Consumer Theory

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Factory Food,
HANNAH FAIRFIELD
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• **Budgets**
  - Budgets without prices
  - Budgets with prices

• **Preferences**
  - Standard preferences
  - Some variants

• **Optimal choice**
  - Constrained optimization
Robinson Crusoe goes to Caltech

Study hours

Budget Line

Budget Set

Hours on Everything else

Everything Else

24

24
• The hours budget
  – Every point in the budget set is feasible
  – Points on the budget line represent allocations without un-used resources
  – Slope of the budget line is -1. (to get one hour of study you have to give up one hour of something else)

• Few people live in Robinson Crusoe-Hours worlds.
  – Hunting vs gathering
    • It takes more time to get a given amount of calories from hunting than it does from gathering (but maybe it tastes better)
  – Prices
    • Things have different prices
Budget Set

\[ p_X x + p_Y y \leq M. \]

M is income
X is one good
Y is the other
Preferences

• To decide what to do with your budget you have to have preferences.

• Complete, reflexive, and transitive

• Continuous, Convex, Monotonic/Locally non-satiation.
Utility

Better than A (Monotonicity)

A=(x,y)

Set of point individual likes just as much as A=Iso-Utility line
Isoquants
Isoquants

• Isoquants are contour sets of the utility function

• Convex preferences means if consumer indifferent between two points, prefers points on the line segment connecting them.
Convex Preferences
Optimal choice

![Diagram showing optimal choice with budget lines and shaded area]
Choice

• *Income is* $M$, *price of* $x=p_x$, *price of* $y=p_y$.

• *Optimal choice*

• *Max* $u(x,y)$ *subject to* $xp_x + yp_y \leq M$

• *Solution?*

• *Substitution*

  $$y = (M-xp_x)p_y$$

• *Lagrange multiplier method*

  $$\text{Max } L(x,y,\lambda) = u(x,y) + \lambda(M-xp_x - yp_y)$$
Two good Maximization by substitution

• Max \( u(x,y) \) s.t. \( \rho_x x + \rho_y y \leq M \)

• Max \( u\left(x, \frac{M - \rho_x x}{\rho_y}\right) \).

\[
0 = \frac{d}{dx} u\left(x, \frac{M - \rho_x x}{\rho_y}\right) = \frac{\partial u}{\partial x} - \frac{\rho_x}{\rho_y} \frac{\partial u}{\partial y}.
\]
First Order Condition

\[ 0 = \frac{d}{dx} u \left( x, \frac{M - p_x x}{p_y} \right) = \frac{\partial u}{\partial x} - \frac{p_x}{p_y} \frac{\partial u}{\partial y}. \]

\[ \frac{p_x}{p_y} = \frac{\partial u / \partial x}{\partial u / \partial y} = \frac{dy}{dx} \bigg|_{u=u_0} = MRS \]

Slope of the budget line = slope of the isoquant
Optimal choice
Second Order Condition

• For future reference

\[ 0 \geq \frac{d^2}{(dx)^2} u \left( x, \frac{M - p_x x}{p_y} \right) = \frac{\partial^2 u}{(\partial x)^2} - 2 \frac{p_x}{p_y} \frac{\partial^2 u}{\partial x \partial y} + \left( \frac{p_x}{p_y} \right)^2 \frac{\partial^2 u}{(\partial y)^2} \]
Lagrange multiplier method

- \( \text{Max } L(x, y, \lambda) = u(x, y) + \lambda(M - x p_x - y p_y) \)

\[
\frac{dL}{dx} = \frac{dU}{dx} - \lambda p_x = 0
\]

\[
\frac{dL}{dy} = \frac{dU}{dy} - \lambda p_y = 0
\]

\[
\frac{dL}{d\lambda} = M - x p_x - y p_y = 0
\]

If you take the ratio of the first two you get back that the ratio of prices is equal to the ratio of marginal utilities.

The third equation is just the constraint.

\( \lambda \) has an interpretation: it’s the marginal utility of income.
Notation

\[(u_1, u_2) = \left( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right)\]

• This is the gradient, direction of steepest ascent of \(u\)
• FOC entails gradient perpendicular to budget line
Cobb-Douglas Example

\[ u(x, y) = x^\alpha y^{1-\alpha} \]

\[
\frac{p_x}{p_y} = \frac{-\frac{dy}{dx}}{\alpha y^\alpha y^{1-\alpha}} = \frac{\partial u/\partial x}{\partial u/\partial y}.
\]

\[
\frac{p_x}{p_y} = \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1 - \alpha) x^\alpha y^{-\alpha}}
\]

\[
\frac{P_x}{P_y} = \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1 - \alpha) x^\alpha y^{-\alpha}}
\]

\[
(1 - \alpha) x P_x = \alpha y P_y = \alpha (M - x P_x)
\]

\[ x = \frac{\alpha M}{p_x}, \quad y = \frac{(1 - \alpha) M}{p_y} \]

• Constant income shares
Substitution Effect

- Quantity may rise when price goes up
- But rare (could be a fashion effect)
Substitution

• Price increase represents a decrease in purchasing power plus a change in relative price
• Substitution and income effects separate these two effects
Substitution Effect Holds Utility Constant

\[ y \]

\[ \text{Initial Choice} \]

\[ x \]

\[ \text{Compensated Choice} \]

\[ p_Y \uparrow \]
Income Effect

- *Engel curve* traces consumption as income rises
Inferior Good
Decomposition into Substitution and Income Effects
Demand and Compensated demand

• Let the demand for good x, given price p and income y be $X(p,y)$,

• Define compensated demand to be the function $H(p,U)$. Such that given price, the optimal consumption of x to reach utility U, is $x^*$.

• Define $e(p,u)$ to be the expenditure function (the minimum income to reach a given level of utility).

• Now at $x^*$ it has to be $H(p,U^*)= X(p, e(p,U^*))= x^*$.

• Differentiate with respect to p.

$$\frac{\partial H}{\partial p} = \frac{\partial X}{\partial P} + \frac{\partial X}{\partial M} \frac{\partial e}{\partial p}$$

$$\frac{\partial X}{\partial p} = \frac{\partial H}{\partial p} - \frac{\partial X}{\partial M} x^*$$
Practical value of Slutzky’s equation

\[
\frac{\partial X}{\partial P} = \frac{\partial H}{\partial P} - \frac{\partial X}{\partial M} x^*
\]

• You can’t measure
• So figuring out how to compensate people say for a sudden shock to heating oil...may be difficult. Except for Slutzky’s equation.
• You can observe \( x^* \) you can measure the total effect \( \frac{\partial X}{\partial P} \) and the income effect \( \frac{\partial X}{\partial M} \)
Labor Supply

Increase in wage

• Increases income
  – Increasing leisure, reducing hours

• Increases price of leisure
  – Decreasing leisure

• Labor supplied may (should) decrease as wages rise
U.S. Hours Worked Have Fallen

True even between 1970 and 1990 when real wages fell, income grew (two earner households)
Intertemporal Choice

- $u(x_1, x_2) = v(x_1) + \delta v(x_2)$
- Budget $(1+r)x_1 + x_2 = (1+r)M_1 + M_2$
- FOC $0 = v'(x_1) - (1+r)\delta v'(x_2)$
Intertemporal Optimization

$\text{Repayment} \quad \left(M_1, M_2\right) \quad \text{Period 1 Borrowing}$