Aggregate Matchings

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What we do:

Revealed preference exercise for matching theory.

Reconcile:

- Theory of stable *individual* matchings.
- Data on *aggregate* matchings.
What we do.

VS.
What we do.

\[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\]
What we do.

\[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{pmatrix}
\quad \begin{pmatrix}
1 & 8 & 0 & 0 \\
0 & 4 & 3 & 0 \\
7 & 3 & 0 & 0 \\
0 & 0 & 9 & 5 \\
\end{pmatrix}
\]
## Marriage Data (Michigan)

<table>
<thead>
<tr>
<th>Age</th>
<th>12-20</th>
<th>21-25</th>
<th>26-30</th>
<th>31-35</th>
<th>36-40</th>
<th>41-50</th>
<th>51-94</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-20</td>
<td>231</td>
<td>47</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>21-25</td>
<td>329</td>
<td>798</td>
<td>156</td>
<td>32</td>
<td>11</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>26-30</td>
<td>71</td>
<td>477</td>
<td>443</td>
<td>136</td>
<td>27</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>31-35</td>
<td>11</td>
<td>148</td>
<td>249</td>
<td>196</td>
<td>83</td>
<td>21</td>
<td>0</td>
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<td>36-40</td>
<td>2</td>
<td>41</td>
<td>105</td>
<td>144</td>
<td>114</td>
<td>51</td>
<td>1</td>
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<tr>
<td>41-50</td>
<td>0</td>
<td>15</td>
<td>42</td>
<td>118</td>
<td>121</td>
<td>162</td>
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</tr>
<tr>
<td>51-94</td>
<td>0</td>
<td>2</td>
<td>11</td>
<td>11</td>
<td>35</td>
<td>137</td>
<td>158</td>
</tr>
</tbody>
</table>
Question:

▶ Given an “aggregate matching table” (data), when are there preferences for individuals s.t. the matching is stable?
▶ In other words, what are the testable implications of stability for aggregate matchings.
General motivation: two sided decision problems

- Standard revealed preference:
  Alice buys tomatoes when carrots are available
  \[ \Rightarrow (T \succ_A C). \]
General motivation: two sided decision problems

- Standard revealed preference:
  Alice buys tomatoes when carrots are available
  $\Rightarrow (T \succ_A C)$.

- Two sided decision:
  Alice chooses Tomás over Carlos
  $\Rightarrow (T \succ_A C)$ or $\text{(C prefers its match to A)}$. 
Broad motivation: two sided decision problems

- Important problem: rationalizing preferences can explain revealed preference and “available sets” (budgets).
Important problem: rationalizing preferences can explain revealed preference and “available sets” (budgets).

Hence direction of revealed preference is affected by the hypothesized rationalizing preferences.

Literature mostly deals with the problem by assuming transferable utility.
Main results

Revealed preference exercise:
Main results

Revealed preference exercise:

- Characterization of rationalizable agg. match.
- Characterization under TU: strictly more restrictive.

Ex:
Main results

Revealed preference exercise:

- Characterization of rationalizable agg. match.
- Characterization under TU: strictly more restrictive.

Ex:

```
5  3  1
0  7  8
9  4  0
```
Main results

Revealed preference exercise:
- Characterization of rationalizable agg. match.
- Characterization under TU: strictly more restrictive.

Ex:

```
5  3  1
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
0  7  8
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
9  4  0
```
Main results

Econometric estimation strategy:

- Moment inequalities
- Set identification parameters in “index” utility model.
- Empirical illustration to US marriage data.
Other results

- Stability for aggregate match. is substantially different from individual match.
- Structure of stable aggregate matchings.
Model

An *aggregate matching market* is described by a triple $\langle M, W, \succ \rangle$, where:

- $M$ and $W$ are disjoint, finite sets. We call the elements of $M$ *types of men* and the elements of $W$ *types of women*.
- $\succ = ((\succ_m)_{m \in M}, (\succ_w)_{w \in W})$ is a profile of strict preferences: for each $m$ and $w$, $\succ_m$ is a linear order over $W \cup \{m\}$ and $\succ_w$ is a linear order over $M \cup \{w\}$.
An aggregate matching market is described by a triple $\langle M, W, > \rangle$, where:

- $M$ and $W$ are disjoint, finite sets. We call the elements of $M$ 
  types of men and the elements of $W$ types of women.
- $> = ((>_m)_{m \in M}, (>_w)_{w \in W})$ is a profile of strict preferences:
  for each $m$ and $w$, $>_m$ is a linear order over $W \cup \{ m \}$ and $>_w$ is a linear order over $M \cup \{ w \}$.

Note: identical preferences within type.
We show that relaxing this assumption in our framework leads to a vacuous theory.
An aggregate matching is a $K \times L$ matrix $X = (X_{ij})$ with $X_{ij} \in \mathbb{N}$. 
Model

- An **aggregate matching** is a $K \times L$ matrix $X = (X_{ij})$ with $X_{ij} \in \mathbb{N}$.
- An aggregate matching $X$ is **canonical** if $X_{ij} \in \{0, 1\}$. 
Model

▶ An aggregate matching is a $K \times L$ matrix $X = (X_{ij})$ with $X_{ij} \in \mathbb{N}$.
▶ An aggregate matching $X$ is canonical if $X_{ij} \in \{0, 1\}$.
▶ A canonical matching $X$ is a simple matching if for each $i$ there is at most one $j$ with $X_{ij} = 1$, and for each $j$ there is at most one $i$ with $X_{ij} = 1$. 
X is \textit{individually rational} if

\[ X_{ij} > 0 \implies w_j >_{m_i} m_i \text{ and } m_i >_{w_j} w_j. \]
Model

- X is *individually rational* if

\[ X_{ij} > 0 \Rightarrow w_j > m_i \text{ and } m_i > w_j. \]

- (\(m_i, w_j\)) is a *blocking pair* if \(\exists\)
  - \(w_k \in W\) with \(X_{ik} > 0\), and \(m_l \in M\) with \(X_{jl} > 0\),
  - s.t. \(w_j > m_i\) \(w_k\) and \(m_i > w_j\) \(m_l\).
Model

- X is *individually rational* if
  \[ X_{ij} > 0 \implies w_j > m_i \quad \text{and} \quad m_i > w_j. \]

- \((m_i, w_j)\) is a *blocking pair* if \(\exists\)
  - \(w_k \in W\) with \(X_{ik} > 0\), and \(m_l \in M\) with \(X_{jl} > 0\),
  - s.t. \(w_j > m_i \quad w_k\) and \(m_i > w_j \quad m_l\).

- X is *stable* if it is individually rational and there are no blocking pairs for X.
Model

Given $X$, construct a canonical aggregate matching $X^c$ by:

- $X_{ij}^c = 0$ when $X_{ij} = 0$ and
- $X_{ij}^c = 1$ when $X_{ij} > 0$.

Observation

An aggregate matching $X$ is stable if and only if $X^c$ is stable.
Example: simple vs. aggregate matching

Let \( \langle M, W, \rangle \) with \( M = \{m_1, m_2, m_3\} \), \( W = \{w_1, w_2, w_3\} \), and

\[
\begin{array}{ccc}
  m_1 & m_2 & m_3 \\
  w_1 & w_2 & w_3 \\
  w_2 & w_3 & w_1 \\
  w_3 & w_1 & w_2
\end{array}
\quad
\begin{array}{ccc}
  w_1 & w_2 & w_3 \\
  m_2 & m_3 & m_1 \\
  m_3 & m_1 & m_2 \\
  m_1 & m_2 & m_3
\end{array}
\]
The following simple matchings are stable:

\[
X^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad X^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}
\]

Sum of \(X^1\) and \(X^2\):

\[
\hat{X} = X^1 + X^2 = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}.
\]

\((m_1, w_2)\) is a blocking pair.
Stability

\[ \langle M, W, > \rangle \text{ defines a graph } (V, E) \text{ where} \]
\begin{itemize}
  \item \( V \) is the set of pairs \((i, j)\)
  \item \(( (i, j), (k, l) ) \in E \) if
    \begin{itemize}
      \item \( w_l >_{m_i} w_j \) and \( m_i >_{w_l} m_k \) or
      \item \( w_j >_{m_k} w_l \) and \( m_k >_{w_j} m_i \).
    \end{itemize}
\end{itemize}

\( X \) is stable iff

\[ ((i, j), (k, l)) \in E \Rightarrow X_{ij}X_{kl} = 0. \quad (1) \]

Otherwise (ie. \( X_{ij} = X_{kl} = 1 \)), either \((i, j)\) or \((k, j)\) is blocking pair.
Stability – Example

3 men and women:

\[
\begin{array}{ccc|ccc}
> m_1 & > m_2 & > m_3 & > w_1 & > w_2 & > w_3 \\
w_1 & w_2 & w_3 & m_2 & m_3 & m_1 \\
w_2 & w_3 & w_1 & m_3 & m_1 & m_2 \\
w_3 & w_1 & w_2 & m_1 & m_2 & m_3 \\
\end{array}
\]

Graph:
Stability – Example

3 men and women:

<table>
<thead>
<tr>
<th>$&gt;m_1$</th>
<th>$&gt;m_2$</th>
<th>$&gt;m_3$</th>
<th>$&gt;w_1$</th>
<th>$&gt;w_2$</th>
<th>$&gt;w_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
<td>$m_2$</td>
<td>$m_3$</td>
<td>$m_1$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$w_3$</td>
<td>$w_1$</td>
<td>$m_3$</td>
<td>$m_1$</td>
<td>$m_2$</td>
</tr>
<tr>
<td>$w_3$</td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$m_1$</td>
<td>$m_2$</td>
<td>$m_3$</td>
</tr>
</tbody>
</table>

Stable matching:
An antiedge is a pair \((i, j), (k, l)\) with \(i \neq k \in M; j \neq l \in W\) s.t. \(X_{ij} = X_{kl} = 1\). Then \(X\) is stable iff

\[(ij), (kl) \text{ is anti-edge } \Rightarrow \begin{cases} d_{ilj}d_{lik} = 0 \\
d_{jki}d_{kjl} = 0 \end{cases} \] (2)

Define: \(d_{ilj} = 1(w_l >_m w_i, w_j)\)
**Structure of Aggregate Stable Matchings**

$X$ dominates $X'$ if

$$X_{ij} = 0 \Rightarrow X'_{ij} = 0.$$  

Proposition

*Let $X$ be a stable aggregate matching. If $X'$ is an aggregate matching, and $X$ dominates $X'$, then $X'$ is stable.*

So all stable matchings are described by set of maximal stable matchings.
(Trivial) Algorithm for maximal stable matching.

Given \((V, E)\)

- Enumerate vertices, \(V = \{1, 2, \ldots, N\}\).
- \(X^0 = \) identically zero.
- For \(v \in V\), \(X^{v-1}\), define \(X^v\) by changing entry \(v\).
  - \(X^v_v = 1\) if 1 is not violated
  - \(X^v_v = 0\) o/w.
- Let \(X = X^N\).
Model

Proposition

Let $X$ be an individual stable matching.

1. If $K = L = 3$ then $X$ is not a maximal stable matching.

2. If $K > 3$, $L > 3$ and $X$ is a maximal stable matching, then one of the following two possibilities must hold:
   
   2.1 For all $(i, j)$, the submatching $X^{-(i,j)}$ is a maximal stable matching in the $-(i,j)$ submarket.
   
   2.2 There is $(h, l)$ with $X_{hl} = 1$, and a maximal stable matching $\tilde{x}$, for which $\tilde{x}_{h,j} = \tilde{x}_{i,l} = 0$ for all $i$ and $j$. 
Rationalizable Matchings

Given: \( M = \{m_1, \ldots, m_K\} \) and \( W = \{w_1, \ldots, w_L\} \).

\( X \) is *rationalizable* if \( \exists \) preference profile \( > \) s.t. \( X \) is a stable aggregate matching in \( \langle M, W, > \rangle \).
Rationalizable Matchings

Given \( X \):
Define a “lattice graph” \((V, L)\) on the matrix \( X \).

- Vertices: \((i, j)\) s.t. \( X_{i,j} = 1 \)
- Edge \((i, j) \rightarrow (i', j')\) if share a column or a row.
Example

Let $X$ be

$$
\begin{pmatrix}
1 & 1 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0
\end{pmatrix}.
$$

$(V, L)$ is:
Rationalizable Matchings

Theorem
An aggregate matching $X$ is rationalizable if and only if the associated graph $(V, L)$ has not two connected distinct minimal cycles.
Rationalizable Matchings

Let $X$ be

\[
\begin{pmatrix}
1 & 1 & 0 \\
1 & 1 & 1 \\
1 & 0 & 1
\end{pmatrix}.
\]

$(V, L)$ is:
Rationalizable Matchings

The following are two minimal cycles that are connected.

\[
\begin{array}{ccc}
1 & 1 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0
\end{array}
\]
Idea: necessity.

Canonical cycle:
Idea: necessity.

Preferences $\Rightarrow$ orientation of edges:

```
1  ->  1
|    |
|    |
1 ---- 1
```
Idea: necessity.

Preferences $\Rightarrow$ orientation of edges:

```
1 → 1
\|    \|
1     1
```
Idea: necessity.

Preferences $\Rightarrow$ orientation of edges:

```
1 ----> 1
|     |
|     |
1 ----> 1
```
Idea: necessity.

Preferences $\Rightarrow$ orientation of edges:

```
1 ⟷ 1
   |
   v
1 ⟷ 1
```
Idea: necessity.

Preferences ⇒ orientation of edges:

\[
\begin{array}{c}
1 \quad \rightarrow \quad 1 \\
\downarrow \\
1 \quad \leftarrow \quad 1
\end{array}
\]
Idea: necessity.

Preferences $\Rightarrow$ orientation of edges:

```
1 ←→ 1
  ↑   ↓
1  1
```
Idea: necessity

So a cycle must be oriented as a flow.
Idea: necessity
Idea: necessity
Idea: necessity
Idea: necessity

- Orientation of a minimal path must then point away from a cycle.
Idea: necessity

- Orientation of a minimal path must then point **away** from a cycle.
- Two connected cycles $\Rightarrow$ connecting path must point away from both.
Idea: necessity

Subsequent edges in a minimal path must be at a right angle:
Idea: necessity

Two connected cycles $\Rightarrow$ connecting path must point away from both.
So connected path does (at some point):

\[ 1 \rightarrow 1 \rightarrow 1 \]
\[ 1 \uparrow \downarrow 1 \]

$\Rightarrow$ no two connected cycles.
Idea: sufficiency

- Given $X$, construct an orientation of $(V, L)$.
- Use orientation to define preferences.
Idea: sufficiency

- Given $X$, construct an orientation of $(V, L)$.
- Use orientation to define preferences.
- Decompose $(V, L)$ in connected components. At most one cycle in each.
Idea: sufficiency

- Given $X$, construct an orientation of $(V, L)$.
- Use orientation to define preferences.
- Decompose $(V, L)$ in connected components. At most one cycle in each.
- Orient cycle as a “flow,” and paths as “flows” pointing away from cycle.
- Uniqueness of cycle within a component ensures transitivity.
Surplus: $\alpha_{i,j} \in \mathbb{R}$.

Surplus generated by matchings of types $i$ and $j$ in $X$ is $X_{i,j}\alpha_{i,j}$. 

X is *TU-rationalizable* by a matrix of surplus $\alpha$ if $X$ is unique sol. to:

\[
\begin{align*}
\max_{\tilde{X}} \sum_{i,j} \alpha_{i,j} \tilde{X}_{i,j} \\
\text{s.t.} \begin{cases} \\
\forall j \sum_i \tilde{X}_{i,j} = \sum_i X_{i,j} \\
\forall i \sum_j \tilde{X}_{i,j} = \sum_j X_{i,j} \end{cases}
\end{align*}
\]
Theorem

An aggregate matching $X$ is TU-rationalizable if and only if the associated graph $(V, L)$ contains no minimal cycles.

Corollary

If an aggregate matching $X$ is TU-rationalizable, then it is rationalizable.
Estimation

Parametrized preferences:

\[ u_{ij} = Z_{ij} \beta + \varepsilon_{ij}, \]  

\[ d_{ijk} \equiv 1(u_{ij} \geq u_{ik}). \]
Recall:

An **antiedge** is a pair \((i, j), (k, l)\) with \(i \neq k \in M\); \(j \neq l \in W\) s.t. \(X_{ij} = X_{kl} = 1\).

Then \(X\) is stable iff

\[(ij), (kl)\) is anti-edge \(\Rightarrow \begin{cases} d_{ilj}d_{lik} = 0 \\ d_{jki}d_{kjl} = 0 \end{cases} \] (5)
Estimation

\[ Pr((ij), (kl) \text{ antiedge}) \leq (1 - Pr(d_{ilj} d_{lik} = 1))(1 - Pr(d_{jki} d_{kjl} = 1)) = Pr(d_{ilj} d_{lik} = 0, d_{jki} d_{kjl} = 0). \]
Estimation

\[ Pr((ij), (kl) \text{ antiedge}) \leq (1 - Pr(d_{ij}d_{lik} = 1))(1 - Pr(d_{jki}d_{kjl} = 1)) = Pr(d_{ilj}d_{lik} = 0, d_{jki}d_{kjl} = 0). \]

Gives a moment inequality:

\[ \mathbb{E} \left[ g_{ijkl}(X_t; \beta) \right] \leq 0. \]

The identified set is defined as

\[ \mathbb{B}_0 = \{ \beta : \mathbb{E}g_{ijkl}(X_t; \beta) \leq 0, \forall i, j, k, l \} . \]
Estimation

Sample analog

$$\frac{1}{T} \sum_t 1((ij), (kl) \text{ is antiedge in } X_t) - 1$$

$$+ \Pr(d_{il}d_{ik} = 0, d_{jki}d_{kjl} = 0; \beta)$$

$$= \frac{1}{T} \sum_t g_{ijkl}(X_t; \beta).$$
Problem: condition in the theorem is violated. Hence no preferences (no betas) rationalize data.
Problem: condition in the theorem is violated. Hence no preferences (no betas) rationalize data.

We relax the model (∃ other solutions).
Estimation – Relaxation of the model

A blocking pair may not form.

\[ \delta_{ijkl} = P(\text{types } (i, j), (k, l) \text{ communicate}). \]

Idea: a BP forms only when types \((i, j), (k, l)\) communicate.
Estimation – Relaxation of the model

Stability inequalities become:

\[
\begin{pmatrix}
(ij), (kl) \text{ is anti-edge} \\
(ij), (kl) \text{ meet}
\end{pmatrix} \implies \begin{cases}
d_{ij}d_{ik} = 0 \\
d_{jki}d_{kjl} = 0
\end{cases}
\]

Assume: two events are independent.
Modified moment inequality:

$$Pr((ij), (kl) \text{ antiedge}) \leq \frac{Pr(d_{ilj}d_{lik} = 0, d_{jki}d_{kjl} = 0; \beta)}{\delta_{ijkl}}$$
Estimation – Relaxation of the model

We suppose $\delta_{ijkl}$ depends on the number of agents in each type.

$$\delta_{ijkl}^t = 1 - (1 - p)^{x_{TM_i,TW_j} + x_{TM_k,TW_l}}$$

where $p$ is the prob. that two agents meet.
Estimation – Relaxation of the model

As a result: we are weighting anti-edges by how many agents are involved.
Sample moment inequalities

\[
\frac{1}{T} \sum_t g_{ijkl}(X_t; \beta) = \left( \frac{1}{T} \sum_t 1((ij), (kl) \text{ is antiedge in } X_t) \ast \delta_{ijkl}^t \right) \\
- (1 - Pr(d_{ilj} = 1; \beta_{1,2}) Pr(d_{lik} = 1; \beta_{3,4})) \\
(1 - Pr(d_{jki} = 1; \beta_{3,4}) Pr(d_{kjl} = 1; \beta_{1,2}))
\]

for all combinations of pairs, \((i, j)\) and \((k, l)\).
Specification of Utilities

\[ \text{Utility}^m = \beta_1 |\text{Age}^m - \text{Age}^w|^- + \beta_2 |\text{Age}^m - \text{Age}^w|^+ + \epsilon^m \]
\[ \text{Utility}^w = \beta_3 |\text{Age}^m - \text{Age}^w|^- + \beta_4 |\text{Age}^m - \text{Age}^w|^+ + \epsilon^w \]
Results: Identified set.

We describe the identified set for different values of $p$.

if $p$ is too high $\Rightarrow$ identified set $= \emptyset$.
if $p$ is too low $\Rightarrow$ identified set is everything.

Idea: choose high $p$ as discipline on our estimates.
Results: Identified set.

Table: Unconditional Bounds of $\beta$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\beta_1$ min</th>
<th>$\beta_1$ max</th>
<th>$\beta_2$ min</th>
<th>$\beta_2$ max</th>
<th>$\beta_3$ min</th>
<th>$\beta_3$ max</th>
<th>$\beta_4$ min</th>
<th>$\beta_4$ max</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0006</td>
<td>-2</td>
<td>2</td>
<td>-2</td>
<td>2</td>
<td>-2</td>
<td>2</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>0.0007</td>
<td>-2</td>
<td>2</td>
<td>-2</td>
<td>1.6</td>
<td>-2</td>
<td>2</td>
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<td>1.2</td>
</tr>
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<td>-2</td>
<td>1.2</td>
<td>-1.6</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>-1.4</td>
<td>0.6</td>
</tr>
<tr>
<td>0.0009</td>
<td>-1.2</td>
<td>0.4</td>
<td>-0.6</td>
<td>0.6</td>
<td>-2</td>
<td>0.4</td>
<td>-0.6</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Joint identified sets

- More anti-edges below the diagonal, where $age^m > age^w$.
- More “downward-sloping” anti-edges than “upward-sloping” ones.

**Downward-sloping anti-edge:**

$$(i, j) \rightarrow (i, l) \rightarrow (k, j) \rightarrow (k, l)$$

**Upward-sloping anti-edge:**

$$(k, j) \rightarrow (k, l) \rightarrow (i, j) \rightarrow (i, l)$$
$\beta_3 = -1$ and $\beta_4 = 1$
$\beta_3 = 0$ and $\beta_4 = 1$
$\beta_3 = 1$ and $\beta_4 = 1$
$\beta_3 = -1$ and $\beta_4 = 0$
\( \beta_3 = 0 \) and \( \beta_4 = 0 \)
\( \beta_3 = 1 \) and \( \beta_4 = 0 \)
$\beta_3 = -1$ and $\beta_4 = -1$
\( \beta_3 = 0 \) and \( \beta_4 = -1 \)
\( \beta_3 = 1 \) and \( \beta_4 = -1 \)
Related literature

TU model:

▶ Theory: Shapley-Shubik (1971)

NTU model:

▶ Theory: Gale-Shapley (1967), Knuth (1971)
▶ Econometrics: Dagsvik (2000)

Aggregate NTU matching: no theoretical results; Dagsvik develops estimation techniques.
Conclusions

- First theoretical characterization of stable aggregate matchings.
- Testable implications of stability for aggregate matchings.
- Econometric estimation technique ← based on moment inequalities implied by stability.
- Illustration to US marriage data.