Homework Policy Goods for all

Study You can study the homework on your own or with a group of fellow students. You should feel free to consult notes, text books and so forth.

The quiz will be available Wednesday at 5pm. Following the Honor code, you should find 20 minutes and do the quiz, by yourself and without using any notes. Paper and pen should be all you need. Then turn it in by Thursday 5pm. (drop off in box in front of Baxter 133). It will include one question from each section

The answers to the whole homework will be available Friday at 2pm.

Definitions
Please explain each term in three lines or less

- Consumer theory
  Sol: Consumer theory is based on what people like and it tries to explain how people make choices.
- Utility
  Sol: Utility refers to the flow of pleasure or happiness that a person enjoys derived as a consequence of a choice.
- Feasible Set
  Sol: For given prices and income, the feasible set is given for the set of bundles that are affordable for a consumer.
- Isoquants
  Sol: It is the utility contour, and it means equal quantity.
- Marginal rate of substitution.
  Sol: Reflects the tradeoff, from the consumer’s perspective, between the goods.
- Substitution effect.
Sol: The substitution effect considers the change in the relative price, with a sufficient change in income to keep the consumer on the same utility.

- Income effect.
  Sol: Is the change in consumption when the real income changes.

- Convex preferences.
  Sol: Mean that a consumer prefers a mix to any two equally valuable extremes

**Word problems**

True or False: Please explain each question in a few sentences.

- If pork chops and mashed potatoes are the only goods consumed and they are perfect complements, then neither can be an inferior good.

  Sol: False. When the income of a consumer increases, then he can increase the consumption either of pork shops or mashed potatoes. Since both goods are complement of each other, then an increase in income must increase the demand of both goods.

- When the consumer maximizes his utility over two goods, the marginal utilities of each good are always equal.

  Sol: False. We know that the optimality condition is given by:

  \[
  \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \Leftrightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}
  \]

  Thus, if \( P_x = P_y \), we get \( MU_x = MU_y \). But in general \( P_x \neq P_y \) and the marginal utilities differs for the goods (at the optimum)
• In the first week of July, Bob’s favorite store charges $2.50 for an ice-cream and $1.50 for a soda. Bob spends all his income on ice-cream and soda and chooses to consume 6 ice-creams and 10 sodas. By the first week of August, prices have risen to $3 per ice-cream and $2 per soda. Bob’s income has also changed. He still only consumes ice-cream and soda, but he now buys 4 ice-creams and 20 sodas. Is Bob’s behavior in the two weeks consistent with valid indifference curves? Can you determine in which week Bob is better off? Explain.

Sol: Not consistent. The price of ice cream has increased by 20% (from 2.5 to $3). The price of soda by 33% (from 1.5 to $2). It seems that Bob is decreasing his consumption of the relatively cheaper good and increasing his consumption of the relatively more expensive. Because Bob consumes less ice cream and more soda, the ratio of the marginal utility of ice cream to the marginal utility of soda must have increased. But the price ratio is going the other way.

Which week is he better off? Revealed preference
First week of July: Prices $P_i = 2.5$, $P_s = 1.5$
Bob’s income = $p_i * 6 + p_s * 10 = 30$
First week of August: Prices $P_i = 3$, $P_s = 2$
Bob’s income = $p_i * 4 + p_s * 20 = 52$
The bundle that Bob chooses in July is also affordable in August. If Bob chose the second in August it must be better than the July choice.

Technical problems

1. Anna spends all of her income on shirts and jeans $J$. Anna’s preferences can be represented by the utility function $U(S, J) = 3\sqrt{SJ}$.

   - Derive the demand functions for shirts and jeans in terms of the price of shirts $P_s$ the price of jeans $P_j$, and income $I$.
   
   Sol: We have to solve the following program
   $$\max \left\{ 3\sqrt{SJ} \quad \text{st} \quad P_s S + P_j J \leq I \right\}$$

   The Lagrangian for this problem is:
   $$L(S, J; P_s, P_j, I) = 3\sqrt{SJ} + \lambda(I - P_s S - P_j J)$$

   Solving we find $S^* = \frac{I}{2P_s}$ and $J^* = \frac{I}{2P_j}$.
• Draw the Engel curve for shirts.

Sol: The solution is straightforward from previous part: fix \( P_s \), then the Engel curve is a line with positive slope which is given by \( \frac{1}{2P_s} \).

• Suppose the price of a shirt is $4, the price of a pair of jeans is $16, and Anna has $128 income. What bundle of shirts and jeans \((S,J)\) maximizes Anna’s utility?

Sol: Using the information for prices and income we find

\[
S^* = \frac{I}{2P_s} = \frac{128}{2*4} = 16 \quad \text{and} \quad J^* = \frac{I}{2P_j} = \frac{128}{2*16} = 4
\]

Then the utility level is \( U(S^*,J^*) = U(4,16) = 3\sqrt{4*16} = 24 \).

• Suppose the price of a shirt increases to $16. What bundle of shirts and jeans will Anna demand now?

Sol: Using the new information for prices and income we find

\[
S^* = \frac{I}{2P_s} = \frac{128}{2*16} = 4 \quad \text{and} \quad J^* = \frac{I}{2P_j} = \frac{128}{2*16} = 4
\]

Then the new utility level is \( U(S^*,J^*) = U(4,4) = 3\sqrt{4*4} = 12 \).

• Devastated by the sudden price increase, Anna decides to ask her brother for money to help finance her clothing purchases. How much total income does Anna need to remain as happy as she was before the price change?

Sol: The original situation: \( S^* = 16 \).

New situation: \( S^* = 4 \).

Then she needs from her brother: \( \Delta I = 12*16 = 192 \). in order to be as happy as the original situation.

• If Anna has exactly enough income to keep her as happy as she was before the price change, what bundle of shirts and jeans will Anna demand?

Sol: this question is related to substitution effect. Using the demand function derived in the first question of this exercise we find:

\[
S^* = \frac{I}{2P_s} = 8 \quad \text{and} \quad J^* = \frac{I}{2P_j} = 8.
\]
Unfortunately, Anna’s brother refuses to give Anna any money. She is now back to the situation in part c: the price of shirts has increased from $4 to $16, and her income remains unchanged at $128. Decompose the total change in consumption of shirts and jeans purchased into a substitution effect and an income effect. In a clearly-labeled diagram with shirts on the horizontal axis, draw the income and substitution effects of the increase in the price of a shirt.

Sol: from previous part we know that (absolute values)
Substitution effect is: 8.
Income effect: 4.
Total effect= 12.

2. Consider the following a consumer with the following utility function: \( U(X, Y, Z) = \frac{1}{2}(XYZ)^2 \).

The prices for the goods are \((P_X, P_Y, P_Z)\) and the income level is \(I\).

- What is the budget set?
Sol: The budget set is given by:
\[
B(P, I) = \{(X, Y, Z): P_X X + P_Y Y + P_Z Z \leq I\}
\]

- Solve the optimization problem using Lagrange’s method.
Sol: The Lagrangian for this problem is given by:
\[
L(X, Y, Z; P, I) = \frac{1}{2}(XYZ)^2 + \lambda \{I - P_X X - P_Y Y - P_Z Z\}
\]

The FOC are:
\[
\begin{align*}
\frac{\partial L}{\partial X} &= 0 \iff X(YZ)^2 - \lambda P_X = 0, \quad (1) \\
\frac{\partial L}{\partial Y} &= 0 \iff Y(XZ)^2 - \lambda P_Y = 0, \quad (2) \\
\frac{\partial L}{\partial Z} &= 0 \iff Z(XY)^2 - \lambda P_Z = 0. \quad (3)
\end{align*}
\]
combining (1), (2), and (3) with the budget constraint we find the solutions for this problem. Thus we get the demand functions for $X,Y,Z$ as

\[
X^* = X(P, I) = \frac{I}{3P_x},
\]
\[
Y^* = Y(P, I) = \frac{I}{3P_y},
\]
\[
Z^* = Z(P, I) = \frac{I}{3P_z}.
\]

- Find the demand functions for: $X,Y,Z$.

Sol: From previous part we know that the demand functions are given by: $X^*, Y^*, Z^*$.

- Compute $\frac{\partial X}{\partial P_x}, \frac{\partial X}{\partial P_y}, \frac{\partial X}{\partial P_z}$. Explain.

Sol: Using the demand functions we get

\[
\frac{\partial X(P, I)}{\partial P_x} = -\frac{I}{3P_x^2},
\]
\[
\frac{\partial X(P, I)}{\partial P_y} = 0,
\]
\[
\frac{\partial X(P, I)}{\partial P_z} = 0.
\]

This means that the demand for good $X$ depends only in its own price. The goods $Y, Z$ are not complements or substitutes.

- Compute $\frac{\partial X}{\partial I}, \frac{\partial Y}{\partial I}, \frac{\partial Z}{\partial I}$. Explain.

Sol: we get,
\[
\frac{\partial X(P, I)}{\partial I} = \frac{1}{3P_x},
\]
\[
\frac{\partial Y(P, I)}{\partial I} = \frac{1}{3P_y},
\]
\[
\frac{\partial Z(P, I)}{\partial I} = \frac{1}{3P_z},
\]

Thus the three goods are normal ones.