A Measure of Bizarreness

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Abstract

We introduce a path-based measure of convexity to be used in assessing the compactness of legislative districts. Our measure is the probability that a district will contain the shortest path between a randomly selected pair of its’ points. The measure is defined relative to exogenous political boundaries and population distributions.

JEL classification: D72, K00, K19

1 Introduction

Hundreds of years ago, legislators discovered that the ultimate composition of a legislature is not independent of the means through which district boundaries are drawn. Hoping to stave off unemployment, legislators learned to master the art of gerrymandering: carefully drawing district boundaries to increase their electoral chances and political power. Like certain forms of painting and ballet, this art became more and more noticeable by the odd shapes it produced.1

Past attempts on the part of political reformers to fight gerrymandering have led to the introduction of vague legal restrictions requiring districts to be “compact and contiguous.”2 The vagueness of these legal terms has led to the introduction of several methods to measure district “compactness.”3 However,
none of these methods is widely accepted, in part because of problems identified by Young [23] and Altman [1]. We argue that these laws were introduced with the aim of eliminating bizarrely shaped districts. To this end we introduce a measure of “bizarreness.”

The primary problem with gerrymandering is that elections become less competitive when legislators draw district lines to strengthen their reelection chances. The “bizarre” shapes which result are merely a side-effect of this process. Reformers have focused on compactness because, while there is no consensus as to how district boundaries should be drawn, bizarre shapes are clearly identifiable as a symptom of gerrymandering.

Part of the difficulty of defining a measure of compactness is that there are many conflicting understandings of the concept. According to one view the compactness standard exists to eliminate elongated districts. In this sense a square is more compact than a rectangle, and a circle may be more compact than a square. According to another view compactness exists to eliminate bizarrely shaped districts. According to this view a rectangle-shaped district would be better than a district shaped like a Rorschach blot.

We follow the latter approach. While it may be preferable to avoid elongated districts, the classic sign of a heavily-gerrymandered district is bizarre shape. To the extent that elongation is a concern, it should be studied with a separate measure. These are two separate issues, and there is no obvious way to weigh tradeoffs between bizarreness and elongation.

The basic principle of convexity requires a district to contain the shortest path between every pair of its’ points. Circles, squares, and triangles are examples of convex shapes, while hooks, stars, and hourglasses are not. (See Figure 1.) The most striking feature of bizarrely shaped districts is that they are extremely non-convex. (See Figure 2.) We introduce a measure of convexity with which to assess the bizarreness of the district.

The path-based measure we introduce is the probability that a district will contain the shortest path between a randomly selected pair of its’ points. This measure will always return a number between zero and one, with one being

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4 However, the U.S. Supreme Court has held that “bizarre shape and noncompactness” of districts is not only evidence of unconstitutional manipulation of district boundaries but also “part of the constitutional problem.” See Shaw v. Reno, 509 U.S. 630 (1993); Bush v. Vera, 517 U.S. 952, 959 (1996).

5 Writing for the majority in Bush v. Vera, Justice O’Connor referred to “bizarre shape and noncompactness” in a manner which suggests that the two are synonymous, or at least very closely related. If so then a compact district is one without a bizarre shape, and a measure of compactness is a measure of bizarreness.

6 The majority opinion in Shaw v. Reno noted that one district had been compared to a “Rorschach ink blot test” by a lower court and a “bug splattered on a windshield” in a major newspaper. 509 U.S. at 633.

7 Note that the term gerrymander was coined in 1812 by a political cartoonist who sought to link then-Massachusetts Governor Elbridge Gerry to a salamander-shaped legislative district. Had the controversial district merely resembled a rectangle, the process of district manipulation would possibly be referred to as a gerytangle.

8 Elongated districts are not always undesirable. See Figure 5.

9 A version of this measure was independently discovered by Ehud Lehrer [10].
perfectly convex. To understand how our measure works, consider a district containing two equally sized towns connected by a very narrow path, such as a road. (See Figure 3(a).) Our method would assign this district a measure of approximately one-half. A district containing \( n \) towns connected by narrow paths would be assigned a measure of approximately \( 1/n \).\(^{10}\) (See Figure 3(b).)

Ideally, a measure of compactness should consider the distribution of the population in the district. For example, consider the two arch-shaped districts depicted in Figure 4. The districts are of identical shape, thus the probability that each district will contain the shortest path between a randomly selected pair of its’ points is the same. However, the populations of these districts are distributed rather differently. The population of district A is concentrated near the bottom of the arch, while that of district B is concentrated near the top. The former district might represent two communities connected by a large forest, while the second district might represent one community with two forests attached.

Population can be incorporated by using the probability that a district will contain the shortest path between a randomly selected pair of its’ residents. In practice our information will be more limited — we will not know the exact location of every resident, but only the populations of individual census blocks. We can solve this problem by weighting points by population density. The population-weighted measure of district A is approximately one-half, while that of district B is nearly one.\(^{11}\)

Alternatively one might use the reciprocal, where the measure represents the equivalent number of disparate communities strung together to form the district. The reciprocal will always be a number greater or equal to one, where one is perfectly convex. A district containing \( n \) towns connected by narrow paths would be assigned a measure of approximately \( n \).

\(^{10}\)Alternatively one might use the reciprocal, where the measure represents the equivalent number of disparate communities strung together to form the district. The reciprocal will always be a number greater or equal to one, where one is perfectly convex. A district containing \( n \) towns connected by narrow paths would be assigned a measure of approximately \( n \).

\(^{11}\)Note that the population-weighted approach measures the compactness of the districts’ populations, and not the compactness of their shapes. A district may have a perfect score even though it has oddly shaped boundaries in unpopulated regions. The ability to draw bizarre boundaries in unpopulated regions is of no help to potential gerrymanderers.
One potential problem is that some districts may be oddly shaped simply because the states in which they are contained are non-convex. Consider, for example, Maryland’s Sixth Congressional District (shown in Figure 5 in gray). Viewed in isolation, this district is very non-convex — the western portion of the district is almost entirely disconnected from the eastern part. However, the odd shape of the district is a result of the state’s boundaries, which are fixed. We solve this problem by measuring the probability that a district will contain the shortest path in the state between a randomly selected pair of its’ points. The adjusted measure of Maryland’s Sixth Congressional District would be close to one.

Our measure considers whether the shortest path in a district exceeds the shortest path in the state. Alternatively, one might wish to consider the extent...
to which the former exceeds the latter. We introduce a parametric family of measures which vary according to the degree that they “penalize” deviations from convexity. At one extreme is the measure we have described; at the other is the degenerate measure, which gives all districts a measure of one regardless of their shape.

1.1 Related Literature

1.1.1 Individual District Compactness Measures

A variety of compactness measures have been introduced by lawyers, social scientists, and geographers. Here we highlight some of basic types of measures and discuss some of their weaknesses. A more complete guide may be found in surveys by Young [23], Niemi et. al. [12], and Altman [1].

Most measures of compactness fall into two broad categories: (1) dispersion measures and (2) perimeter-based measures. Dispersion measures gauge the extent to which the district is scattered over a large area. The simplest dispersion measure is the length-to-width test, which compares the ratio of a district’s length to its’ width. Ratios closer to one are considered more compact. This
test has had some support in the literature, most notably Harris [7].

Another type of dispersion measure compares the area of the district to that of an ideal figure. This measure was introduced into the redistricting literature by Reock [15], who proposed using the ratio of the area of the district to that of the smallest circumscribing circle. A third type of dispersion measure involves the relationship between the district and its’ center of gravity. Measures in this class were introduced by Boyce and Clark [2] and Kaiser [9]. The area-comparison and center of gravity measures have been adjusted to take account of district population by Hofeller and Grossman [8], and Weaver and Hess [22], respectively.

Dispersion measures have been widely criticized, in part because they consider districts reasonably compact as long as they are concentrated in a well-shaped area. (See Young [23].) We point out a different (although related) problem. Disconnection-sensitivity requires the measure to consider the combined region less compact than at least one of the original communities. None of the dispersion measures are disconnection-sensitive. An example is shown in Figure 6.

Perimeter measures use the length of the district boundaries to assess compactness. The most common perimeter measure, associated with Schwartzberg [18], involves comparing the perimeter of a district to its’ area. Young [23] objected to the Schwartzberg measure on the grounds that it is overly sensitive to small changes in the boundary of a district. Jagged edges caused by the arrangement of census blocks may lead to significant distortions. While a perfectly square district will receive a score of 0.785, a square shape superimposed upon a diagonal grid of city blocks will have a much longer perimeter and a lower score, as shown in Figure 7(a).

Figure 7 shows four shapes, arranged according to the Schwartzberg ordering from least to most compact. Taylor [19] introduced a measure of indentation which compared the number of reflexive (inward-bending) to non-reflexive (outward-bending) angles in the boundary of the district. Taylor’s measure is similar to ours in that it is a measure of convexity. Figure 8 shows six districts and their Taylor measures, arranged from best to worst.

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12 The length-to-width test seems to have originated in early court decisions construing compactness statutes. See *In re Timmerman*, 100 N.Y.S. 57 (N.Y. Sup. 1906).

13 The length-width measure is the ratio of width to length of the circumscribing rectangle with minimum perimeter. See Niemi et. al. [12]. All measures are transformed so that they range between zero and one, with one being most compact. The Boyce-Clark measure is $q = \frac{1}{1 + bc}$, where $bc$ is the original Boyce-Clark measure [2]. The Schwartzberg measure used is the variant proposed by Polsby and Popper [14] (originally introduced in a different context by Cox [3]), or $(\frac{1}{sc})^2$, where $sc$ is the measure used by Schwartzberg [18].

14 This idea was first introduced by Cox [3] in the context of measuring roundness of sand grains. The idea first seems to have been mentioned in the context of district plans by Weaver and Hess [22] who used it to justify their view that a circle is the most compact shape. Polsby and Popper [14] have also supported the use of this measure.

15 The score of the resulting district will decrease as the city blocks become smaller, reaching 0.393 in the limit.
Figure 6: District II is formed by connecting district I to a copy of itself. Disconnection-sensitivity implies that I is more compact.

**Compactness Measures**

<table>
<thead>
<tr>
<th>Dispersion Measures</th>
<th>District:</th>
<th>I</th>
<th>II</th>
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<tr>
<td>Length-Width</td>
<td></td>
<td>0.63</td>
<td>1.00</td>
</tr>
<tr>
<td>Area to Circumscribing Circle</td>
<td></td>
<td>0.32</td>
<td>0.44</td>
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<td>Area to Convex Hull</td>
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<td>0.70</td>
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<tr>
<td>Boyce-Clark</td>
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<table>
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<th>Other Measures</th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Path-Based Measure</td>
<td></td>
<td>0.84</td>
<td>0.42</td>
</tr>
<tr>
<td>Schwartzberg</td>
<td></td>
<td>0.29</td>
<td>0.14</td>
</tr>
<tr>
<td>Taylor</td>
<td></td>
<td>0.40</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Lastly, Schneider [16] introduced a measure of convexity using Minkowski addition. For more on the relationship between convex bodies and Minkowski addition, see Schneider [17].

1.1.2 Districting-Plan Compactness Measures

In addition to these measures of individual legislative districts, several proposals have been introduced to measure entire districting plans. The “sum-of-the-perimeters” measure, found in the Colorado Constitution, is the “aggregate linear distance of all district boundaries.”\(^{16}\) Smaller numbers indicate greater compactness. An alternative method was introduced by Papayanopoulous [13]. His proposal can be described through a two-stage process. First, in each district, the sum total of the distances between each pair of residents is calculated. The measure for the plan is then the sum of these scores across the districts. Smaller numbers again indicate greater compactness. More recently, Fryer and Holden [5] have proposed a related measure which uses quadratic distance and which is normalized so that an optimally compact districting plan has a score of one.

A potential problem, raised by Young [23], is that these measures penalize

\(^{16}\)Colo. Const. Art. V, Section 47
deviations in sparsely populated rural areas much more severely than deviations in heavily populated urban areas. For example, Figure 9 shows five potential districting plans for a four-district state with sixteen equally sized population centers (represented by dots). The upper portion of the state represents an urban area with half of the population concentrated into one-seventeenth of the land. Papayanopoulos scores are given, although we note that the sum-of-the-perimeters and Fryer-Holden measures give identical ordinal rankings of these districting plans.

According to these measures, the ideal districting plan divides the state into four squares (Figure 9(a)). The plan with triangular districts is less compact (Figure 9(b)), and the plan with wave-shaped districts fares the worst (Figure 9(c)). However, the measure is more sensitive to deviations in areas with lower population density. The plan in Figure 9(d), which divides the rural area into perfect squares and the urban area into low-scoring wave-shape districts, is considered more compact than the plan in Figure 9(e), which divides the rural area into triangles and the urban area into perfect squares.

An alternative approach is to rank state-wide districting plans using the
scores assigned to individual districts. Examples include the *utilitarian* criterion, which is the average of the districts’ scores (see Papayanopoulos [13]), and the *maxmin* criterion, which is simply the lowest of the scores awarded the districts under the plan. This approach allows for the ranking of both individual districts and entire districting plans as required by Young [23].

The ideal criterion depends in large part on the individual district measure with which it is used. We advocate the use of the maxmin criterion with our path-based measure on the grounds that it will restrict gerrymandering the most. The maxmin criterion is also consistent with the U.S. Supreme Court’s focus on analyzing individual districts as opposed to entire districting plans. However, if some districts must necessarily be non-compact (a common problem with the Schwartzberg measure) then the utilitarian criterion may be more appropriate.

### 1.1.3 Other literature

Vickrey [21] showed that restrictions on the shape of legislative districts are not necessarily sufficient to prevent gerrymandering. In Vickrey’s example there is a rectangular state in which support for the two parties (white and gray) are distributed as shown in Figure 10. With one district plan, the four legislative seats are divided equally; with the other district plan, the gray party takes all four seats. In both plans, the districts have the same size and shape.

Compactness measures have been touted both as a tool for courts to use in determining whether districting plans are legal and as a metric for researchers to use in studying the extent to which districts have been gerrymandered. Other methods exist to study the effect of gerrymandering – the most prominent of

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17This focus might stem from the Court’s understanding of the right to vote as an individual right, and not a group or systemic right. This understanding may have influenced other measures used in the redistricting context, such as the ‘total deviation’ test. See Edelman [4].
these is the seats-votes curve, which is used to estimate the extent to which the district plan favors a particular party as well as the responsiveness of the electoral system to changes in popular opinion. For more see Tufte [20].

2 The model and proposed family of measures

2.1 The Model and Notation

Let $\mathcal{K}$ be the collection of compact sets in $\mathbb{R}^n$ whose interiors are path-connected (with the usual Euclidean topology) and which are the closure of their interiors. Elements of $\mathcal{K}$ are called parcels. For any set $Z \subseteq \mathbb{R}^n$ let $\mathcal{K}_Z \equiv \{K \in \mathcal{K} : K \subseteq Z\}$ denote the restriction of $\mathcal{K}$ to $Z$.

Consider a path-connected set $Z \subseteq \mathbb{R}^n$ and let $x, y \in Z$. Let $\mathcal{P}_Z(x, y)$ be the set of continuous paths $g : [0, 1] \to Z$ for which $g(0) = x$, $g(1) = y$, and $g([0, 1]) \subseteq Z$. For any path $g$ in $\mathcal{P}_Z(x, y)$, we define the length $l(g)$ in the usual way. We define the distance from $x$ to $y$ within $Z$ as:

$$d(x, y; Z) \equiv \inf_{g \in \mathcal{P}_Z(x, y)} l(g).$$

We define $d(x, y; \mathbb{R}^n) \equiv d(x, y)$. This is the Euclidean metric.

Let $\mathcal{F}$ be the set of density functions $f : \mathbb{R}^n \to \mathbb{R}_+$ such that $\int_K f(x)dx$ is finite for all parcels $K \in \mathcal{K}$. Let $f_u \in \mathcal{F}$ refer to the uniform density. For any density function $f \in \mathcal{F}$, let $F$ be the associated probability measure so that $F(K) \equiv \int_K f(x)dx$ represents the population of parcel $K$.

We measure compactness of districts relative to the borders of the state in which they are located. Given a particular state $Z$, we allow the measure to consider two factors: (1) the boundaries of the legislative district, and (2) the population density. Thus, a measure of compactness is a function $s_Z : \mathcal{K}_Z \times \mathcal{F} \to \mathbb{R}_+$. 

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18 That is, suppose $g : [0, 1] \to Z$ is continuous. Let $k \in \mathbb{N}$. Let $(t_0, ..., t_k) \in \mathbb{R}^{k+1}$ satisfy for all $i \in \{0, ..., k - 1\}$, $t_i < t_{i+1}$. Define $l_t(g) = \sum_{i=1}^k \|g(t_i) - g(t_{i-1})\|$. The length (formally, the arc length) of $g$ is then defined as $l(g) = \sup_{k \in \mathbb{N}} \sup_{\{t \in [0, 1] : t_i < t < t_{i+1}\}} l_t(g)$.

19 We define $f_u(x) = 1$.

20 Similarly, the uniform probability measure $F_u(K)$ represents the area of parcel $K$.

21 The state $Z$ is typically chosen from set $\mathcal{K}$ but is allowed to be chosen arbitrary; this allows the case where $Z = \mathbb{R}^n$ and the borders of the state do not matter.

22 The latter factor can be ignored by assuming that the population has density $f_u$. 

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(a) 2 gray, 2 white
(b) All gray, no white
2.2 The basic family of compactness measures

As a measure of compactness we propose to use the expected relative difficulty of traveling between two points within the district. Consider a legislative district \( K \) contained within a given state \( Z \). The value \( d(x, y; K) \) is the shortest distance between \( x \) and \( y \) which can be traveled while remaining in the parcel \( K \). To this end, the shape of the parcel \( K \) makes it relatively more difficult to get from points \( x \) to \( y \) the lower the value of \( d(x, y; K) \).

\[
\frac{d(x, y; Z)}{d(x, y; K)}. \tag{1}
\]

Note that the maximal value that expression (1) may take is one, and its smallest (limiting) value is zero. Alternatively, any function \( g(d(x, y; Z), d(x, y; K)) \) which is scale-invariant, monotone decreasing in \( d(x, y; K) \), and monotone increasing in \( d(x, y; Z) \) is interesting; expression (1) can be considered a canonical example. The numerator \( d(x, y; Z) \) is a normalization which ensures that the measure is affected by neither the scale of the district nor the jagged borders of the state. We obtain a parameterized family of measures of compactness by considering any \( p \geq 0 \); so that \[
\left[ \frac{d(x, y; Z)}{d(x, y; K)} \right]^p
\]
is our function under consideration, defining

\[
\left[ \frac{d(x, y; Z)}{d(x, y; K)} \right]^{\infty} = \begin{cases} 
1, & \text{if } \frac{d(x, y; Z)}{d(x, y; K)} = 1 \\
0, & \text{otherwise}
\end{cases}.
\]

Note that for \( p = 0 \), the measure is degenerate. This expression is a measure of the relative difficulty of travelling from points \( x \) to \( y \). Our measure is the expected relative difficulty over all pairs of points, or:

\[
\mathcal{S}_Z^p(K, f) = \int_K \int_K \left[ \frac{d(x, y; Z)}{d(x, y; K)} \right]^p \frac{f(y)}{f(x)} \left( \frac{1}{F(K)} \right)^2 dy \, dx. \tag{2}
\]

We note a few important cases. First, the special case of \( p = +\infty \) corresponds to the measure described in the introduction, which considers whether the district contains the shortest path between pairs of its points.\(^{23}\) Second, we can choose to measure either the compactness of the districts’ shapes (by letting \( f = f_u \) or the compactness of the districts’ populations (by letting \( f \) describe the true population density). Third, if \( Z = \mathbb{R}^n \), our measure describes the compactness of the legislative district without taking the state’s boundaries into consideration.

\(^{23}\)Mathematically, there may be two shortest paths in a parcel connecting a pair of residents. The issue arises when one state is not simply connected. For example, two residents may live on opposite sides of a lake which is not included in the parcel. In this general case, our measure is the probability that at least one of the shortest paths is contained in the district for any randomly selected pair of residents.
2.3 Discrete Version

Our measure may be approximated by treating each census block as a discrete point. This may be useful if researchers lack sufficient computing power to integrate the expression described in (2).

Let \( Z \in \mathbb{R}^n \) be a state as described in subsection 2.1 and let \( K \in \mathcal{K}_Z \) be a district. Let \( \mathcal{B} \equiv \mathbb{R}^n \times \mathbb{Z}_+ \) be the set of possible census blocks, where each block \( b_i = (x_i, p_i) \) is described by a point \( x_i \) and a non-negative integer \( p_i \) representing its' center and population, respectively. Let \( Z^* \in \mathcal{B}_m \) describe the census blocks in state \( Z \) and let \( K^* \subset Z^* \) describe the census blocks in district \( K \). The approximate measure is given by:

\[
\kappa_{Z^*}(K^*) \equiv \left[ \sum_{b_i \in K^*} \sum_{b_j \in K^*} \left[ \frac{d(x_i, x_j; Z)}{d(x_i, x_j; K)} \right]^p p_i p_j \right]^{-1} \sum_{b_i \in K^*} \sum_{b_j \in K^*} p_i p_j .
\]

3 Data

To illustrate our measure we have calculated scores for all districts in Connecticut, Maryland, and New Hampshire during the 109th Congress. (See Figures 11, 12, and 13.) Because of limitations in computing power we use the approximation described in Section 2.3.

Dark lines represent congressional district boundaries, while shading roughly follows population distributions. Table 1 contains scores for both our path-based measure as well as the Schwartzberg measure. The small numerals in parentheses give the ordinal ranking of the district according to the respective measure. Thus, according to our measure, Connecticut’s Fourth District is the most compact, with a nearly perfect score of 0.977, followed by Maryland’s Sixth District (0.926). Maryland’s Third District is the least compact with a score of 0.140, which makes it slightly less compact than seven equally sized communities connected with a narrow path. (See Figure 3). The Schwartzberg measure ranks Connecticut’s Second District as most compact and Maryland’s First District as least compact. The ordinal rankings agree on fewer than seventy percent of the pairwise comparisons.

The measures give strikingly different results with respect to Connecticut’s Fifth District and Maryland’s Sixth District. Both assign a high rank to one of the districts and a low rank to the other, but the order is reversed. The difference primarily stems from two factors: state boundaries and population.

Maryland’s Sixth District has a very low area-perimeter ratio owing to its’ location in the sparsely populated panhandle of western Maryland and to the

\footnote{To calculate perimeters for the Schwartzberg measure we summed the lengths of the line segments that form the district boundary. In some cases, natural state boundaries (such as the Chesapeake Bay) added significantly to the total length. The Census data we used did not allow us to calculate district tri-junctions (as recommended by Schwartzberg [18]), although it seems unlikely that this would have a substantial effect on the calculation in this case. We do not know whether practitioners use a different method to calculate these scores.}
Table 1: Legislative District Scores

<table>
<thead>
<tr>
<th>District</th>
<th>Measure</th>
<th>Path-Based</th>
<th>Schwartzberg</th>
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<tr>
<td><strong>Connecticut:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>0.609 (8)</td>
<td>0.223 (8)</td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td>0.860 (4)</td>
<td>0.499 (1)</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>0.891 (3)</td>
<td>0.301 (5)</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>0.977 (1)</td>
<td>0.378 (3)</td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>0.481 (12)</td>
<td>0.383 (2)</td>
<td></td>
</tr>
<tr>
<td><strong>Maryland:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>0.549 (10)</td>
<td>0.024 (15)</td>
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</tr>
<tr>
<td>2nd</td>
<td>0.294 (14)</td>
<td>0.026 (14)</td>
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<tr>
<td>3rd</td>
<td>0.140 (15)</td>
<td>0.040 (13)</td>
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<tr>
<td>4th</td>
<td>0.366 (13)</td>
<td>0.125 (10)</td>
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<tr>
<td>5th</td>
<td>0.517 (11)</td>
<td>0.109 (12)</td>
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<tr>
<td>6th</td>
<td>0.926 (2)</td>
<td>0.121 (11)</td>
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<tr>
<td>7th</td>
<td>0.732 (6)</td>
<td>0.211 (9)</td>
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<tr>
<td>8th</td>
<td>0.657 (7)</td>
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<td><strong>New Hampshire:</strong></td>
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<td>0.375 (4)</td>
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<tr>
<td>2nd</td>
<td>0.561 (9)</td>
<td>0.267 (7)</td>
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</table>

ragged rivers which makes up its’ southern and eastern borders. Our path-based measure, however, takes the state boundaries into account and thus gives this district a high score.

Connecticut’s Fifth District, however, has a much higher area-perimeter ratio: the generally square shape of the district compensates for the two appendages protruding from its’ eastern side. However, the appendages reach out to incorporate several urban areas into the district. (See for example, the southeastern portion of the northern appendage and the eastern part of the southern appendage.) Because the major population centers are relatively disconnected from each other, our path-based measure assigns this district a low score of 0.481, which is slightly less compact than two equally sized communities connected with a narrow path. (See Figure 3).

4 Conclusion

We have introduced a new measure of district compactness: the probability that the district contains the shortest path connecting a randomly selected pair of its’ points. The measure can be weighted for population and can take account of the exogenously determined boundaries of the state in which the district is located. It is an extreme point in a parametric family of measures which vary according to the degree that they “penalize” deviations from convexity.
References


Figure 11: Connecticut
Figure 12: Maryland

Figure 13: New Hampshire