Homework Policy Goods for all

Study You can study the homework on your own or with a group of fellow students. You should feel free to consult notes, text books and so forth.

The quiz will be available Wednesday at 5pm. Following the Honor code, you should find 20 minutes and do the quiz, by yourself and without using any notes. Paper and pen should be all you need. Then turn it in by Thursday 5pm. (drop off in box in front of Baxter 133). It will include one question from each section

The answers to the whole homework will be available Friday at 2pm.

Definitions
Please explain each term in three lines or less

- Certainty equivalent
  The certainty equivalent is an amount of money that provides equal utility to the random payoff of the gamble.

- Risk premium
  The *risk premium* is defined to be the difference between the expected payoff and the certainty equivalent.

- Exogenous risk
  Sets of events to which we can assign probabilities

- Asymmetric information
  What one party to a transaction knows is different from the other

- Free insurance
  The insurance company pools the risk and eliminates it in aggregate (if the number of buyers are large, risks are independent and industry is competitive then insurance is close to free)

- Diversifiable risk
In order to distribute the risk, agent may want to invest some part \((\alpha \in (0,1))\) of his wealth to some asset, the rest to another, etc

- **Risk aversion**

  Risk aversion is the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain with more certain, but possibly lower, expected payoff

- **Concave function**

  \[ u(\alpha x + (1 - \alpha)y) > \alpha u(x) + (1 - \alpha)u(y) \]

- **Adverse selection**

  It refers to a market process in which "bad" results occur when buyers and sellers access to different information: the "bad" products or customers are more likely to be selected.

**Word problems**

Please explain each question in a few sentences.

- Draw the risk-averse and risk-loving people’s utility functions.

Utility function of risk averse person is concave, i.e.,

\[ u(px_i + (1-p)x_h) > pu_i + (1-p)u(x_h) \]
Utility function of risk averse person is convex, i.e.,
\[ u(px_t + (1-p)x_h) < pu(x_t) + (1-p)u(x_h) \]

- Suppose you have to pay $2 to play a gamble. You earn $19 with probability 1/3 and nothing with probability 2/3. Your current wealth is $10 and your utility function is \( u(x) = \log(x) \). What is the certainty equivalence of this gamble? What is risk premium? Should you play the gamble or not?

Expected utility:
\[ \frac{1}{3} u(19 - 2 + 10) + \frac{2}{3} u(10 - 2) = \frac{1}{3} \log(27) + \frac{2}{3} \log(8) \]
\[ = \log(3) + \log(4) = \log(12) = u(CE) \]
Since certainty equivalence of the lottery is 12 > 10, you should play the gamble.
Risk premium is \( \frac{1}{3} (19 - 2 + 10) + \frac{2}{3} (10 - 2) - CE = 14 \cdot \frac{1}{3} - 12 = 2 \left( \frac{1}{3} \right) \)

- True or false. The more risk averse person is the higher she would like to pay for the insurance.
True. The higher risk aversion a person has the higher she can pay for the insurance.

Technical problems

1. There is an agent with utility function \( u(w) = w^{0.5} \) where \( w \) is agent’s current wealth. There is a 5% chance of burglary with a loss of $1000. Suppose the initial wealth is $2000.

   - What is this person’s expected utility?
     \[ U(\text{no insurance}) = 0.05 \cdot u(1000) + 0.95 \cdot u(2000) = 44.06 \]
   - If insurance is possible, what is fair premium amount? What is his utility when he buys insurance, is he better off?
     Expected loss=0.05*1000=50=fair amount
     Utility (with insurance)=U(2000-50)=44.16
     Yes he is better off with insurance, 44.16>44.06.
   - Now, suppose that insurance company wants to make profit, what is the maximum premium is this person willing to pay to insurance company?
     \[ U(\text{no insurance}) = 44.06 = U(2000-m) = \sqrt{2000 - m} \]
     \[ m = 58.15 \]
     This is higher than the fair amount.
   - Suppose now the agent’s utility function is \( u = w^{0.8} \). How does maximum willingness to pay for the insurance change for the agent? Is it higher or lower? Interpret.
     Then expected utility when he is not insured
     \[ = 0.05 \cdot 1000^{0.8} + 0.95 \cdot 2000^{0.8} = 428.04 \]
Insurance company will maximize its utility when $428.04 = U(2000-m)\]
$m=53.05$
This is lower because this individual is less risk-averse comparison to the
person with utility function $u = w^{0.5}$.

2. Suppose there are two types of cars, good or bad with a market share $p$ and $1-p$ in the
market. Sellers won’t sell the good cars lower than $1800$ and bad cars lower than $800$. Analogously, risk neutral buyers won’t pay higher than $2000$ for the god cars and $1000$ for bad cars.

- If the quality of the cars were observed, what would be the trading price
for good and bad cars?
  Seller would sell from the highest price that the buyers accept to buy,
which is $2000$ for the good cars and $1000$ for the bad cars
- If the quality is unobserved what would be the price buyers would pay?
  Buyers expected value for a car (when the quality is unobserved)
  $p \times 2000 + (1-p) \times 1000 = 1000p + 1000$
- What should be the share of good cars in order to sellers of good cars are
willing to sell their cars?
  Good car sellers would accept to sell if
  $1000p + 1000 > 1800$
  Hence good car seller will sell only if $p > 0.8$.

- Draw the market price/good(bad) car percentage in a graph?
  Below $p=0.8$ the market price constant and equal to $1000$, the price of bad cars, if
the share is higher than 0.8, then the market price can be in $(1800,2000)$