The First Use of a Combined-Value Auction for Transportation Services

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Combined-value auctions (CVAs) allow participants to make an offer of a single amount for a collection of items. These auctions provide value to both buyers and sellers of goods or services in a number of environments, but they have rarely been implemented, perhaps because of lack of knowledge and experience. Sears Logistics Services (SLS) is the first procurer of trucking services to use a CVA to reduce its costs. In 1993, it saved 13 percent over past procurement practices. Experimental economics played a crucial role in the development, sale, and use of the CVA.

(Economics. Games/group decisions: bidding/auctions. Transportation: costs.)

Sears, Roebuck and Co. is one of the largest procurers of trucking services in the world through its wholly-owned subsidiary, Sears Logistics Services (SLS). SLS (http://www.slslogistics.com) controls elements of the supply chain that connect the vendor (manufacturer), distribution centers, retail stores, and cross-dock facilities (similar to airline hubs for redistributing cargo). A major portion of SLS operations is contracting for truck and carrier services. SLS sought to consolidate its use of trucking services and reduce its costs. In 1992, SLS engaged the consulting firm of Joseph Swanson and Company (JSOC) to help it consolidate its trucking services. JSOC identified as promising the combined-value trading technology being developed within the California Institute of Technology (Caltech) by the founders of Net Exchange (NEX). This work was based on the work of Banks et al. (1989). At the time, the only other published work on combined-value auctions was by Rassenti et al. (1982). SLS, JSOC (http://www.JSCO.com), and NEX (http://www.nex.com) formed a team to execute the project. The initial auction would involve 854 lanes with a current service cost of approximately $190 million per year. SLS implemented a combined value iterative auction that reduced this cost to $165 million per year, a 13 percent savings. Subsequent auctions maintained that saving rate.

Background
Historically SLS contracted for trucking services through a series of bilateral negotiations. A large group of SLS agents worked with many carriers to negotiate services on individual lanes (single transportation paths, for example, from Chicago to Los Angeles) or groups of lanes. For a large enterprise like Sears, this negotiation process was time consuming and expensive.
In the early 1990s, SLS sought to lower its truckload carrier costs by consolidating its acquisition of trucking services so that truckload carriers could better deploy their assets and share the savings with SLS. SLS put together a strategy that, it believed, would save logistics costs and would encourage carriers to participate in the consolidation process. SLS had begun to experiment with procurement auctions. By allowing carriers to offer it simple single classes of transportation services through single-round sealed-bid single-lane auctions, it was saving on costs. The idea was to extend this approach to the larger consolidation effort.

What would it mean if a carrier won?

SLS and JSCO recognized that the trucking firms might rebel if SLS put up a sizable piece of their regular and profitable business for competitive auction. Some of the more reliable firms might even refuse to participate, and therefore, the next auction might not have enough capacity to serve the needs of SLS.

JSCO identified a small number of partners that it could give exclusive rights to bid in the auction and whom it would help in their planning and participation. By limiting the number of partners, SLS provided an incentive for participation, the opportunity to obtain the business. Using a complex proprietary procedure, JSCO and SLS qualified the carriers to insure that SLS could be confident it could rely on the carriers selected. This also reassured the invited partners that other participants were peers able to provide a similar level of service. In the end, 14 national and regional carriers qualified and participated. SLS informed the candidates of the names of the competitors that qualified.

In 1992, it was not known precisely how to organize an auction of this magnitude. What was to be auctioned? That is, what would it mean if a carrier won? Would it have to deliver anything Sears requested for the term of the contract? Would it have to handle only a fixed maximum weekly set of loads, leaving SLS to find others to handle excess shipments? JSCO provided the answers to these questions. Should SLS auction one lane at a time; and if so, in what order and how fast? Or should it auction them all at the same time? If so, how could it coordinate all the bidding?

These issues occur in many auction design problems; for example, in the Federal Communication Commission (FCC) auctions of the electromagnetic spectrum. Caltech and Net Exchange provided the answers to these questions.

Two innovations offered the promise of better asset deployment, which could result in shared savings for the carrier and SLS if implemented through a properly designed auction:

1. Letting three-year contracts that included surge demand contingencies; and
2. Letting contracts on a large number of lanes simultaneously through a process that solicited single offers for multiple lanes, thereby allowing carriers to coordinate SLS business with other business and reduce related empty or low-value back-haul movements.

A standard one-sided procurement auction, increasingly referred to as a reverse auction, was the process of choice for implementing the first innovation. To implement the second, one must use a combined-value procurement auction. In a combined-value auction (CVA), carriers can put together orders that comprise multiple items and make offers that express their combined value for the group rather than cobbling together individual trades and facing the risk that they will miss links in a desired chain.

Combined-Value Design

If the carriers were to create the saving for themselves, thus creating value to share with SLS, SLS would have to assure them of steady, nearly risk-free business, and they would need to coordinate their bids across multiple lanes. To assure the carriers, SLS decided to auction three-year contracts with contingencies for surge and slack demand.

Allowing carriers to coordinate their bids was more difficult. To use their assets efficiently, by reducing the number of miles trucks travel empty, carriers must solve a fairly complex minimization problem. The carrier must coordinate SLS shipments and shipments...
they expect to have under contract to others on lanes SLS may not even be auctioning. What a firm is willing to supply services for on a specific Chicago-to-Los Angeles lane depends critically on what it has committed in the LA-to-Chicago direction (which may itself involve multiple lanes). If an SLS lane (say St. Louis to Chicago) is part of that return cartage, then the amount the trucking firm is willing to accept for the Chicago-to-LA lane depends on how much it will be paid for the St. Louis-to-Chicago lane. That is, the amount it is willing to accept to do both the Chicago-to-LA and the St. Louis-to-Chicago lanes depends on the combined value, which will generally be less than the sum of the individual parts (Appendix).

The problem was to design an auction that would reveal and take advantage of these combined-value opportunities. At that time, we did not know of any auctions being used that allowed combined-value bidding. However, research using laboratory experiments, described by Banks et al. (1989), had demonstrated the potential power of combined-value bidding over traditional methods. Combined-value auctions differ primarily in one aspect from traditional auctions. They allow participants to make a bid of a single amount for a collection of items. In the SLS case, a carrier would be able to say “I ask $1 million for the Chicago-to-Los Angeles lane and the St. Louis-to-Chicago lane,” meaning “I am willing to service the two lanes for a fee of at least $1 million if and only if

The stopping rule is absolutely critical.

I can service both lanes.” Once this new type of procurement bid is considered, the rest of the design follows. First, SLS auctioneers determine the winners by accepting the bids that minimize the total cost of procuring the services when it allows only one carrier per lane. Second, SLS will pay all winning bidders their asking prices. Suppose there are three bids for lanes A and B: bid one is $10 for lane A, bid two is $35 for lane B, and bid three is $40 for both lanes A and B. Bid three is the winning bid, and that bidder will be paid $40, since the $40 of bid three is less than the $45 combined bids of bidders one and two (Appendix).

A critical, if seemingly innocuous, part of the auction design is the stopping rule. The stopping rule for an auction is absolutely crucial to its performance, both in the final cost of acquisition and in the time to completion, because it affects the incentives and the information of the bidders. One option is to let all bidders submit as many bids as they wish, up to a specified time. At that time, winners are determined and the auction is over. This is a sealed-bid procurement auction using combined-value bids. The sealed-bid auction presents problems because it requires bidders to consider all contingencies and to evaluate all of the business implications of winning each subset of lanes. It also encourages submission of all possible bids by all bidders, and it has been shown to result in a higher final cost of procurement. Previous experimental results (Banks et al. 1989; McCabe et al. 1993) had shown that allowing the bidders to update their bids would improve the allocation. In complex environments, past experience has shown that iterations with some sort of commitment are needed to stabilize response and to speed convergence. Iterations allow feedback, reaction, and learning about the possibilities.

Iterative auctions (also called progressive auctions) have proven to be highly efficient and easy to understand and have been successfully applied in a number of settings. An English auction is an example of an iterative auction. In an English auction, bidders submit bids verbally to an auctioneer. The first bidder’s bid becomes the standing bid that other bidders must beat to establish a new standing bid or to win the item. The auction continues until no one is willing to submit a higher bid than the standing bid. The winner is the bidder with the last standing bid (that is, the highest bid submitted). In this auction, the winner pays a price equal to its winning bid. Comparisons of sealed-bid auctions and English auctions reported by Coppinger et al. (1980; reprinted by Smith 1991) and by Cox et al. (1982; reprinted by Smith 1991) show that the English auction is more efficient than the sealed-bid auction.
(Kagel and Roth (1995, Chapter 7) provide a comprehensive survey.) The principal advantages of the progressive auction are that the optimal bidding strategy is transparent and that the successive bids are known to all. Bidders do not need to be publicly identified, which makes collusion difficult. (Smith (2000) and DeMartini et al. (1999) discuss the advantages and disadvantages of CVA iterative auctions.)

The design team from SLS, JSCO, and NEX chose an iterative version of the sealed-bid procurement auction, in which bidding proceeded in rounds. At the end of each round, the auctioneer announced provisional winners. Going into the next round, the auctioneer held all of the provisional winning bids from the previous round, and carriers submitted new bids against that set. This requirement is extremely important because it imposes a commitment on the bidders; each bid may be viewed as a contract proposal to which the bidder will adhere if the auctioneer accepts it. Allowing bidders to withdraw provisional winning bids extends the auction and creates bad incentives. Without such commitment, bidders face no penalty from, say, bidding randomly or from bidding to attack an opponent. Laboratory test runs had revealed that this iterative process increased cost savings because it allowed the firms to concentrate their efforts on those

SLS saved more than $84.75 million by running six combined-value auctions.

lanes that gave them a cost advantage. The stopping rule used was of this form; if total acquisition cost did not decline by at least x percent from the previous round, then the just-completed round is declared to have been the final round.

Selling Combined Value to SLS

The first step was to demonstrate to SLS that running such an auction was feasible. Its senior management team had a recent history of imaginative approaches to multiple logistics solutions but was insistent on constructive demonstrations of "new approaches" to service contracting. NEX created a test-bed environment and then ran combined-value auctions in that scaled-down world in the Caltech Economics Laboratory. In this case, a test bed is a scaled-down version of the items to be auctioned and the incentive structure of the participants. Using such an environment is similar to using scale models and wind tunnels in automotive and aircraft design. The SLS team readily subscribed to this concept. JSCO, working with SLS participants, provided a model of a truckload transportation network with approximately correct representations of loads, unit costs, and lane profitabilities (Appendix).

The experimental test bed operated over a local area network of computers in the Caltech Economics Laboratory. Undergraduate students at Caltech were recruited, all of whom had experience in other market experiments. They acted as trucking firms in the auction and earned money based on their decisions. They earned money by winning bids on lanes for amounts above their costs, which we gave them (Appendix). For example, if the cost of providing service on lane AB was 300 cents and a student won the auction with a bid of 350 cents for lane AB, he or she earned 50 cents. On average, students earned $30 to $40 for a two-hour session. The experiments also tested the optimization program in real time; we were able to correct errors that had escaped the debugging process.

In the meantime, the NEX team scaled up the auction part of the test bed so that it could handle over 1,000 lanes. This potentially huge, combinatoric problem is daunting (even with today's technology), but it was almost overwhelming in 1993. In theory, each bidder could submit a huge number (2 raised to the 854th power) of bids, possibly more than the number of stars in the universe. Practical factors, however, limited the number of bids submitted. In particular, in the actual auction, bidders submitted bids via spreadsheets on floppy disks. At the time, these spreadsheets limited what a bidder could do. The maximum number of bids submitted was 4,595; and bidders did not complain that they could not submit enough bids. Starting from standard algorithms with some front-end sorting and culling of orders, we created an algorithm that easily handled problems of this scope. The algorithm had no trouble finding the optimal solution in less than an hour. (Using today's computers, that would be less than a minute.)
Once we had designed an acceptable CVA, we had to explain it to the SLS team and get its approval. The experimental test bed was an important demonstration tool. We took the test bed to SLS so the team members could participate in a CVA. The goal of the demonstration was to show that trucking firms could understand the auction procedures and that SLS would incur savings. The demonstration convinced the SLS team that a CVA was workable.

The final step was to take this test bed to the trucking firms. It served as a sales device to interest them in participating in the ultimate auction and as a training device to teach teams how to participate successfully. JSCO used materials developed in the laboratory to do most of the sales and training with the firms that had been prequalified. In 1993, the sales and training processes were time intensive. Today, creative use of the Web and interactive software could easily cut the time needed for training. In similar environments, explaining such processes takes less than an hour. Ultimately, the firms accepted the CVA as a new way of doing business.

The First Auction Results

SLS initially committed to a single auction. The auction ran for five rounds with about one month between rounds (Table 1)

In 50 percent of the bids, bidders used the multilane-bid capability of the combined-value auction, and 30 percent of the package bids submitted in the last round were winners, indicating the usefulness of this feature.

In addition, contrary to some people’s expectations that combinatoric calculations would be a problem, our software took only 15 to 30 minutes to calculate winning bids.

In observing the bidding behavior, we found that many bidders wanted to submit bids of the following form: “I will supply lanes A, B, and C for $100 or supply lanes D, E, and F for $120 but not both.” Although this feature, called XOR, is now standard in combined-value auctions, the SLS auction did not allow such bids. Nevertheless, the bidders found a clever way to make such bids. They would submit two overlapping bids, including a small, inexpensive lane, say G, in both: “I bid $101 for A, B, C, and G,” and “I bid $121 for D, E, F, and G.” Both bids couldn’t win. However, this tactic had the unintended result of raising the cost of supplying lane G.

SLS contacted all the participants two to three weeks after it distributed the final results requesting observations on both the process and outcome. The carriers’ overall reaction to this type of auction was favorable. They were happy to execute the outcome and to participate in future auctions of the same format. Carriers found that the redistribution did not negatively affect the volume of business they were carrying, nor did market participants trash the rates. Carriers met the business requirements of the traffic awarded and still made an acceptable margin on the business. They particularly liked the format and the level of detail in the traffic information provided. They stated that they were reasonably happy with their outcomes. Each carrier had lost one or two lanes that they would have liked to retain, but they recognized the loss as a reality of the business and the process.

Most of the negative issues concerned execution of the auction. Most bidders thought the auction should have gone fewer rounds, and that the process took too long. Carriers differed as to whether they had enough time to respond. Some looked at every lane in every round and wanted more time (the national carriers). Others focused on fewer lanes in their bidding and would have been happy with less turnaround time. The carriers differed in their level of sophistication in analysis. Some used detailed modeling, while others worked out their strategies using pencil and paper.

One bidder thought that the process was purely price driven but shouldn’t have been. It felt that SLS
gave no consideration to the service capabilities of the carriers and that incumbents on particular lanes had no advantage. However, SLS considered service and performance capabilities during the participant-selection process. It had considered incumbency benefits and thought they would realistically be implicit because incumbent carriers would have been able to develop supporting business to support those lanes during their tenure.

The Market Test

Following the success of this first SLS procurement auction, SLS bought software and hardware from NEX with the capability of running similar auctions. With the help of JSCO, SLS ran five auctions during 1995 and 1996 (Table 2).

SLS acquired services for 536 lanes for about $102.2 million. Its total savings were about $13.3 million (13 percent). These cost-savings estimates are based on estimates of the carrier prices that SLS would have had to pay without the auction. For lanes with established prices, SLS used the actual realized rates. For lanes without established prices, SLS used state-to-state-matrix rates (a set of backup rates that carriers provide to a company that are intended for noncontract loads or extremely low-volume lanes). Comparing these rates with those from the first auction shows that, over a three-year period, SLS saved more than $84.75 million by running six combined-value auctions. It acquired truckload transportation services for 1,390 lanes for $587 million. SLS adapted CVAs for procuring transportation services and still uses them today, with the full support of management.

<table>
<thead>
<tr>
<th>Number of carriers</th>
<th>Number of lanes</th>
<th>Acquisition cost</th>
<th>Estimated savings</th>
<th>Savings percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>$1,200,000</td>
<td>$80,000</td>
<td>6.2/3</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>$15,000,000</td>
<td>$3,000,000</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>$49,000,000</td>
<td>$5,000,000</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>$10,000,000</td>
<td>$2,500,000</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>$27,000,000</td>
<td>$2,000,000</td>
<td>7.1/2</td>
</tr>
</tbody>
</table>

Table 2: In this table are some facts about and the results for each of the next five combined-value auctions run by SLS during 1995 and 1996.

Conclusion: Lessons Learned

The key measure of success for any new auction design is whether it is used. The CVA implemented by SLS has been a success. It has been a success in other regards also. As designers of economic systems, we learned a considerable amount about designing and implementing complex auctions and markets. Of the many things we learned, one was the importance of including an XOR bid (I want A or B but not both) in the auction. The XOR bid allows bidders to specify a larger range of preferences in a single round of bidding. An important feature of the auction was the prequalification of participants. It ensured informed and serious bidding. Finally, bidders complained about the length of time the overall process took. The same issues arose in the design and operation of the FCC auctions of the electromagnetic spectrum. In retrospect, we see no reason why one can't reduce the time between rounds.

A whole new field of research and development devoted to designing smart markets for complex commercial transactions has emerged. Other potential application areas, such as markets for natural gas, water consumption, electric power, and financial assets, abound in both the public and private sectors. Bossaerts et al. (forthcoming), Olson et al. (2000), and Geoffrion and Krishnan (2001) make the point that the combination of operations research and economics (computation and incentives) is needed for success in complex auction design. This was also our experience. New processes providing better feedback to bidders now exist, and they seem more user friendly and faster than the path-breaking but simple process SLS used (DeMartini et al. 1999). We believe that the pioneering SLS auction opens the door to constructive applications in other market areas.

Acknowledgments

The authors thank Takashi Ishikida, Charles Polk, and David Swan-son for helpful comments.

APPENDIX

A Stylized Example of Combined Value

A retail company requires different loads on three lanes: from Los Angeles to Chicago, five truckloads;
from Chicago to New Orleans, seven truckloads; and from New Orleans to Los Angeles, 10 truckloads (Figure 1). It could buy or lease 10 trucks and allow a lot of empty back-hauls, but that would be costly. It might find a cheaper solution by outsourcing.

Suppose three trucking firms have the same costs but different customer bases. Each has a current contract for a lane and has some (usually uncertain) revenue from the return trip (Table 3).

Firm 1 would be happy to carry the retailer’s required five loads from Los Angeles to Chicago for any price greater than X, because it can take advantage of the combined value for servicing both directions on the Los Angeles-to-Chicago lane. If X < C (LA to Chicago), the cost of shipping five loads from LA to Chicago, then the retailer and Firm 1 both could gain if the retailer outsourced the transportation services for that lane. At any price P such that X < P < C (LA to Chicago), both would gain.

One can continue the analysis. Firm 2 would carry the 10 loads from Chicago to New Orleans for any price greater than Y, and Firm 3 would carry the 10 loads from LA to New Orleans for a price greater than Z. Finally, Firm 3 would also be willing to carry the five loads from LA to Chicago and the seven from Chicago to New Orleans for a price greater than W = Z + Cost (LA to Chicago to New Orleans) – Cost (LA to New Orleans). In this case, Firm 3 would be able to reap the combined value from the triple-lane combination of LA to Chicago to New Orleans to Chicago.

If the retailer knew the values of X, Y, Z, and W, what should it do? The answer is easy. If X + Y + Z < W + C (LA to New Orleans), then it should hire 1 to handle the LA-to-Chicago loads, hire 2 to handle the Chicago-to-New Orleans loads, and hire 3 to handle the LA-to-New Orleans loads. However, if X + Y + Z > W + C (LA to New Orleans), then it should hire 3 to handle both the LA-to-Chicago loads and the Chicago-to-New Orleans loads. The retailer would provide its own service on the LA-to-New Orleans lane. That is, in the second case, it would not outsource the whole thing.

In practice, the retailer would not know the value of W, X, Y, or Z, and the optimization problem could involve 854 lanes instead of three. Signing contracts one at a time could interfere with the retailer’s ability to take advantage of the combined values available. That is why a combined-value auction can provide value to both the retailer and the trucking firms.

### The SLS Experimental Setup

The goal in designing the experiments for the SLS auction was to provide a test bed that would exhibit the benefits of combined value in an example that would look familiar to truckers and that would enable us to demonstrate how easy and productive a combined-value auction would be. Taking a lane map of the United States with 854 lanes and reducing it to something manageable was the first step. We chose to focus on seven locations and nine lanes. We chose a structure that would represent most combined-value opportunities known to exist (Figure 2).

<table>
<thead>
<tr>
<th>Firm #</th>
<th>Contract lane</th>
<th>Current loads</th>
<th>Revenue from return trip</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Chicago to Los Angeles</td>
<td>5</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>New Orleans to Chicago</td>
<td>10</td>
<td>Y</td>
</tr>
<tr>
<td>3</td>
<td>New Orleans to Los Angeles</td>
<td>10</td>
<td>Z</td>
</tr>
</tbody>
</table>

Table 3: In this table are the load requirements and the return trip revenue possibilities from other loads for the three lanes of the stylized example in Figure 1 that we use to illustrate the basis of combined value in shipping.
Figure 2: This is the network of lanes we used as the basis for our test bed experiments with load requirements and other fundamentals as detailed in Table 4.

To put meat on this structure, we added network loads, one-way times, and costs. This information was common knowledge, available and known to all (Table 4).

The carriers’ private information included other traffic revenue (both its own and possibly six revenues for others), units (trucks) at its disposal, and opportunity costs (Table 5). Finally, to help the carriers, we calculated the combined value of several packages and reported them as the values of “some packages that you may want to consider” (Table 6).

Table 4: In this table are details of the “lanes” to be auctioned by the “shipper” in our test bed experiments, illustrated in Figure 2, including data on the one-way distance of a lane, the cost of carrying a load one-way on that lane, the number of loads required to be carried, and the “shipper’s” reserve price for that “lane.”

<table>
<thead>
<tr>
<th>Lane</th>
<th>Miles</th>
<th>Cost</th>
<th>Loads</th>
<th>Reserve price</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>300</td>
<td>450</td>
<td>5</td>
<td>5,000</td>
</tr>
<tr>
<td>AC</td>
<td>300</td>
<td>450</td>
<td>15</td>
<td>13,000</td>
</tr>
<tr>
<td>AF</td>
<td>400</td>
<td>520</td>
<td>20</td>
<td>11,500</td>
</tr>
<tr>
<td>AG</td>
<td>200</td>
<td>380</td>
<td>10</td>
<td>7,000</td>
</tr>
<tr>
<td>CF</td>
<td>100</td>
<td>210</td>
<td>5</td>
<td>2,100</td>
</tr>
<tr>
<td>DE</td>
<td>100</td>
<td>210</td>
<td>10</td>
<td>4,000</td>
</tr>
<tr>
<td>FA</td>
<td>400</td>
<td>520</td>
<td>15</td>
<td>13,000</td>
</tr>
<tr>
<td>GA</td>
<td>200</td>
<td>380</td>
<td>15</td>
<td>10,000</td>
</tr>
<tr>
<td>GD</td>
<td>500</td>
<td>800</td>
<td>15</td>
<td>15,000</td>
</tr>
<tr>
<td>FG</td>
<td>300</td>
<td>450</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>EF</td>
<td>200</td>
<td>380</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5: In this table we provide the typical data on fundamentals given to “carriers” in our test bed experiments including facts about their capacities and potential revenue as well as indications (network averages) of the capabilities and potential revenues of the carriers they would be bidding against.

<table>
<thead>
<tr>
<th>Lane</th>
<th>Firm’s potential loads per week</th>
<th>Network average loads per week</th>
<th>Firm’s potential revenue per load</th>
<th>Network average revenue per week</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA</td>
<td>5</td>
<td>5</td>
<td>300</td>
<td>325</td>
</tr>
<tr>
<td>CA</td>
<td>10</td>
<td>12</td>
<td>400</td>
<td>300</td>
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<td>FA</td>
<td>5</td>
<td>5</td>
<td>500</td>
<td>300</td>
</tr>
<tr>
<td>AG</td>
<td>5</td>
<td>5</td>
<td>160</td>
<td>75</td>
</tr>
<tr>
<td>FC</td>
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<td>100</td>
</tr>
<tr>
<td>ED</td>
<td>8</td>
<td>8</td>
<td>190</td>
<td>100</td>
</tr>
<tr>
<td>AF</td>
<td>5</td>
<td>5</td>
<td>500</td>
<td>300</td>
</tr>
<tr>
<td>DG</td>
<td>15</td>
<td>15</td>
<td>650</td>
<td>450</td>
</tr>
<tr>
<td>EF</td>
<td>8</td>
<td>10</td>
<td>390</td>
<td>350</td>
</tr>
</tbody>
</table>

Table 6: This is an example of the computational help we provided to the “carriers” in our test bed experiments, wherein we calculated the potential profits of various package bids they could consider submitting in the combined-value auction.

The Simple Mathematics of a Combined-Value Auction

In the experiments, a bid was a detailed specification of a package of lanes a firm was willing to service along with the minimal amount of revenue the firm required for that package. (Carriers served lanes in all-or-none fashion.) A bid consisted of four items:

1. The list of lanes,
2. The minimum amount the firm needed to supply those lanes per week,

<table>
<thead>
<tr>
<th>Package firm would bid for</th>
<th>Other revenue</th>
<th>Operating cost</th>
<th>Required units</th>
<th>Break-even bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>BA $1,500</td>
<td>$4,500</td>
<td>2</td>
<td>$3,000</td>
</tr>
<tr>
<td>AG GA</td>
<td>AG $800</td>
<td>$10,800</td>
<td>3</td>
<td>$10,000</td>
</tr>
<tr>
<td>GD</td>
<td>DG $9,750</td>
<td>$18,000</td>
<td>3</td>
<td>$10,000</td>
</tr>
<tr>
<td>AF FA</td>
<td>FA $2,500</td>
<td>$20,800</td>
<td>8</td>
<td>$18,300</td>
</tr>
<tr>
<td>AC CF</td>
<td>FC $1,500</td>
<td>$19,800</td>
<td>6</td>
<td>$14,300</td>
</tr>
<tr>
<td>DE CF</td>
<td>ED $1,520</td>
<td>$15,600</td>
<td>4</td>
<td>$9,460</td>
</tr>
<tr>
<td>EF</td>
<td>EF $3,120</td>
<td>$15,000</td>
<td>4</td>
<td>$9,000</td>
</tr>
<tr>
<td>FC</td>
<td>FC $1,500</td>
<td>$15,000</td>
<td>4</td>
<td>$9,460</td>
</tr>
</tbody>
</table>
(3) An estimated utilization of capacity in whole units for each package, and

(4) A capacity constraint that specified the total units the carrier would supply across all of its accepted bids.

The last two features of a bid allowed each carrier firm to submit more bids than it would actually be able to service, since the algorithm would not accept more bids than the firm could service. These two features were not included in the actual SLS auction.

One can think of a bid for firm \( j \) as \( \langle (x, b, u_l), U \rangle \), where \( x(j,k,l) = 1 \) if firm \( j \) wants package \( k \) and lane \( l \) is part of that package, and \( x(j,k,l) = 0 \) if lane \( l \) is not part of \( j \)'s package \( k \), \( b(j,k) \) is the minimum dollar amount needed by firm \( j \) to supply package \( k \), \( u(j,k) \) is the unit capacity estimated by firm \( j \) for package \( k \), and \( U(j) \) is firm \( j \)'s total capacity.

For each lane \( l \), there is a reserve price \( R_l \). The reserve price can be the price that the shipper could contract for that lane individually. Reserve prices allowed us to structure the optimization so that there were no degeneracies. The reserve price \( R_k \) of a package \( k \) was the sum of the reserve prices of the lanes in that package.

Given a collection of bids, the combined-value-auction algorithm finds \( d_{jk} \) to maximize

\[
\sum_{jk} (R_k - b_{jk})d_{jk}
\]

subject to

\[
\sum_{jk} x_{jk,l}d_{jk} \leq 1 \quad \forall \text{ lane } l,
\]

\[
\sum_{k} u_{jk}d_{jk} \leq f(j) \quad \forall \text{ firm } j,
\]

(this constraint was not included in the SLS auction), and

\[
d_{jk} = 0 \text{ or } 1.
\]

If \( d_{jk} = 1 \) in the solution, then firm \( j \) was (provisionally) awarded lane \( k \) at a price of \( b_{jk} \). If \( d_{jk} = 0 \) for all \( jk \) such that \( x(j,k,l) = 1 \), then lane \( l \) is not awarded and must be contracted at a later time.

References


