SS 214, HMWK 4, Due Monday, Feb 13, 2006

1. In the Black-Scholes model write down the PDE for the price of the option with payoff

$$\left(S^2(T) - \frac{1}{T}\int_0^T S^2(t)dt\right)^+$$

and the expression for the replicating portfolio in terms of the price function (both for stock holdings and bank holdings). Hint: Think of this as an exchange option on two assets. The price will be a function of t and two variables, so the PDE will bee two-dimensional.

2. Consider the following model under the risk-neutral probability, with interest rate equal to zero:

$$dS_1(t) = S_1(t)\sigma_1 dB_1^*(t)$$

$$dS_2(t) = S_2(t)[\gamma_1 dB_1^*(t) + \gamma_2 dB_2^*(t)]$$

where B_1^* and B_2^* are independent.

(a) Find $d(S_2/S_1)$.

(b) Find the Brownian Motions corresponding to the probability under which S_2/S_1 is a martingale.

(c) Find the price of the claim with payoff equal to the product $S_1(T)S_2(T)$. Hints: If you wish, you can use (b) and the change of numeraire technique, with S_1 as the numeraire; or, you can price directly under the usual risk-neutral probability.

3. Assume that the exchange rate process $Q(t)$, denoting the value in dollars of one unit of foreign currency at time t, is given by

$$dQ = Q[\mu_Q dt + \sigma_Q dB] \quad .$$

We consider trading in the risk-free accounts of the two currencies, with interest rate r for the domestic, and r_f for the foreign account. Find $d\tilde{Q}$, where $\tilde{Q}$ is the dollar value of one unit of the foreign account: $\tilde{Q}(t) = Q(t)e^{r_f t}$. Use this to argue that a self-financing wealth process (in domestic currency) corresponding to π dollars held in the foreign risk-free account, and the rest in the domestic risk-free account, has the dynamics

$$dW = [rW + \pi(\mu_Q + r_f - r)]dt + \pi\sigma_Q dB. \tag{1}$$

Find the relation between B and B^*, where B^* is the Brownian motion under the probability measure which makes the discounted wealth We^{-rt} a martingale, and write dQ under this measure.

4. Consider the American contingent claim with payoff $\sqrt{S_\tau}$. Assume the interest rate is zero. At what time should the holder of this claim exercise? What is the price equal to when $r = 0$?

5. Read Chapter 9.