Homework 2

1. Failure of the first fundamental welfare theorem with overlapping generations

Construct the following “overlapping generations” economy. Let \( t = \{0, 1, 2, \ldots \} \) be time, indexed by the natural numbers. At every period in time, there is one commodity that agents may consume. Moreover, at every period of time, an agent is born. Each agent lives for only two periods.

We model this as follows: the agent set \( N = \{0, 1, 2, \ldots \} \). Consumption space \( \mathbb{R}_+^{\infty} = \prod_{t=0}^{\infty} \mathbb{R}_+ \). Agent \( t \) has an endowment \( \omega_t^t = 3/4, \omega_t^{t+1} = 1/4 \). For all \( j \neq t, t+1, \omega_j^t = 0 \). This models the idea that in period \( t \), agent \( t \) has 3/4 units of the commodity, and in period \( t+1 \), she is endowed with 1/4. Her preferences are represented by the utility function \( u_t(x_0, x_1, \ldots, x_t, x_{t+1}, x_{t+2}, \ldots) = x_t x_{t+1} \). (Thus, she only cares about consumption in the periods she is alive).

a) Demonstrate that the price vector and allocation \((p^*, x^*)\) is a Walrasian equilibrium, where \( p^*_t = 3^t \), and for all agents \( t, x_i^t = 3/4, x_i^{t+1} = 1/4 \), and for all \( j \neq t, t+1, x_j^t = 0 \).

b) Show that the equilibrium allocation \( x^* \) is Pareto inefficient by demonstrating an allocation \( y^* \) that all agents strictly prefer to \( x^* \).

c) Going through the proof of the first fundamental welfare theorem, find out where it breaks down.

2. Read Varian 17.6-17.7. Complete exercise 17.3.

3. Say that a price vector and allocation \((p^*, x^*)\) is a Walrasian quasiequilibrium for an economy \((N, \{\succeq_i, \omega_i\})\) if i) for all \( i \in N \), if \( x^i \succeq_i x_\ast^i \), then \( p^* \cdot x^i \geq p^* \cdot x^\ast_i \), and ii) \( \sum_{i \in N} x_i^t \leq \sum_{i \in N} \omega_i \).

a) Show that if \((p^*, x^*)\) is a Walrasian equilibrium, then it is a Walrasian quasiequilibrium.

b) In Varian’s exercise 17.3, verify that there are prices which support \( x^* \) as a Walrasian quasiequilibrium.

c) Find out where the proof of the Second Fundamental Welfare Theorem breaks down for Arrow’s example. Show that with the same assumptions on preferences, we can always conclude that an efficient allocation can be supported as a Walrasian quasiequilibrium.

3. Complete Varian 17.5 (here, equal division refers to the allocation \( x^* = \frac{1}{|N|} \)).

4. Complete Varian 17.8