1. Look at the di Finetti’s theorem in Kreps (Theorem 11.4, p. 158) and compare it to the special case result he states on p. 154. Explain how to derive this special case from the general result.

2. Explain what Kreps means when he states that it is necessary that \( \{ Z_n \} \) be infinite for the theorem to be true. In other words, define a probability space and a finite list of random variables \( \{ Z_n \} \) having the exchangeability property of Kreps for which the conclusion of di Finetti’s Theorem does not hold.

3. An agent that satisfies Savage’s P2 admits a well-defined notion of ‘conditional preference,’ whereby for acts \( f, g \in \mathcal{F} \) and all events \( A, fR_A g \) if and only if there exists \( h \) such that

\[
\begin{bmatrix}
    f(s) & \text{for } s \in A \\
    h(s) & \text{for } s \notin A 
\end{bmatrix}
\sim
\begin{bmatrix}
    g(s) & \text{for } s \in A \\
    h(s) & \text{for } s \notin A 
\end{bmatrix}.
\]

Prove that if \( A \) and \( B \) are disjoint events, then \( fR_A g \) and \( fR_B g \) imply \( fR_{A \cup B} g \). Which of Savage’s axioms are necessary for this conclusion?

4. Imagine a decision maker who has two choices: a choice as to how much money to invest in a particular opportunity, as well as a choice over acts. Such a decision maker’s choice space is represented by \( \mathbb{R}_+ \times \mathcal{F} \). The decision maker’s preferences over such pairs can be represented as follows:

\[
U(\alpha, f) = \int_S u_\alpha(f(s)) \, d\mu(s)
\]

for some probability measure \( \mu \) (thus, for fixed \( \alpha \), the decision maker obeys Savage’s axioms, say).

If the decision maker has to make a choice as to how much to invest before realizing the state of the world, then her utility from any act \( f \in \mathcal{F} \) can be represented by

\[
W(f) = \sup_{\alpha \in \mathbb{R}_+} \int_S u_\alpha(f(s)) \, d\mu(s).
\]

Show that the preference relation represented by \( W \) does not generally obey the Savage axioms (thus, find an example of such a preference that violates at least one of Savage’s axioms–NOT P1, P5, or P6). However, such a decision maker appears to have a subjective probability in mind when making decisions. How can you explain this?