or the relevant probabilities cannot be computed.
    – So you know you care but you are in a bind.
Mitigating risk

• Matters for most of us who are risk averse
• Exogenous risk
  – What we are going to work on next
• Endogenous risk
  – Contracts and institutions
• Uncertainty
  – Well that remains an open question
  – Connection to robust systems in CS
Insurance

• Lets take a partial equilibrium approach
• Individual risk averse
  \(- U(pX_l+(1-p)X_h) > pU(X_l)+(1-p)U(X_h)\)
• It follows that there must exist \(y\) such that
  \(- U(pX_l+(1-p)X_h-y) = pU(X_l)+(1-p)U(X_h)\)
• \(y\) is the price an individual is willing to pay to insure against the risk of the low outcome
• Now we need to deal with the other side of the market
Insurers

• Individuals get good news with probability \((1-p)\) and bad news with probability \(p\)
• An insurance contract implies they receive average return \(X\) so on net the insurance company pays \(X-X_l\) to those with bad news and gets \(X_h - X\) from those with good news.
• Assume there are \(n\) individuals in such a situation and that risk is independently distribute so that we can write
• So returns are \(\{(1-p)n(X_h-X) - pn(X-X_l)\}\)
  – Or \(n\{(1-p)X_h+p(X_l)-X\}\)
  – If the industry is competitive there will be zero profits so \(X=(1-p)X_h+pX_l\)
  – Insurance is free \((y=0)\)
Free insurance

• If the number of buyers are large, risks are independent, industry is competitive then insurance is close to free
  – the insurance company pools the risks and eliminates it in the aggregate

• So key here is whether risks are independent
  – Life insurance?
  – Accident insurance
  – Unemployment
  – Mortgage insurance
  – Hurricanes, Earthquakes
Diversification

• Risk we saw above are un-diversifiable
  – Either I am alive or I am not
  – Either I have an accident or I do not
  – So I really care to have insurance

• But some risks are diversifiable
  – This is more true for firms (multiproduct)
  – But works for individual wealth
• Assume to assets X and Y
• \( X = X_l \) with probability \( p \) and \( X_h \) with prob \((1-p)\)
• \( Y = Y_l \) with probability \( q \) and \( Y_h \) with prob \((1-q)\)
• What should I do. Let me put \( \alpha \) of my wealth in X and \((1- \alpha)\)
• What is optimal \( \alpha \)? (tradeoff between return and risk)

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<td>( Y )</td>
<td>Good ((1-p))</td>
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<tr>
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<td>( \alpha X_h + (1- \alpha)Y_h )</td>
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<tr>
<td>Bad ((q))</td>
<td>( \alpha X_h + (1- \alpha)Y_l )</td>
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• But we can’t say anything about what is optimal unless we know how X and Y co-vary
• Lets look at three simple cases
Perfect correlation (p=q)

- Optimal $\alpha$?
- Either $\alpha=1$ or $\alpha=0$
- Why

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<td>$\text{Prob}=0$</td>
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- Suppose $Y_l<X_l$ and $Y_h<X_h$
- Then $X$ is better than $Y$ no matter what $\alpha=1$
- Suppose $X_l<Y_l$ and $Y_h<X_h$
- Then $X$ is better in good time and $Y$ in bad times if you are risk averse enough you pick $Y$ $\alpha=0$
- otherwise $X$ ($\alpha=1$)
- You can check all the other possibilities
Independence

- \( q=p \) (prob good-good \( \left(p^*q\right)=p^2 \))
- Optimal \( \alpha \)?
- If risk averse \( 0<\alpha<1 \)
- Why

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- \( \alpha=1 \) or \( \alpha=0 \)
- Means you bounce between two extreme outcome
- \( 0<\alpha<1 \) means you put less weight on the extreme outcomes and more on the middling ones (one good one bad)
- If fact \( p=\frac{1}{2} \) means you only get both bad outcomes with probability \( \frac{1}{4} \)
Perfect negative corelation

- Y=Y₁ if and only if (X=X₉)
- Optimal α?
- If risk averse 0<α<1
- Why

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- Suppose Y<Y₉<X₉<X₉
- Then X is better than Y no matter what α=1
- But if X<Y₉
- X<α X₉+(1- α)Y₉
- If risk averse would like to insure so α>0 makes sense
- Of course you pay something on the other side because X₉>α X₉+(1- α)Y₉
- The closer α is to ½ the more insurance you are buying
  - If X=Y then perfectly insured
Optimal portfolios

• Whenever $\alpha$ is greater than 0 and less than 1 the individual is buying insurance.

• Optimal portfolio if $X$ is the asset with a greater expected value, as $\alpha<1/2$ is optimal and the more risk averse you are the closer you get to a balanced portfolio.

• In general the more risk averse you are the more diversified you should be

• Why might risk aversion vary?
Insurance and incentives

• The problem of insurance is that it dampens incentives.
• Automobile driver likes to drive fast, and but she is risk averse.
• Accidents occur at an increasing rate as she drives faster.
• Assume first a world without insurance and the driver can drive either fast or safely (slower).
Insurance and incentives

• If she drives fast,
  – probability $p$ she has accident and must pays $C$
  – probability $(1-p)$ she does not.
  – $U_F = U(F) + pU(Y-C) + (1-p)U(Y)$

• If she drives safely
  – probability $q<p$ has accident and must pays $C$
  – probability $(1-q)$ she does not.
  – $U_s = U(S) + qU(Y-C) + (1-q)U(Y)$

• $U_s - U_F = U(S) - U(F) + (q-p)U(Y-C) + (p-q)U(Y)$
• $U_s - U_F = U(S) - U(F) + (q-p)(U(Y) - U(Y-C))$
Insurance

• Assume $U_s - U_F = U(S) - U(F) + (q - p)(U(Y) - U(Y-C)) > 0$
• So our driver drives slowly.
  • $U_s = U(S) + qU(Y-C) + (1-q)U(Y)$
• Now enters the insurance company which offers to cover the cost of an accident at a price $qC + \Delta < pC$
• The driver buys the insurance so her utility is now
  • $U_s = U(S) + (Y-qC-\Delta) > U(S) + qU(Y-C) + (1-q)U(Y)$
  • Will she continue to drove slowly?
  • If she drives fast $U(F) + U(Y-qC-\Delta) > U(S) + U(Y-qC-\Delta)$
• Insurance interferes with the incentive to be safe
Market failure

- If she drives fast she has more accidents in fact at a rate P so the insurance company gets $qC + \Delta$ but pays out $pC$ in claims. Market Fails

- **Solution 1:** company charges $pC + \Delta$, so driver much decide between
  - $U_s = U(S) + qU(Y-C) + (1-q)U(Y)$
  - $U_f = U(F) + U(Y-pC-\Delta)$
  - It could be that she does not buy insurance.

- **Solution 2** insurance company offers a good driver bonus.
  - $U_s = U(S) + U(Y-qC-\Delta)$ vs $U_f = U(F) + U(Y-pC-\Delta) + (p-q)U(p)$
  - Driver might buy this policy but its incomplete insurance
Incentives and risk

• The problem of insurance is that it makes people insensitive to outcomes.
• So it’s easy to insure for exogenous shocks, you just have to price it right.
• The problem is that in many cases individuals have some control over outcomes.
  – Accidents and speed of driving
  – Life insurance and health
  – Unemployment insurance and work
Beyond insurance

• Back to the problem of the firm
• From Last Thursday
• Let the production function be $F(L,K)$ where $L$ is discretionary labor input and $K$ is discretionary capital input.
• Here what matters is what is contractible.
  – Farmer has discretion on how hard he work
  – Landowner has discretion on capital improvement
What if not contractible

• What happens if there is a fixed rent for the land (and the farmer gets the net profit)
  – Landowner’s return is just the rent, he will make 0 discretionary capital investment.
  – Farmer max F(L,K)-L  FOC F_l’ = 1 Efficient investment on discretionary labor

• What happens if the farmer is hired for the year?
  – Farmer’s return is just the wage, she will make 0 discretionary investment
  – landowner max F(L,K)-K  FOC F_K’ = 1 Efficient investment on discretionary capital
Incentives require people to bear risk

• In the previous slides we assumed the ‘owner’ would make the efficient investment.
• But what if investment is risky?
• You can’t insure the owner and give him incentives.
• So if you want to put the owner in a high risk situation you have to pay him a lot
Best Insurance movie

• Double Indemnity (1944).
  – Happens in Glendale

• Remake: Body Heat (1981)
  – Happens in Florida