Demand, price effects, income effects

Demand more!
Demand

• Back to preferences
• Price effects
  – Normal goods
  – Perfect complements
  – Perfect substitutes
• Income effects
  – Normal goods
  – Giffen goods (?)
• intervention
• Observable implications
Price effects (for standard goods)

- $\frac{\partial X(p, M)}{\partial p} < 0$.
  - Changes in demand with respect to own price is negative.

- Elasticity $-\frac{\partial X}{\partial p} \frac{p}{X}$ (in this case its positive but it can be either greater or smaller than 1)

- Who cares about this?
  - Consumer?
  - Producers?
    - Revenue
  - Government?
    - Think of taxes to discourage behavior
    - Think of taxes to raise revenue
Back to Cobb-Douglas

\[ u(x, y) = x^\alpha y^{1-\alpha} \]

\[ X(p_x) = \frac{\alpha M}{p_x}, \quad Y = \frac{(1-\alpha)M}{p_y} \]

\[ \frac{\partial X}{\partial p_x} = -\frac{\alpha M}{(p_x)^2}, \quad \frac{\partial X}{\partial p_y} = 0 \]

\[ \varepsilon = -\left(-\frac{\alpha M}{(p_x)^2}\right) \frac{p_x}{x} = \frac{\alpha M}{(p_x)^2} \frac{p_x}{\alpha M} = 1 \]

Cobb-Douglass very practical because it’s a one parameter per good, constant elasticity demand curve. It’s a very good place to start.
Decomposition into Substitution and Income Effects
Demand and Compensated demand

- Cobb-Douglas $U$
  \[ u(x, y) = x^\alpha y^{1-\alpha} \]
- Demand for good $x$ is $X(p, M)$.
  -- Cobb-Douglas
  \[ X(p_x, M) = \frac{\alpha M}{p_x} \]

- $e(p, u)$ is the expenditure function
  -- minimum income to reach a given level of utility.
  -- Min $p_x x + p_y y$ s.t. $x^\alpha y^{(1-\alpha)} \geq u$
  \[ x = y \frac{\alpha}{(1-\alpha) p_x} \]
  \[ x^\alpha y^{(1-\alpha)} - u = 0 \]
  \[ p_x \left( \frac{\alpha}{(1-\alpha) p_x} \right)^\alpha y^{\alpha y^{(1-\alpha)} - u} = 0 \]
  \[ e(p_x, p_y, u) = u \left( p_x \left( \frac{\alpha}{(1-\alpha) p_x} \right)^{1-\alpha} + p_y \left( \frac{1-\alpha}{\alpha} \right)^\alpha \right) \]
  -- Notice we get back the homothetic property (to double utility you must double expenditures)
Demand and Compensated demand

- Demand for good $x$
  \[ X(p_x, M) = \frac{\alpha M}{p_x} \]

- $e(p, u)$ Expenditure function
  \[ e(p_x, p_y, u) = u \left( p_x \left( \frac{\alpha}{(1 - \alpha)} \frac{p_y}{p_x} \right)^{(1-\alpha)} + p_y \left( \frac{(1 - \alpha) p_x}{\alpha} \right)^\alpha \right) \]

- Compensated demand
  \[ H_x(p_x, p_y, u) = u \left( \frac{\alpha}{(1 - \alpha)} \frac{p_y}{p_x} \right)^{(1-\alpha)} \]

- So we can now find Slutzky’s equation
  \[ \frac{\partial X}{\partial P} = \frac{\partial H}{\partial P} - \frac{\partial X}{\partial M} x^* \]
Slutzky’s equation

- Let's evaluate this

\[ \frac{\partial X(.)}{\partial p_x} = -\frac{\alpha M}{p_x^2} \]

\[ \frac{\partial H(.)}{\partial p_x} = (1 - \alpha) U \left( \frac{\alpha}{p_y (1 - \alpha)} \right)^{(1-\alpha)} p_x^{2+\alpha} \]

\[ \frac{\partial H(.)}{\partial p_y} = (1 - \alpha) \left( \frac{\alpha M}{p_x} \right)^\alpha \left( \frac{(1 - \alpha) M}{p_y} \right)^{(1-\alpha)} \left( \frac{p_y}{(1 - \alpha)} \right)^{(1-\alpha)} p_x^{-2+\alpha} \]

\[ -\frac{\partial X(.)}{\partial M} p_x = -\frac{\alpha}{p_x} \frac{\alpha M}{p_x} = M \frac{\alpha^2}{p_x^2} \]

\[ \frac{\partial X(.)}{\partial p_x} = -\frac{\alpha M}{p_x^2} = M (1 - \alpha) \frac{\alpha}{p_x^3} - M \frac{\alpha^2}{p_x^2} \]
Practical value of Slutzky’s equation

\[
\frac{\partial X}{\partial P} = \frac{\partial H}{\partial P} - \frac{\partial X}{\partial M} x^* - \frac{\partial H}{\partial P}
\]

• You can’t measure \( \frac{\partial H}{\partial P} \)

• So figuring out how to compensate people say for a sudden shock to heating oil...may be difficult. Except for Slutzky’s equation.

• You can observe \( x^* \) you can measure the total effect \( \frac{\partial X}{\partial P} \) and the income effect \( \frac{\partial X}{\partial M} \) So you can figure out \( \frac{\partial H}{\partial P} \) by the formula
Substitutes

• Blu-ray vs DVD
• Blu-ray is better but DVD is cheaper. They are used for the exact same thing.
• Fix the price of DVD player $32 (amazon sells a Sony DVD)
• Assume rentals of disc cost the same (Netflix $2 more per month rather than 8.99, so we are not too far off)
• Now let's worry about how to price a Blu-ray player
• Clearly if we sell it at $31, No one buys a DVD player.
• Clearly if we sell it at $3000 on those who really care about the increased quality are going to buy it.
Substitutes

• So \( U(X, DVD, BR) \) where \( X \) depends on everything else \( BR \) is whether one has Blu-ray and DVD is whether one has DVD.

• Because they are substitutes no one wants both, for now assume they are a small part of the budget. So we can simply consider that choice

• \( U(BR,DVD) = DVD + \alpha BR \)
  – What do indifference curves look like?

• \( \text{Max} \ U(BR,DVD) = DVD + \alpha BR \) sbjt to \( P_D \) DVD + \( P_B \) BR < M

So buy Blu-ray if \( P_B < \alpha P_D \)
Consumer choice

Budget set when BR is expensive

Budget set when BR is Cheap

Indifference curves
Slope $-1/\alpha$

BR

DVD
Individual Demand

\[ Q_{BR} \]

Buy Blu-ray

\[ P_{BR} \]

\[ \alpha^{*}31 \]

Buy DVD

\[ 31 \]

\[ 1 \]

\[ Q_{BR} \]
Aggregte Demand

Customers with $\alpha > 10$

Customers with $\alpha = 1$

$P_{BR}$

$BR$
Complements

• Normal goods $\partial X/\partial p_x < 0$, $\partial X/\partial p_y \geq 0$
• Substitutes goods changes are non linear
• Complements goods $\partial X/\partial p_x < 0$, $\partial X/\partial p_y < 0$

• Example DVD players and DVD rentals
  – $U = (\text{DVD}, \text{DISC}) = \text{DVD}^*\text{DISC}$
  – But does that quite capture the idea?
• Strict complement $X = \beta Y$
  – So by definition $\partial X/\partial p_x = \beta \partial Y/\partial p_x < 0$
  – and $\partial X/\partial p_y = \beta \partial Y/\partial p_y < 0$
Summary of Price effects

• Expect own price effect to be negative
• Expect responses to other prices to vary quite a bit
  – Some cases you buy more (because this good has become relatively cheaper)
  – Some cases you buy less (because you need to reallocate to afford the complement).
• Figuring these characteristics of demand is critical for businesses and government
• The other argument from the budget set is income.
Income effect

• Normal goods $\frac{\partial X}{\partial M} > 0$,
• Inferior goods $\frac{\partial X}{\partial M} < 0$,
• Standard empirical case
  – Up to some income $M \frac{\partial X}{\partial M} > 0$, then past $M \frac{\partial X}{\partial M} < 0$, as you switch to higher quality goods.
Demand for calories as a function of income

Trevon Logan JEH 2007

These are annual per person calories:
13 = 442,000 calories a year (1212 year)
Range 12 = 572/day
14 = 3300/day
From Logan JEH 2009,
Note that this elasticity is declining over time. So there is both cross sectional and time variation

<table>
<thead>
<tr>
<th>Expenditure Elasticity of Calories</th>
<th>Income Elasticity of Calories</th>
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<tbody>
<tr>
<td><strong>Nation</strong></td>
<td><strong>Method</strong></td>
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</tbody>
</table>
Energy consumption and Birth rates over time
Income Effect

- *Engel curve* traces consumption as income rises
Inferior Good
Intertemporal Choice

• First two slides from last time
• \( u(x_1, x_2) = v(x_1) + \delta v(x_2) \)
• Budget \( (1+r)x_1 + x_2 = (1+r)M_1 + M_2 \)

• FOC \( 0 = v'(x_1) - (1 + r)\delta v'(x_2) \)
In this case you are poor when young rich when old
Interest Rate Increase

Borrower’s Income Falls

\((M_1, M_2)\)
Interest Rate Increase

Creditor’s Income Rises

$$(M_1, M_2)$$
Neither Borrower Nor Lender Be
Differences in Borrowing/Lending Rates

$(M_1, M_2)$
100% propensity to consume

\[(M_1, M_2)\]
Conclusion From Week 2

• Demand framework useful for a variety of purposes
• We did the basics
  – No asymmetric information (you always know everything about the market and the goods)
  – No risk (you are sure what you want and what you get)
  – Continuity
  – NO TRANSACTION COSTS
• More of this in most Econ classes
  – But EC121ab, EC 106 come back heavily to those topics