SS201c Assignment 7 (due in one week)

1. Say that a PAR is a “weighted voting rule” if there exists a vector \( w \in \mathbb{R}^N_+ \) and \( \alpha \) such that for all \( \theta \in \Theta \), \( xP(\theta)y \) if and only if

\[
\sum_{\{i \in N : xP_i(\theta)y\}} w_i > \alpha.
\]

i) When are weighted voting rules well-defined?
ii) What examples of weighted voting rules have we studied so far in class?
iii) Which axioms do the weighted voting rules satisfy?
iv) Can you provide a characterization of the weighted voting rules?

2. We have always assumed a PAR to map to the set of complete binary relations (so that social preference is always complete). Reconsider the proof of the Arrow’s Theorem. Suppose we require that the PAR is always transitive, satisfies the Pareto condition, and IIA; yet social preference need not be complete.

a) Give an example of a non-dictatorial incomplete PAR satisfying Arrow’s remaining axioms.

b) Where exactly is completeness used in the proof of Arrow’s Theorem? Does this help you to establish a characterization of PAR’s satisfying all of Arrow’s axioms except for completeness?

3. Pareto, in Arrow’s Theorem, is always formulated as \( xP_i(\theta)y \) for all \( i \) implies \( xP_j(\theta)y \). In other words, it is formulated for strict preference. Suppose we require instead the condition that \( xR_i(\theta)y \) for all \( i \) implies \( xR_j(\theta)y \). What other PAR’s satisfy the remaining Arrow axioms and this version of Pareto? What about dropping Pareto altogether?