Economics 11 Caltech Spring 2010

Problem Set 5

Homework Policy Goods for all

Study You can study the homework on your own or with a group of fellow students. You should feel free to consult notes, text books and so forth.

The quiz will be available Wednesday at 5pm. Following the Honor code, you should find 20 minutes and do the quiz, by yourself and without using any notes. Paper and pen should be all you need. Then turn it in by Thursday 5pm. (drop off in box in front of Baxter 133). It will include one question from each section

The answers to the whole homework will be available Friday at 2pm.

Definitions
Please explain each term in three lines or less

- Variable cost:
  Costs of production that vary with the quantity of production.

- Diseconomies of scale
  A situation in which long run average total cost increases as the output of firm increases.

- Free entry exit
  Movement of firms into and out of an industry that is not blocked by regulation, other firms, or any other barriers.

- Long run equilibrium
  A situation in which entry into and exit from and industry are complete and economic profits are zero, with price equals to average total cost.

- Long run competitive equilibrium model
  A model of firms in an industry in which free entry and exit produce and equilibrium such that price equals the minimum of average total cost.

- External economies of scale
  A situation in which growth in an industry causes average total cost for the individual firm; it corresponds to a downward-sloping long-run industry supply curve.

- Long run total cost
  The curve that traces out the short-run average cost curves, showing the lowest average total cost for each quantity produced as the firm expands in the long run.

- Shutdown point
  When P=min AVC it is better for a firm not to produce anymore.
Word problems

Please explain each question in a few sentences.

- Draw typical ATC, AVC, and MC curves for a profit maximizing, price taking firm. Show the case where price equals to average total cost and the rectangles that represent fixed costs and variable costs. What happens to the size of these areas as the market price increases? Show this in your diagram.

- If Price<ATC, firm should shut down. True or false? False, if this is the case firm is not making profit, but in order to shut down it needs to wait for P<AVC.

- Consider a cost function of producing an output q of the form c(q) = q² + 2q + 64.
  Determine:
  a. Marginal cost
     \[
     \frac{dc(q)}{dq} = 2q + 2
     \]
  b. Average cost
     \[
     AC = \frac{C(Q)}{Q} = Q + 2 + 64/Q
     \]
  c. Average variable cost
     \[
     VC = Q + 2
     \]
  d. Assume this product is produced only by that firm. What is the long run equilibrium price? Average cost is minimized when q=8. Price should be equal to MC in profit maximizing firm, hence p=18.

- Suppose corn farmers in the US can be represented by a competitive industry with no economies or diseconomies of scale. Describe how this industry would adjust to an increase in demand for corn. Explain your answer graphically; showing the cost curves for the typical farmer as well as the market supply and demand curves for short run and long run.
In the short run, an increase in demand for corn will increase profit. Hence, supply will shift to the right. New firms enter the industry. In the long run, since profit is positive, the supply curve will shift as well, and the price will fall.

In the long run, an increase in demand for corn will increase the price. New firms enter the business, increasing supply. The market price will fall as more firms enter, increasing supply and decreasing price. The price will fall to the level where \( \text{min} \text{ ATC} = p \) (none of the firms are making positive profit).
Technical problems

1. Each firm in an industry has the long run cost function

$$TC(y) = \begin{cases} 0 & \text{if } y = 0 \\ 1000 + 10y^2 & \text{if } y > 0. \end{cases}$$

The aggregate demand function for the output of the firms is $Q = 5000 - 2p$. Find the long run equilibrium price, number of firms, and output of each firm.

$$ATC = \frac{TC}{y} = \frac{1000}{y} + 10y$$

Minimum of ATC will give us the equilibrium price and quantity, $y$ that needs to be produced at this price by a firm.

$$\frac{d(\text{ATC})}{dy} = -\frac{1000}{y^2} + 10 = 0$$

$$y = 10$$

The price of the good $y$ needs to be $p=\min \text{ATC}=1000/10+10\times10=200$
It is given that aggregate demand is $Q = 5000 - 2p=5000-400=4600$
Hence if the quantity level of each firm producing is 10, there needs to be $4600/10=460$ firms to supply that amount of aggregate demand.

2. The market for xylophones is perfectly competitive, and all firms have the production function,

$$x = K^{1/3}L^{2/3},$$

where $x$ is the quantity of xylophones produced, $K$ is the capital input, and $L$ is the labor input.
Cost of labor, $w=4$ and cost of capital, $r=16$. The market demand curve for xylophones is given by

$$X_d = 2500-5(p_x)^2;$$

(a) If a firm’s capital stock is fixed at one unit in the short run, $K = 1$, find the equation for the firm’s short-run supply curve.
(b) Suppose that, in the short run, there are 720 firms, each with one unit of capital, $K = 1$. Calculate the short-run equilibrium values of $p_x$ and $x$.
(c) In the long run, firms can choose the values of both $K$ and $L$, and can freely enter or exit the industry. Calculate the long-run equilibrium values of $p_x$ and $x$.

a) Fixed capital, $K=1$

Short run production function is $x = L^{2/3}$. In other words, $L=x^{3/2}$. So short run total cost function will be $4L+16K=4x^{3/2}+16$. Marginal cost for that total cost function will give the supply function equation in the short run. i.e., short run supply function is:

$$p_x = MC = 6x^{1/2} \implies x = (p_x)^2/36$$
b) 720 firms, each producing \( x = (p_x)^2 / 36 \), and supply=demand

\[
\frac{720 \times (p_x)^2}{36} = 2500 - 5(p_x)^2
\]

\( p_x = 10 \) and \( x = 2000 \).

c) In the long run, a firm can choose \( K \) and \( L \) to minimize its cost.

Write \( K \) in terms of \( L \) and \( x \), i.e. \( x = \frac{1}{3} L^\frac{2}{3}, K = \frac{q^3}{L^2} \) for some constant \( q \).

Total cost = \( 4L + 16\frac{q^3}{L^2} \)

\( ATC = \frac{4L + 16\frac{q^3}{L^2}}{q} \)

we want to find minimum of this function, which will give us long run equilibrium price. I.e.,

\[
\frac{d(AC)}{dL} = \frac{4}{q} - \frac{32q^2}{L^3} = 0
\]

\( L = 2q \), and \( K = \frac{q^3}{L^2} = q/4 \). If we write those to total cost function we obtain.

\( 4L + 16K = 4 \times 2q + 16 \times q/4 = 12q \), hence average total cost will be a constant \( 12 = p \).

Equilibrium output = \( 2500 - 5(p_x)^2 = 2500 - 5 \times 144 = 1780 \)