Chapter 3

Fixed Income Securities

Road Map

Part A Introduction to finance.

Part B Valuation of assets, given discount rates.

- Fixed-income securities.
- Stocks.
- Real assets (capital budgeting).

Part C Determination of risk-adjusted discount rates.

Part D Introduction to derivatives.

Main Issues

- Fixed-Income Markets
- Term Structure of Interest Rates
- Interest Rate Risk
- Inflation Risk
- Credit Risk

1 Fixed-Income Markets

Definition: Fixed-income securities are financial claims with promised cash flows of fixed amount paid at fixed dates.

Classification of Fixed-Income Securities:

1. Treasury Securities:
   - U.S. Treasury securities (bills, notes, bonds).
   - Bunds, JGBs, U.K. Gilts ...
2. Federal Agency Securities:
   - Securities issued by federal agencies (FHLB, FNMA ...).
3. Corporate Securities:
   - Commercial paper.
   - Medium-term notes (MTNs).
   - Corporate bonds ...
4. Municipal Securities.
5. Mortgage-Backed Securities.
6. ...

Overview of Fixed-income Markets

Composition of U.S. Debt Markets (2005)

	Market value (in trillion dollars)	%
Treasury	4.17	16.5
Corporate	4.99	19.7
Mortgage	5.92	23.4
Agency	2.60	10.3
Munies	2.23	8.8
Asset-Backed	1.96	7.7
Money Market	3.47	13.7
Total	25.33	

Current Trends

	T.	Corp.	MBS	Agency	ABS	Munies	MM	Total
1995	3.31	1.94	2.35	0.84	0.32	1.29	1.18	11.23
1996	3.44	2.12	2.49	0.93	0.40	1.30	1.39	12.01
1997	3.44	2.36	2.68	1.02	0.54	1.32	1.69	13.05
1998	3.34	2.71	2.96	1.30	0.73	1.40	1.98	14.42
1999	3.27	3.05	3.33	1.62	0.90	1.46	2.34	15.96
2000	2.95	3.36	3.56	1.85	1.07	1.48	2.66	16.95
2001	2.97	3.84	4.13	2.15	1.28	1.60	2.57	18.53
2002	3.20	4.09	4.70	2.29	1.54	1.73	2.55	20.15
2003	3.57	4.46	5.31	2.64	1.69	1.89	2.53	22.10
2004	3.94	3.70	5.47	2.75	1.83	2.02	2.87	23.58
2005	4.17	4.99	4.92	2.60	1.96	2.23	3.47	25.33

Organization of Fixed Income Market

ISSUERS:

1. Government
2. Corporations
3. Commercial banks
4. States and municipalities
5. Special purpose vehicles
6. Foreign institutions

INTERMEDIARIES:

1. Primary dealers
2. Other dealers
3. Investment banks
4. Credit rating agencies
5. Credit and liquidity enhancers

INVESTORS:

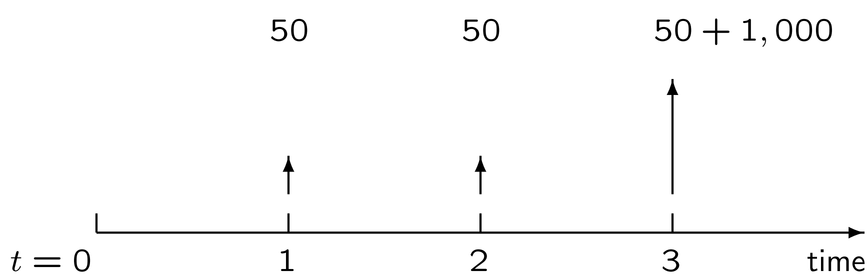
1. Governments
2. Pension funds
3. Insurance companies
4. Commercial banks
5. Mutual funds
6. Foreign institutions
7. Individual investors

Cash Flow and Valuation of Fixed-Income Securities

Cash flow:

1. Maturity
2. Principal
3. Coupon.

Example. A 3-year bond with principal of $1,000 and annual coupon payment of 5% has the following cash flow:



Valuation:

1. Time-value.
   - Interest rates.
2. Risks:
   - Inflation.
   - Credit.
   - Timing (callability).
   - Liquidity.
   - Currency . . .

2 Term Structure of Interest Rates

Our objective here is to value riskless cash flows.

Given the rich set of fixed-income securities traded in the market, their prices provide the information needed to value riskless cash flows at hand.

In the market, this information on the time value of money is given in several different forms:

1. Spot interest rates.
2. Prices of discount bonds (e.g., zero-coupon bonds and STRIPS).
3. Prices of coupon bonds.
4. Forward interest rates.

2.1 Spot Interest Rates

Definition: Spot interest rate, r_t, is the (annualized) interest rate for maturity date t.

- r_t is for payments only on date t.
- r_t is the "average" rate of interest between now and date t.
- r_t is different for each different date t.

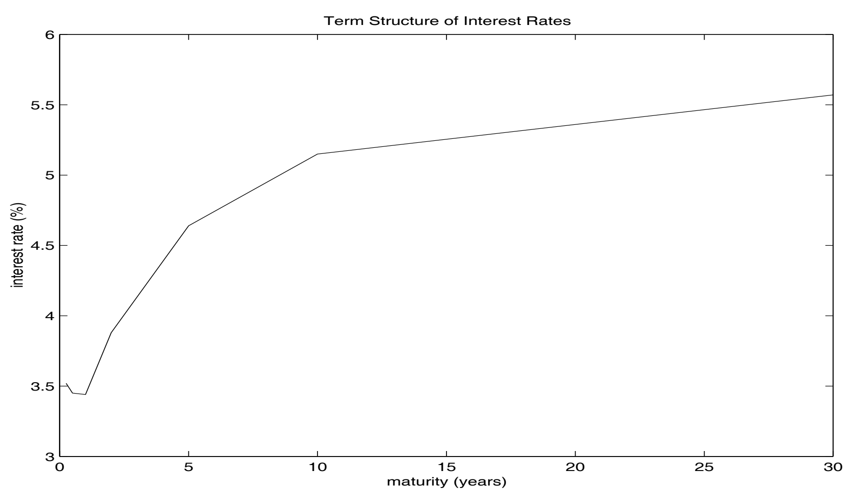
Example. On 2001.08.01., the spot interest rates are:

Maturity (year)	1/4	1/2	1	2	5	10	30
Interest Rate (%)	3.52	3.45	3.44	3.88	4.64	5.15	5.57

Definition: The set of spot interest rates for different maturities

$$\{r_1, r_2, \ldots, r_t, \ldots\}$$

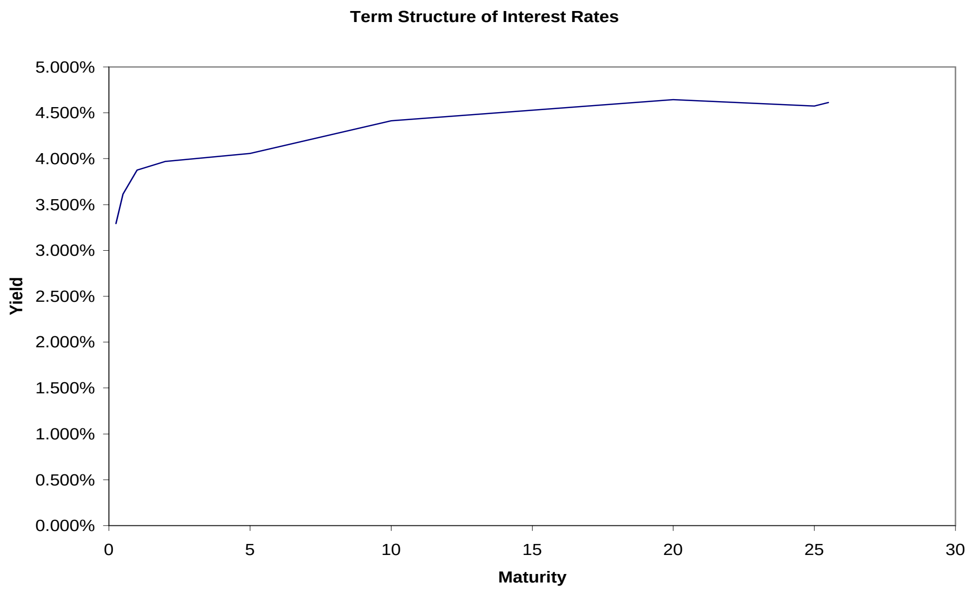
gives the term structure of (spot) interest rates, which refers to the relation between spot rates and their maturities.



2001.08.01 (WSJ)

Term structure of interest rates on 2005.08.01

Maturity (year)	1/4	1/2	1	2	5	10	20	25	25.5	(longest)
Interest Rate (%)	3.29	3.61	3.87	3.97	4.06	4.41	4.65	4.57	4.61	



2.2 Discount Bonds

Discount bonds (zero coupon bonds) are the simplest fixed-income securities.

<u>Definition</u>: A discount bond with maturity date t is a bond which pays \$1 only at t.

Example. STRIPS are traded at the following prices:

Maturity (year)	1/4	1/2	1	2	5	10	30
Price	0.991	0.983	0.967	0.927	0.797	0.605	0.187

For the 5-year STRIPS, we have

$$0.797 = \frac{1}{(1+r_5)^5} \quad \Rightarrow \quad r_5 = \frac{1}{(0.797)^{1/5}} - 1 = 4.64\%.$$

Let B_t denote the current price (at date 0) of a discount bond maturing at t. Then

$$B_t = \frac{1}{(1+r_t)^t} \quad \text{or} \quad r_t = \frac{1}{B_t^{1/t}} - 1.$$

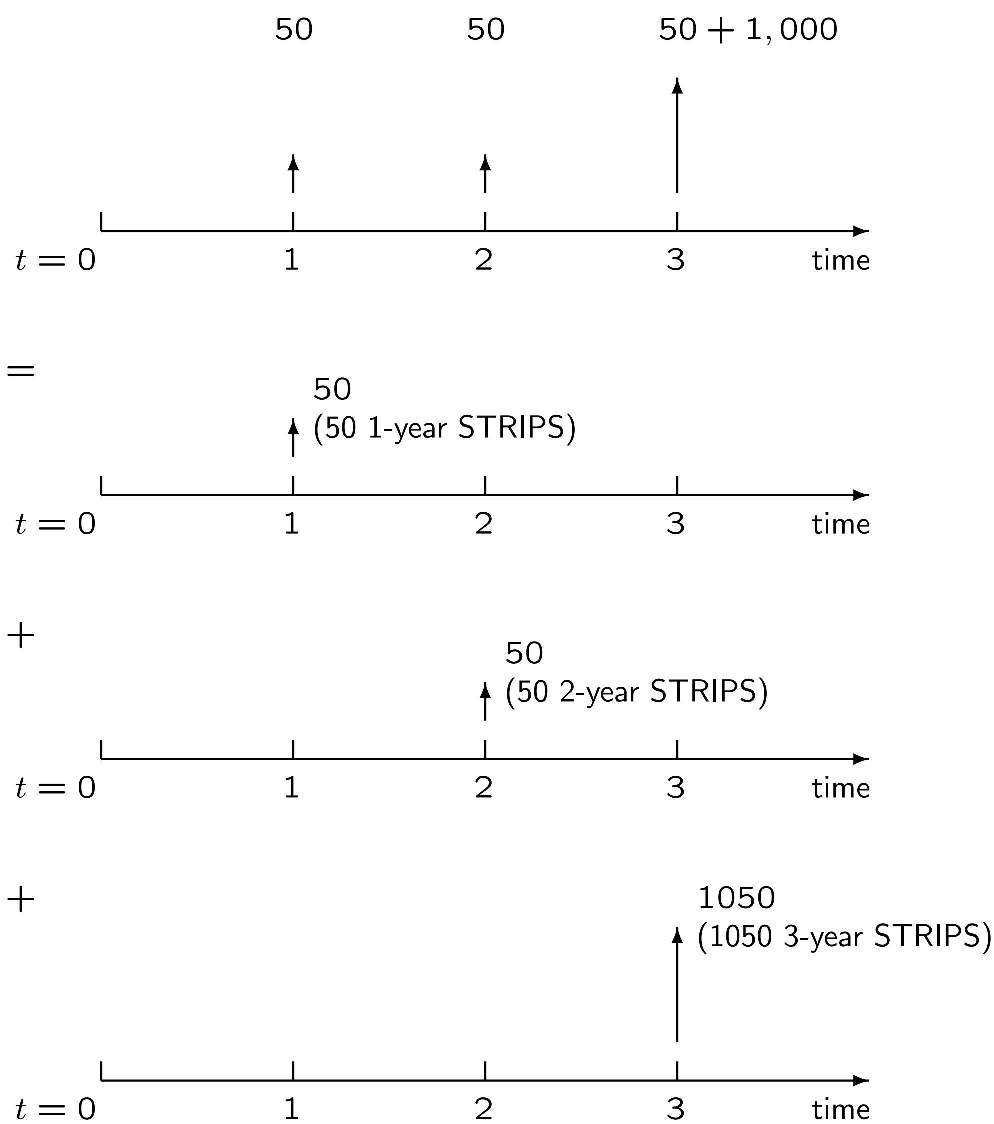
- ► Prices of discount bonds provide information about spot interest rates and vise versa.

2.3 Coupon Bonds

A coupon bond pays a stream of regular coupon payments and a principal at maturity.

Observation: A coupon bond is a portfolio of discount bonds.

Example. A 3-year bond of $1,000 par and 5% annual coupon.



Suppose that the discount bond prices are as follows

t	1	2	3	4	5
B_t	0.952	0.898	0.863	0.807	0.757

What should the price of the coupon bond be?

$$\begin{aligned}\text{Price} &= (50)(0.952) + (50)(0.898) + (1050)(0.863) \\ &= 998.65.\end{aligned}$$

What if not?

Thus, a bond with coupon payments $\{C_1, C_2, \ldots, C_T\}$ and a principal P at maturity T is composed of

- C_t units of discount bonds maturing at t, $t = 1, \ldots, T$
- P units of discount bond maturing at T.

The price of a coupon bond must be

$$\begin{aligned}B &= \sum_{t=1}^{T} (C_t \times B_t) + (P \times B_T) \\ &= \frac{C_1}{1+r_1} + \cdots + \frac{C_{T-1}}{(1+r_{T-1})^{T-1}} + \frac{C_T + P}{(1+r_T)^T}.\end{aligned}$$

Example. Measuring term structure from coupon bond prices.

Years to Maturity	1	2
Face Value	1,000.0	1,000.0
Coupon Rate (%)	5.0	8.0
Current Price	997.5	1048.0

(a) Price of a 1-year bond:

$$\begin{aligned} 997.5 &= B_1 \times (50 + 1000) \quad \Rightarrow \\ B_1 &= \frac{997.5}{1050} = 0.95 \\ r_1 &= 5.26\%. \end{aligned}$$

(b) Price of a 2-year bond:

$$\begin{aligned} 1048 &= (80)(0.95) + B_2 \times 1080 \quad \Rightarrow \\ B_2 &= \frac{972}{1080} = 0.90 \\ r_2 &= 5.41\%. \end{aligned}$$

How to replicate 1- and 2-year discount bonds from coupon bonds?

2.4 Yield-to-Maturity (YTM)

Definition: Yield-to-maturity of a bond, denoted by y, is given by

$$B = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{P}{(1+y)^T}.$$

Given its maturity, the principle and the coupon rate, there is a one to one mapping between the price of a bond and its YTM.

In the market, it is conventional to quote bond prices in YTM.

Example. Current 1- and 2-year spot interest rates are 5% and 6%, respectively. The price of a 2-year Treasury coupon bond with par value of $100 and a coupon rate of 6% is

$$B = \frac{6}{1+0.05} + \frac{106}{(1+0.06)^2} = 100.0539.$$

Its YTM is 5.9706%:

$$100.0539 = \frac{6}{1+0.059706} + \frac{106}{(1+0.059706)^2}.$$

Note the difference between YTM definition and bond pricing formula.

2.5 Forward Interest Rates

So far, we have focused on spot interest rates: rates for a transaction between today, 0, and a future date, t.

Now, we study forward interest rates: rates for a transaction between two future dates, for instance, t_1 and t_2.

For a forward transaction to borrow money in the future:

- Terms of the transaction are agreed on today, $t = 0$.
- Loan is received on a future date t_1.
- Repayment of the loan occurs on date t_2.

Note:

► Future spot rates can be different from current corresponding forward rates.

Example. As the CFO of a U.S. multinational, you expect to repatriate \$10 M from a foreign subsidiary in 1 year, which will be used to pay dividends 1 year later. Not knowing the interest rates in 1 year, you would like to lock into a lending rate one year from now for a period of one year. What should you do?

The current interest rates are

time to maturity t (years)	1	2
spot interest rater_t	0.05	0.07

Strategy:

1. Borrow \$9.524 M now for one year at 5%
2. Invest the proceeds \$9.524 M for two years at 7%.

Outcome (in million dollars):

Year	0	1	2
1-yr borrowing	9.524	−10.000	0
2-yr lending	−9.524	0	10.904
Repatriation	0	10.000	0
Net	0	0	10.904

The locked-in 1-year lending rate 1 year from now is 9.04%, which is the *forward rate* for year 2.

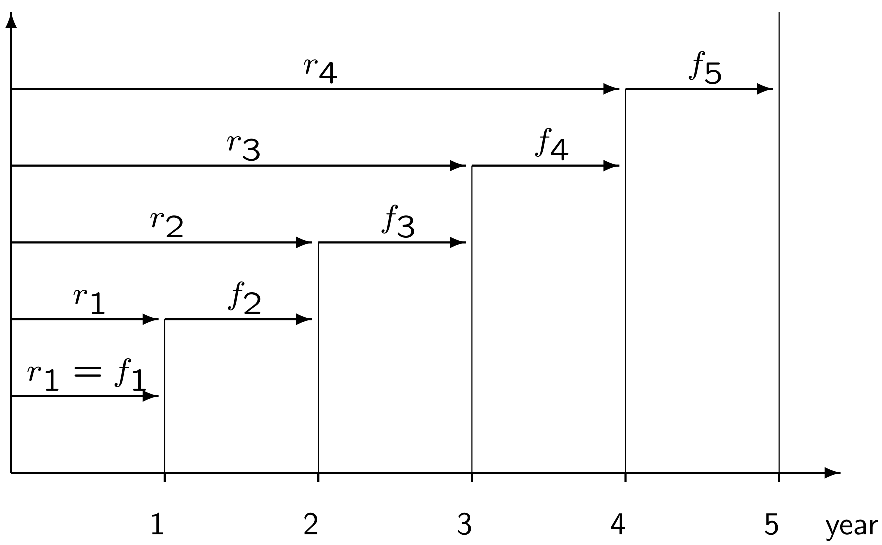
Definition: The forward interest rate between time $t-1$ and t is

$$(1+r_t)^t = (1+r_{t-1})^{t-1}(1+f_t)$$

or

$$f_t = \frac{B_{t-1}}{B_t} - 1 = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}} - 1.$$

Spot and forward rates



Example. Suppose that discount bond prices are as follows:

t	1	2	3	4
B_t	0.9524	0.8900	0.8278	0.7629
YTM_t	0.05	0.06	0.065	0.07

A customer wants a forward contract to borrow \$20 M three years from now for one year. Can you (a bank) quote a rate?

$f_4 = 8.51\%.$

What should you do today to lock-in these cash flows?

Strategy:

1. Buy 20,000,000 of 3 year discount bonds, costing

$$(20,000,000)(0.8278) = \$16,556,000.$$

2. Finance this by selling 4 year discount bonds of amount

$$16,556,000/0.7629 = \$21,701,403.$$

3. This creates a liability in year 4 in the amount \$21,701,403.

Cash flows from this strategy (in million dollars):

Year	0	1–2	3	4
Purchase of 3 year bonds	-16.556	0	20.000	0
Sale of 4 year bonds	16.556	0	0	-21.701
Total	0	0	20.000	-21.701

The yield to maturity for the "bond" is given by:

$$\frac{21,701,403}{20,000,000} - 1 = 8.51\%.$$

3 Hypothesis on Interest Rates

What determines the term structure of interest rates?

1. Expected future spot rates.
2. Risk of long bonds.

Expectations Hypothesis: Forward rates predict future spot rates, $f_t = \mathrm{E}[r_1(t)]$.

Implications:

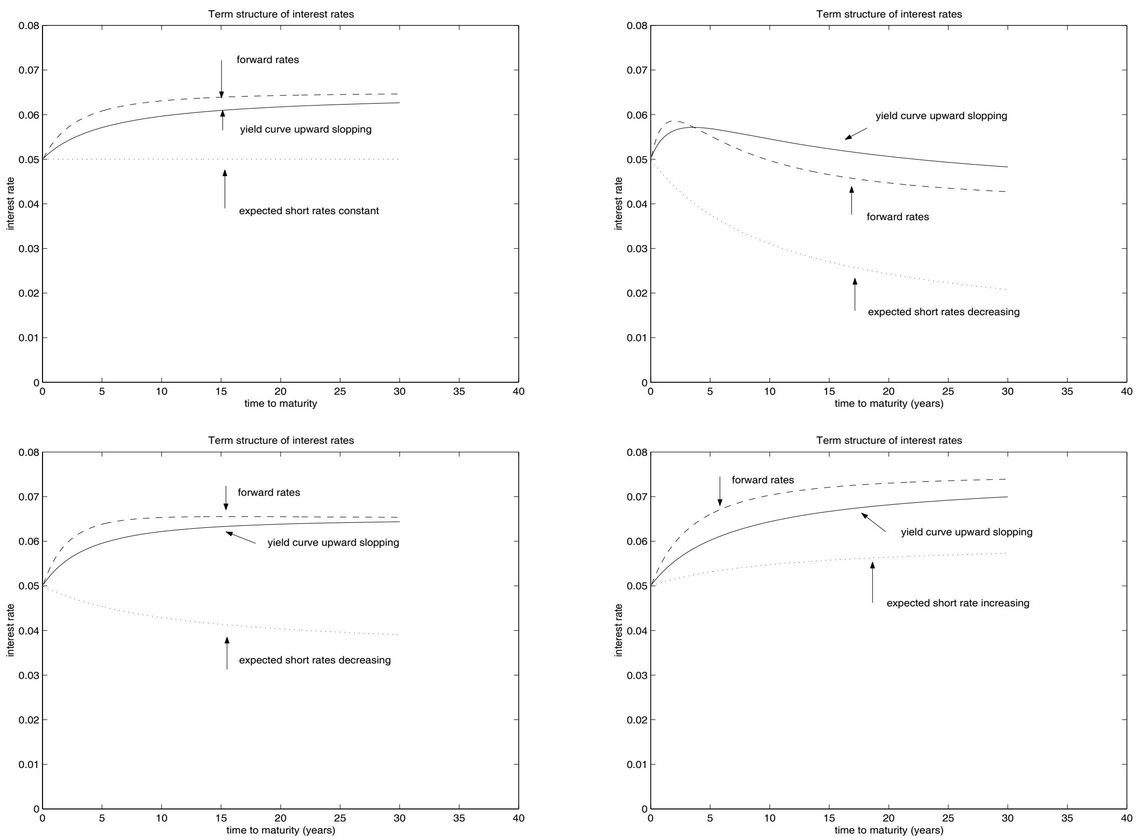
- The slope of the term structure reflects the market's expectations of future short-term interest rates.

Liquidity Preference Hypothesis: Investors regard long bonds as riskier than short bonds.

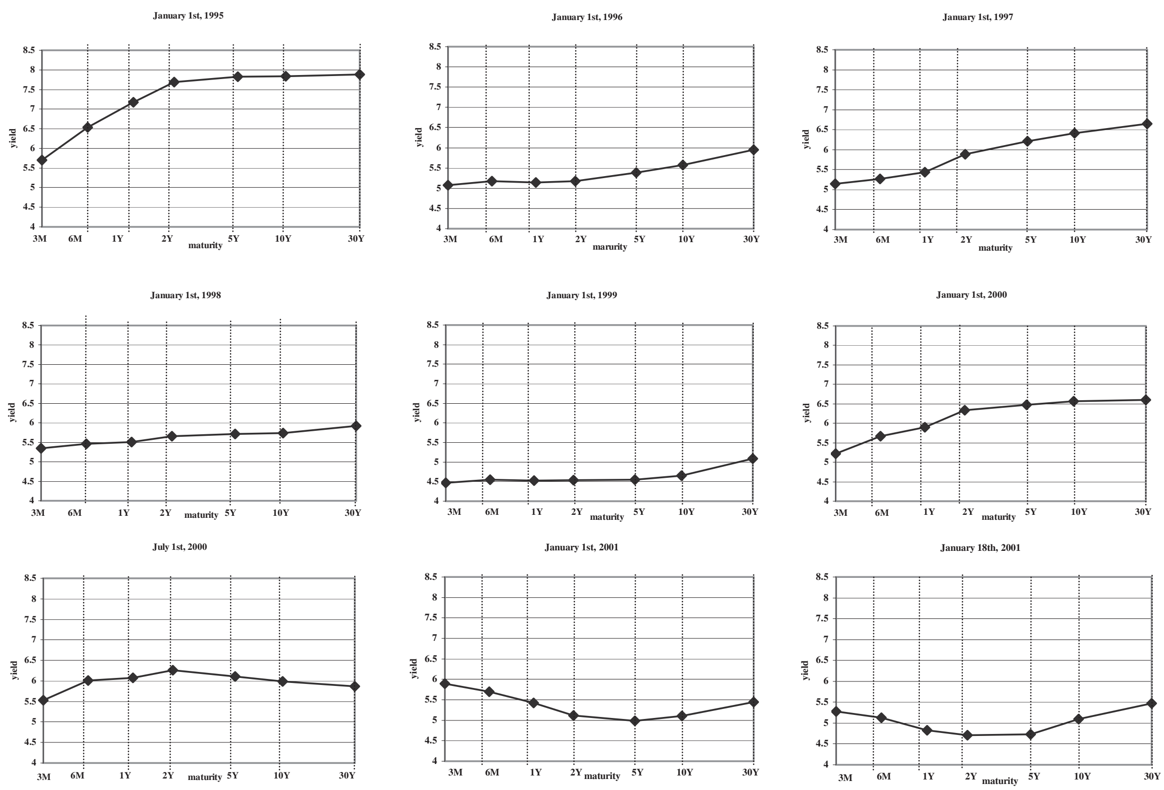
Implications:

- Long bonds on average receive higher returns than short bonds.
- Forward rate on average "over-predict" future short-term rates.
- Term structure reflects
  (a) expectations of future interest rates, and
  (b) risk premium demanded by investors in long bonds.

Yield curves with different expectations of future short rates



Treasury yield curves



Average rates of return on treasuries, 1926 – 2005

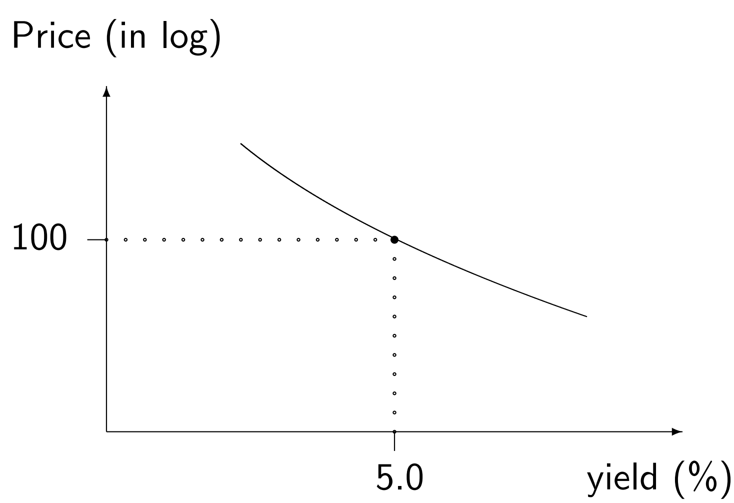
(Source: Ibbotson Associates, 2006 Yearbook)

	Long-term	Bills
Nominal	5.8%	3.8%
Real	2.9%	0.7%

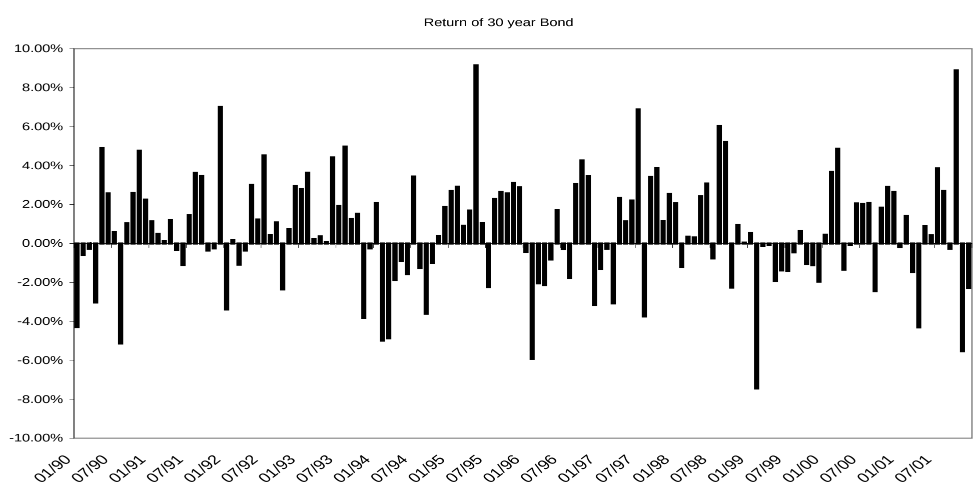
(Inflation is 3.1% per year.)

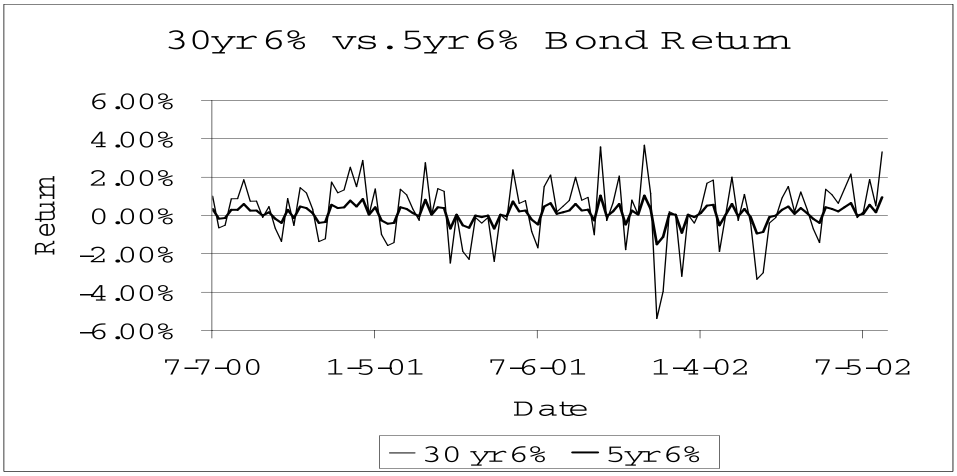
4 Interest Rate Risk

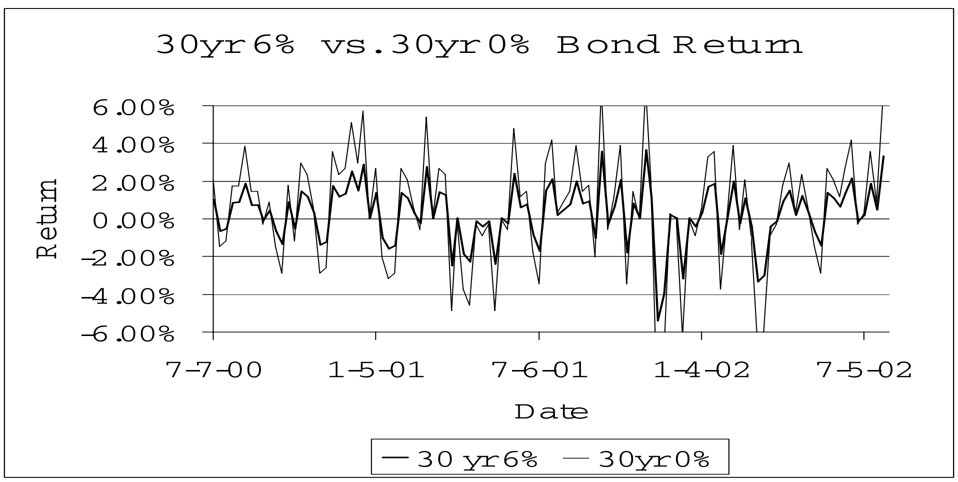
As interest rates change (stochastically) over time, bond prices also change. The value of a bond is subject to interest rate risk.



Interest rate risk of a 6%, 30-year T-bond




30yr 6% vs. 5yr 6% Bond Return
6.00%
4.00%
2.00%
0.00%
-2.00%
-4.00%
-6.00%
Return
7-7-00
1-5-01
7-6-01
1-4-02
7-5-02
Date
30 yr 6%
5yr 6%


30yr 6% vs. 30yr 0% Bond Return
6.00%
4.00%
2.00%
0.00%
-2.00%
-4.00%
-6.00%
Return
7-7-00
1-5-01
7-6-01
1-4-02
7-5-02
Date
30 yr 6%
30yr 0%

4.1 Measures of Interest Rate Risk

Assume flat term structure at $r_t = y$.

Duration and Modified Duration

A bond's interest rate risk can be measured by its relative price change with respect to a change in yield.

$$\mathrm{MD} = -\frac{1}{B}\frac{\Delta B}{\Delta y}.$$

This is called a bond's modified duration or volatility.

The term modified duration comes from its link to duration:

Definition: A bond's duration is the weighted average of the maturity of individual cash flows, with the weights being proportional to their present values:

$$\mathrm{D} = \sum_{t=1}^{T} \frac{\mathrm{PV}(CF_t)}{B} \times t = \frac{1}{B}\sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} \times t.$$

Duration measures the average time taken by a bond, on a discounted basis, to pay back the original investment.

Duration and MD satisfy the following relation:

$$\mathrm{MD} = \frac{\mathrm{D}}{1+y}$$

Example. Consider a 4-year T-note with face value \$100 and 7% coupon, selling at \$103.50, yielding 6%.

For T-notes, coupons are paid semi-annually. Using 6-month intervals, the coupon rate is 3.5% and the yield is 3%.

t	CF	PV(CF)	$t \cdot$ PV(CF)
1	3.5	3.40	3.40
2	3.5	3.30	6.60
3	3.5	3.20	9.60
4	3.5	3.11	12.44
5	3.5	3.02	15.10
6	3.5	2.93	17.59
7	3.5	2.85	19.92
8	103.5	81.70	653.63
		103.50	738.28

- Duration (in 1/2 year units) is

  $\mathrm{D} = (738.28)/103.50 = 7.13.$

- Modified duration (volatility) is

  $\mathrm{MD} = \mathrm{D}/(1+y) = 7.13/1.03 = 6.92.$

If the semi-annual yield moves up by 0.1%, the bond price decreases roughly by 0.692%.

Convexity

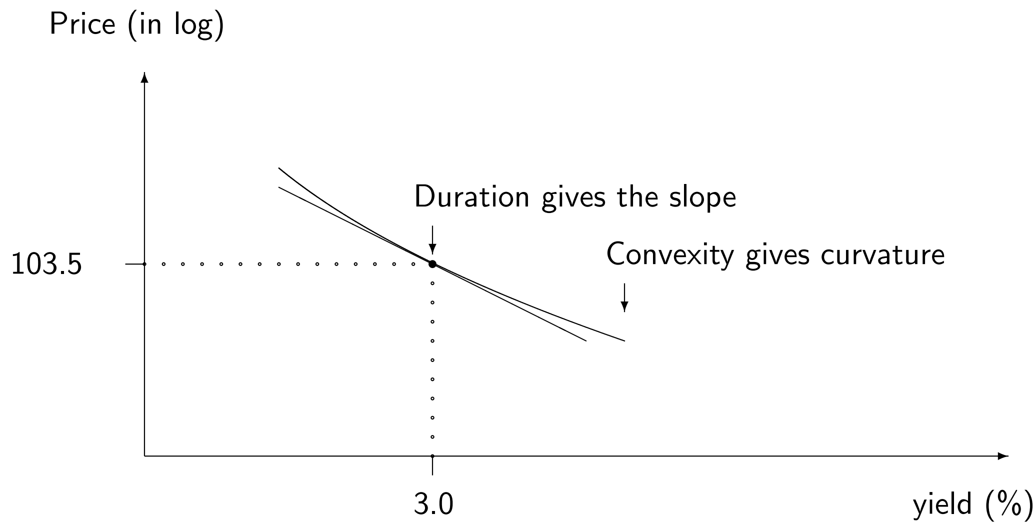
Example. (Continued.) 4-year T-note with 7% coupon and 6% flat yield curve.

- Duration is D = 7.13.
- Volatility is MD = 6.92.

As the yield changes, the bond price also changes:

Yield	Price	Using MD	Difference
0.040	96.63	96.35	0.29
0.035	100.00	99.93	0.07
0.031	102.79	102.79	0.00
0.030	103.50	-	-
0.029	104.23	104.23	0.00
0.025	107.17	107.09	0.08
0.020	110.98	110.67	0.32

- For small yield changes, pricing by MD is accurate.
- For large yield changes, pricing by MD is inaccurate.



Bond price is not a linear function of the yield. For large yield changes, the effect of curvature (i.e., nonlinearity) becomes important.

Approximate price change:

$$\begin{aligned}(\Delta B) &= \frac{\Delta B}{\Delta y}(\Delta y) + \frac{1}{2}\frac{\Delta^2 B}{\Delta y^2}(\Delta y)^2 + \cdots \\ &\approx -\mathsf{MD} \times (\Delta y) + \mathsf{CX} \times (\Delta y)^2\end{aligned}$$

where convexity, CX, measures the curvature of the bond price (per \$) as a function of the yield:

$$\mathsf{CX} = \frac{1}{2}\frac{1}{B}\frac{\Delta^2 B}{\Delta y^2}.$$

4.2 Managing Interest Rate Risks

Given a fixed-income position, we can take another fixed-income position to offset the interest rate risk of the original position. Thus, the interest rate risk of the total position (the portfolio) is reduced. Such a strategy is called hedging.

Assume a flat term structure. Consider a bond portfolio consisting of n_A units of bond A and n_B units of bond B, and

Bond	Price	Duration	Volatility
A	B_A	D_A	MD_A
B	B_B	D_B	MD_B

The value of the portfolio is

$$V_P = V_A + V_B = n_A B_A + n_B B_B.$$

When interest rates change,

$$\begin{aligned}\Delta V_P &= \Delta V_A + \Delta V_B = n_A \Delta B_A + n_B \Delta B_B \\ &= -(V_A \mathsf{MD}_A + V_B \mathsf{MD}_B)(\Delta y).\end{aligned}$$

Thus,

$$\mathsf{MD}_P = \frac{V_A}{V_A + V_B}\mathsf{MD}_A + \frac{V_B}{V_A + V_B}\mathsf{MD}_B.$$

Example. Suppose that you are long in 4-year bonds and you want to use 3-year bonds to hedge the interest rate risk. The data on these bonds are

Bond	Yield	Duration	Volatility (%)
3-year	0.10	2.75	2.50
4-year	0.10	3.52	3.20

To hedge the long position in 4-year bond, we need to sell 3-year bond. How much to sell?

For each dollar worth of 4-year bond, short δ dollars worth of 3-year bond such that the total portfolio has zero volatility:

$$\mathrm{MD}_4 + \delta \times \mathrm{MD}_3 = 0.$$

δ is called the "hedge ratio". Thus

$$\text{hedge ratio} = \frac{\mathrm{MD}_4}{\mathrm{MD}_3} = \frac{3.20}{2.50} = 1.28.$$

For the hedged portfolio, we have

Position	Value change when yields $\uparrow$ 0.1%
Long \$1000 4-year bond	$-(1000)(3.20)(0.001) = -3.20$
Short \$1280 3-year bond	$(1280)(2.50)(0.001) = +3.20$
Net	0

5 Inflation Risk

Most bonds give nominal payoffs. In the presence of inflation risk, real payoffs are risky even when nominal payoffs are safe.

Example. Suppose that inflation next year is uncertain ex ante, with equally possible rate of 10%, 8% and 6%. The real interest rate is 2%.

The 1-year nominal interest rate will be (roughly) 10%.

Consider the return from investing in a 1-year Treasury security:

Year 0 value	Inflation rate (%)	Year 1 nom. payoff	Year 1 real payoff
1000	0.10	1100	1000
1000	0.08	1100	1019
1000	0.06	1100	1038

6 Credit Risk and Corporate Bonds

Fixed-income securities have promised payoffs of fixed amount at fixed times. Excluding government bonds, other fixed-income securities, such as corporate bonds, carry the risk of failing to pay off as promised.

Definition: Default risk (credit risk) refers to the risk that a debt issuer fails to make the promised payments (interest or principal).

Bond ratings by rating agencies (e.g., Moody's and S&P) provide indications of the likelihood of default by each issuer.

Description	Moody's	S&P
Gilt-edge	Aaa	AAA
Very high grade	Aa	AA
Upper medium grade	A	A
Lower medium grade	Baa	BBB
Low grade	Ba	BB

- Investment grade bonds: Aaa – Baa by Moody's or AAA – BBB by S&P.
- Speculative (junk) bonds: Ba and below by Moody's or BB and below by S&P.

Default Premium and Risk Premium

Example. Suppose all bonds have par value \$1,000 and

- 10-year Treasury strip is selling at \$463.19, yielding 8%
- 10-year zero issued by XYZ Inc. is selling at \$321.97
- Expected payoff from XYZ's 10-year zero is \$762.22.

For the 10-year zero issued by XYZ:

$$\text{Promised YTM} = \left(\frac{1000.00}{321.97}\right)^{1/10} - 1 = 12\%$$

$$\text{Expected YTM} = \left(\frac{762.22}{321.97}\right)^{1/10} - 1 = 9\%$$

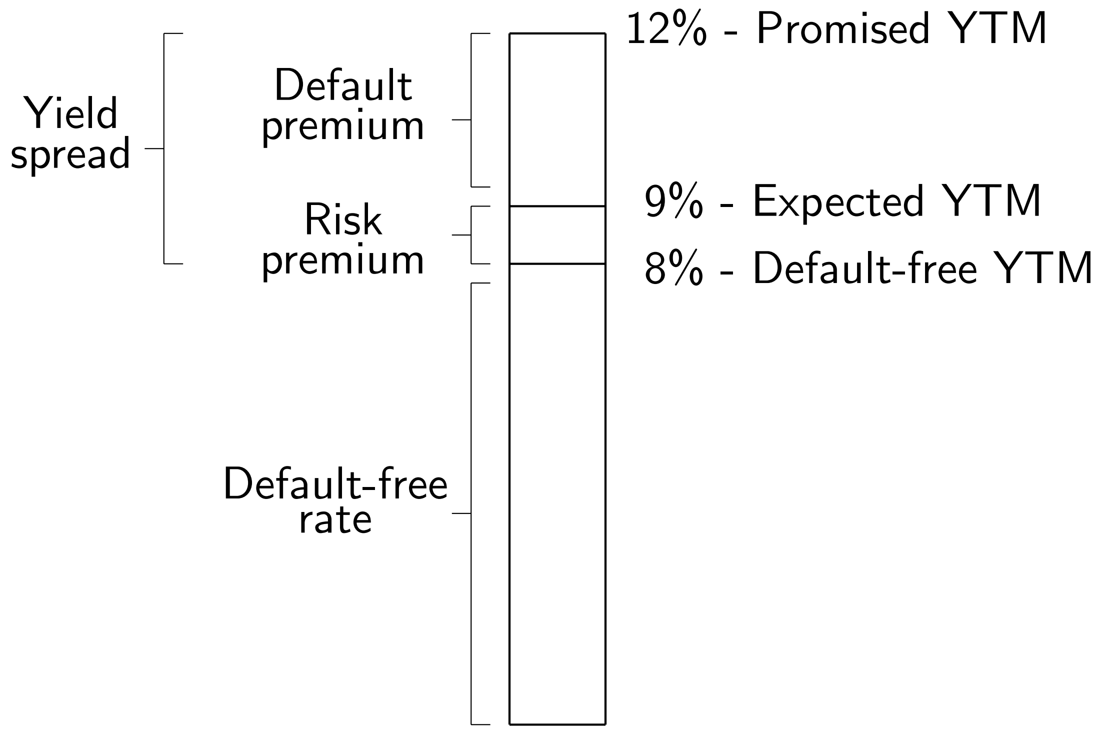
and

$$\begin{aligned}\text{Default Premium} &= \text{Promised YTM} - \text{Expected YTM} \\ &= 12\% - 9\% = 3\%\end{aligned}$$

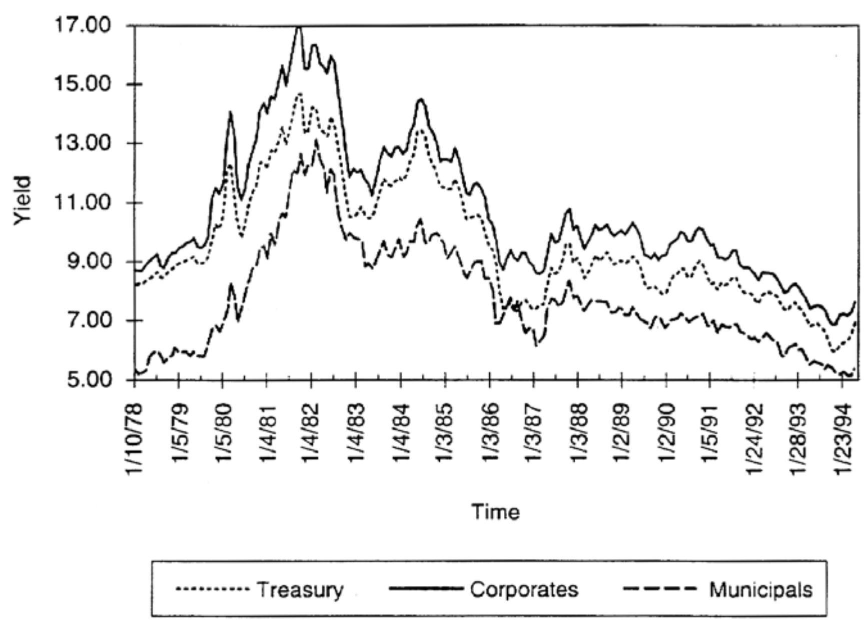
$$\begin{aligned}\text{Risk Premium} &= \text{Expected YTM} - \text{Default-free YTM} \\ &= 9\% - 8\% = 1\%.\end{aligned}$$

- **Promised YTM** is the yield if default does not occur.
- **Expected YTM** is the probability-weighted average of all possible yields.
- **Default premium** is the difference between promised yield and expected yield.
- **Bond risk premium** is the difference between the expected yield on a risky bond and the yield on a risk-free bond of similar maturity and coupon rate.

Yield-to-maturity for a risky bond



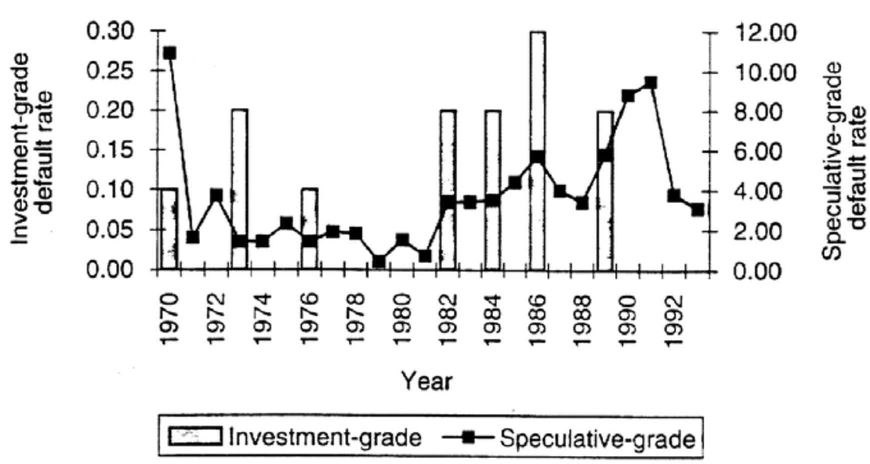
Yields of corporates, treasury, and munies



One-year default rates

Year	*Aaa*	*Aa*	*A*	*Baa*	*Ba*	*B*	*Investment Grade*	*Speculative Grade*
1970	0.00	0.00	0.00	0.30	8.40	21.60	0.10	10.90
1971	0.00	0.00	0.00	0.00	1.50	0.00	0.00	1.60
1972	0.00	0.00	0.00	0.00	0.50	11.80	0.00	3.70
1973	0.00	0.00	0.00	0.50	0.50	3.40	0.20	1.40
1974	0.00	0.00	0.00	0.00	0.00	6.90	0.00	1.40
1975	0.00	0.00	0.00	0.00	1.60	3.00	0.00	2.30
1976	0.00	0.00	0.00	0.00	1.10	0.00	0.10	1.40
1977	0.00	0.00	0.00	0.30	0.60	8.80	0.00	1.90
1978	0.00	0.00	0.00	0.00	1.10	5.30	0.00	1.80
1979	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.40
1980	0.00	0.00	0.00	0.00	0.00	4.40	0.00	1.50
1981	0.00	0.00	0.00	0.00	0.00	4.10	0.00	0.70
1982	0.00	0.00	0.20	0.30	2.60	2.20	0.20	3.40
1983	0.00	0.00	0.00	0.00	1.00	6.00	0.00	3.40
1984	0.00	0.00	0.00	0.60	0.50	7.30	0.20	3.50
1985	0.00	0.00	0.00	0.00	2.00	8.70	0.00	4.40
1986	0.00	0.00	0.00	1.10	1.90	11.60	0.30	5.70
1987	0.00	0.00	0.00	0.00	2.60	5.30	0.00	4.00
1988	0.00	0.00	0.00	0.00	1.50	5.70	0.00	3.40
1989	0.00	0.30	0.00	0.50	2.70	8.60	0.20	5.80
1990	0.00	0.00	0.00	0.00	3.30	12.90	0.00	8.80
1991	0.00	0.00	0.00	0.20	5.10	13.10	0.00	9.50
1992	0.00	0.00	0.00	0.00	0.20	6.40	0.00	3.80
1993	0.00	0.00	0.00	0.00	0.50	5.20	0.00	3.10

Source: Moody's Investors Services (1994)



7 Homework

Readings:

- BKM Chapters, 14, 15, 16.
- BMA Chapters 3, 24.
- Salomon Brothers, "Understanding Duration and Volatility."

Assignment:

- Problem Set 2.
- Case write-up (group project): Rensselaer Advisors.