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## A CONDITIONALLY HETEROSKEDASTIC TIME SERIES MODEL FOR SPECULATIVE PRICES AND RATES OF RETURN

Tim Bollerslev\*

*Abstract*—The distribution of speculative price changes and rates of return data tend to be uncorrelated over time but characterized by volatile and tranquil periods. A simple time series model designed to capture this dependence is presented. The model is an extension of the Autoregressive Conditional Heteroskedastic (ARCH) and Generalized ARCH (GARCH) models obtained by allowing for conditionally  $t$ -distributed errors. The model can be derived as a simple subordinate stochastic process by including an additive unobservable error term in the conditional variance equation. The descriptive validity of the model is illustrated for a set of foreign exchange rates and stock price indices.

### I. Introduction

The distributional properties of speculative prices, stock returns and foreign exchange rates have important implications for several financial models, including models for capital asset prices and models for the pricing of contingent claims. For example, in empirical tests of mean-variance portfolio theories, such as the Sharpe-Lintner Capital Asset Pricing Model, the variances and covariances of the asset returns are used as measures of "dispersion" or "risk." However, depending on the distribution of the returns the variance may not be a valid or sufficient statistic to use. The distribution of the returns also plays an important role in the Black-Scholes option pricing formula, and in the pricing of forward contracts in the foreign exchange market. Furthermore, most tests of the efficient market hypothesis rely critically on the underlying distributional assumptions.

Following the seminal works by Mandelbrot (1963) and Fama (1965), where it is shown that the first differences of the logarithm of cotton and common stock prices generally have fatter tails than are compatible with the normal distribution, a voluminous literature has been addressed to this question. Without attempting an exhaustive list of these studies, particularly interesting are the original papers by Mandelbrot (1963) and Fama (1965) where the stable Paretian family of distributions is suggested to characterize the stochastic properties of speculative prices. In Praetz (1972) and Blattberg and Gonedes (1974) it is argued that for both

stock price indices and individual stock prices the scaled  $t$ -distribution, derivable as a continuous variance mixture of normals, has greater descriptive validity. Other variance mixture models proposed in the literature include the compound events model by Press (1967), the lognormal-normal model by Clark (1973), the subordinate normal mixture model in Westerfield (1977) and the discrete mixture of normal distributions in Kon (1984). For a recent list of references on the subject see also Fielitz and Rozelle (1983) and Boothe and Glassman (1985).

The general conclusion to emerge from most of these studies is that speculative price changes and rates of return are approximately uncorrelated over time and well described by a unimodal symmetric distribution with fatter tails than the normal. However, even though the time series are serially uncorrelated, they are not independent. As noted by Mandelbrot (1963, p. 418),

"... , large changes tend to be followed by large changes—of either sign—and small changes tend to be followed by small changes, ... ."

The same regularity has also been observed for both stock price changes and foreign exchange rate changes, see for instance Fama (1965, 1970) and Mussa (1979). Indeed, this behavior might very well explain the recent rejection of an independent increments process for daily stock returns in Hinich and Patterson (1985).

The Autoregressive Conditional Heteroskedastic (ARCH) model introduced in Engle (1982) explicitly recognizes this type of temporal dependence. According to the ARCH model the *conditional* error distribution is normal, but with *conditional* variance equal to a linear function of past squared errors. Thus, there is a tendency for extreme values to be followed by other extreme values, but of unpredictable sign. Furthermore, the unconditional error distribution of the ARCH model is leptokurtic, reconciling the previous empirical findings.

In the present paper a simple extension of the ARCH model to allow for conditionally  $t$ -distributed errors is given.<sup>1</sup> This development permits a distinction between conditional heteroskedasticity and a conditional leptokurtic distribution, either of which could account for the observed unconditional kurtosis in the data. In

<sup>1</sup>Engle (1982) recognizes that other conditional error distributions could be imposed, but to the author's knowledge nobody has yet done that in practice.

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addition to this extension, we also allow the current conditional variance to be a function of past conditional variances as in the Generalized ARCH (GARCH) model developed in Bollerslev (1986).

### II. The Standardized $t$ -distribution

Since the econometric model characterizing the distributional properties outlined above applies in a much broader context, the discussion in this section will be somewhat general. Thus, let the conditional distribution of  $y_t$ ,  $t = 1, \dots, T$ , be standardized  $t$  with mean  $y_{t|t-1}$ , variance  $h_{t|t-1}$  and degrees of freedom  $\nu$ , i.e.,

$$\begin{aligned}
 y_t &= E(y_t|\psi_{t-1}) + \epsilon_t = y_{t|t-1} + \epsilon_t \\
 \epsilon_t|\psi_{t-1} &\sim f_\nu(\epsilon_t|\psi_{t-1}) \\
 &= \Gamma\left(\frac{\nu+1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)^{-1}((\nu-2)h_{t|t-1})^{-1/2} \\
 &\quad \times \left(1 + \epsilon_t^2 h_{t|t-1}^{-1}(\nu-2)^{-1}\right)^{-(\nu+1)/2}, \\
 &\quad \nu > 2 \quad (1)
 \end{aligned}$$

where  $\psi_{t-1}$  denotes the  $\sigma$ -field generated by all the available information up through time  $t-1$  and  $f_\nu(\epsilon_t|\psi_{t-1})$  the conditional density function for  $\epsilon_t$ .

The  $t$ -distribution is symmetric around 0, and from Kendall and Stuart (1969) the variance and the fourth moment are equal to

$$\begin{aligned}
 \text{Var}(\epsilon_t|\psi_{t-1}) &= h_{t|t-1} \\
 E(\epsilon_t^4|\psi_{t-1}) &= 3(\nu-2)(\nu-4)^{-1}h_{t|t-1}^2, \quad \nu > 4.
 \end{aligned}$$

It is also well known that for  $1/\nu \rightarrow 0$  the  $t$ -distribution approaches a normal distribution with variance  $h_{t|t-1}$ , but for  $1/\nu > 0$  the  $t$ -distribution has "fatter tails" than the corresponding normal distribution.

A particularly appealing feature of the model given in (1) is the fact that it can be derived as a subordinate stochastic process from the conditional normal model:

$$\begin{aligned}
 y_t &= E(y_t|\psi_{t-1}) + \epsilon_t = y_{t|t-1} + \epsilon_t \\
 \epsilon_t|\psi_{t-1}, u_t &\sim (2\pi h_t)^{-1/2} \exp(-1/2\epsilon_t^2 h_t^{-1}), \quad (2)
 \end{aligned}$$

where  $u_t$  denotes the prediction error in the conditional variance equation

$$h_t = E(h_t|\psi_{t-1}) + u_t = h_{t|t-1} + u_t. \quad (3)$$

The decomposition in (3) is analogous to the decomposition in (1) of  $y_t$  into its conditional expectation,  $y_{t|t-1}$ , and a prediction error,  $\epsilon_t$ . Note, however, that unlike  $y_t$ , the conditional variance at time  $t$ ,  $h_t$ , is in general unobservable. Specifying the inverted distribution of  $u_t$  to be a transformed inverted gamma-1

distribution:

$$\begin{aligned}
 u_t|\psi_{t-1} &\sim g_\nu(u_t|\psi_{t-1}) \\
 &= \Gamma\left(\frac{\nu}{2}\right)^{-1}\left(\left(\frac{\nu}{2}-1\right)h_{t|t-1}\right)^{\nu/2}(u_t+h_{t|t-1})^{-1-\nu/2} \\
 &\quad \times \exp\left(\left(1-\frac{\nu}{2}\right)h_{t|t-1}(u_t+h_{t|t-1})^{-1}\right) \\
 u_t &> -h_{t|t-1}, \quad \nu > 2,
 \end{aligned} \quad (4)$$

and following the same line of arguments as in Raiffa and Schlaifer (1961), it is possible to show that the  $g_\nu(u_t|\psi_{t-1})$  mixture of  $\epsilon_t|\psi_{t-1}$ ,  $u_t$  equals the standardized  $t$ -distribution given in (1).

Let  $\theta$  denote all the unknown parameters in model (1), including the degree of freedom parameter  $\nu$ . By the prediction error decomposition the loglikelihood function for the sample  $y_1, \dots, y_T$  is then, apart from some initial conditions, given by

$$L_T(\theta) = \sum_{t=1}^T \log f_\nu(\epsilon_t|\psi_{t-1}),$$

and standard inference procedures regarding  $\theta$  are immediately available.

However, when testing against the interesting null hypothesis of conditionally normal errors, i.e.,  $1/\nu = 0$ ,  $1/\nu$  is on the boundary of the admissible parameter space, and the usual test statistics will likely be more concentrated towards the origin than a  $\chi^2$  distribution. Some preliminary Monte Carlo evidence confirms this conjecture. For moderately sized samples the correct 5% critical value for the Likelihood Ratio (LR) test statistic for the null hypothesis  $1/\nu = 0$  is approximately 2.7. Also, the bias in the maximum likelihood estimates for  $1/\nu$  is very small for sample sizes one hundred or larger. For a more detailed discussion of the Monte Carlo findings the reader is referred to Bollerslev (1985) and Blattberg and Gonedes (1974) for a similar small scale simulation study.

### III. The GARCH( $p, q$ ) Model

As discussed above, several studies indicate that the change in speculative prices and rates of return are approximately uncorrelated over time, but characterized by tranquil and volatile periods. To allow for such a dependence we shall here take the conditional mean,  $y_{t|t-1}$ , as constant,

$$y_t = \mu + \epsilon_t \quad (5)$$

along with a GARCH( $p, q$ ) model for the conditional variance

$$\begin{aligned}
 E(\epsilon_t^2|\psi_{t-1}) &= h_{t|t-1} \\
 &= \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j|t-1-j}
 \end{aligned} \quad (6)$$

where  $\omega > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_j \geq 0$ . It is obvious that in the model given by (5) and (6) there is a tendency for large (small) residuals to be followed by other large (small) residuals but of unpredictable sign.

Rearranging terms in (6), the GARCH( $p, q$ ) model is readily interpreted as a generalization of Engle's (1982) ARCH( $q$ ) model obtained by including  $p$  "moving average terms" of the form  $(\epsilon_{t-j}^2 - h_{t-j|t-1-j})$  in the autoregressive equation for the conditional expectation of  $\epsilon_t^2$ , cf. Bollerslev (1986). The orders of  $p$  and  $q$  can therefore be identified by applying the traditional Box and Jenkins (1976) time series techniques to the autocorrelations and partial autocorrelations for the squared process,  $\epsilon_t^2$  (see Bollerslev (1987) for a more detailed discussion along these lines).

Even though the unconditional distribution corresponding to the GARCH( $p, q$ ) model with conditionally normal errors is leptokurtic, it is not clear whether the model sufficiently accounts for the observed leptokurtosis in financial time series. In particular, a "fat-tailed" conditional distribution might be superior to the conditional normal. The econometric model presented in the previous section with conditionally  $t$ -distributed errors, (1), combined with (5) and (6) allows for this possibility.

**IV. Empirical Example and Concluding Remarks**

The first set of data consists of daily spot prices from the New York foreign exchange market on the U.S. dollar versus the British pound and the Deutschmark from March 1, 1980 until January 28, 1985 for a total of 1,245 observations excluding weekends and holidays.<sup>2</sup> The spot prices,  $S_t$ , are converted to continuously compounded rates of return,

$$y_t = \log_e(S_t/S_{t-1}).$$

As documented by many previous studies the  $y_t$  series are approximately uncorrelated over time.

In particular, for the British pound the Ljung-Box (1978) portmanteau test statistic for up to tenth order serial correlation in  $(y_t - \hat{\mu})$  takes the value  $Q(10) = 7.64$ , cf. table 1, which is not significant at any reasonable level in the corresponding asymptotic  $\chi_{10}^2$  distribution. On the other hand,  $(y_t - \hat{\mu})^2$  is clearly not uncorrelated over time, as reflected by the highly significant Ljung-Box test statistic for absence of serial correlation in the squares,  $Q^2(10) = 74.79$  distributed asymptotically as a  $\chi_{10}^2$  distribution, see McLeod and Li (1983). This absence of serial dependence in the conditional first moments along with the dependence in the condi-

TABLE 1.—SUMMARY STATISTICS

	$y_t - \mu$			GARCH(1,1)- $t$		
	$Q(10)$	$Q^2(10)$	$\kappa$	$Q(10)$	$Q^2(10)$	$\kappa$
<b>Exchange Rates</b>						
Britain	7.64	74.79	4.81	4.34	8.60	4.63
Germany	9.16	133.08	4.21	7.70	9.51	3.76
<b>Stock Price Indices</b>						
500 Composite	15.00	26.36	4.06	14.08	8.08	3.89
Industrial	14.87	25.03	3.98	14.11	7.75	3.84
Capital Goods	9.28	17.92	4.17	8.84	9.29	4.06
Consumer Goods	19.20	49.03	5.70	12.81	9.27	4.32
Public Utilities	26.57	115.51	5.41	19.61	7.69	4.08

Note:  $Q(10)$  and  $Q^2(10)$  denote the Ljung-Box (1978) portmanteau tests for up to tenth order serial correlation in the levels and the squares, respectively.  $\kappa$  is the usual measure of kurtosis given by the fourth sample moment divided by the square of the second moment.

tional second moments is one of the implications of the GARCH( $p, q$ ) model given by (5) and (6). Furthermore, the unconditional sample kurtosis for the British pound  $\kappa = 4.81$ , cf. table 1, exceeds the normal value of three by several asymptotic standard errors,  $\sqrt{24/1244} = .139$ . Of course, this is also in accordance with the implications of the GARCH( $p, q$ ) model.

As mentioned above, the appropriate orders for  $p$  and  $q$  could now be identified by standard Box-Jenkins methodology applied to the squared residuals,  $(y_t - \hat{\mu})^2$ , cf. Bollerslev (1987) and Engle and Bollerslev (1986). However, for illustrative purposes we shall here consider the simple GARCH(1,1)- $t$  model only:

$$\begin{aligned}
 y_t &= \mu + \epsilon_t \\
 h_{t|t-1} &= \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1|t-2} \\
 \epsilon_t | \psi_{t-1} &\sim f_\nu(\epsilon_t | \psi_{t-1}).
 \end{aligned}
 \tag{7}$$

As we shall see, it just so happens that this particularly simple model fits the time series chosen here quite adequately.

Maximum likelihood estimates of the parameters in model (7) are presented in table 2 along with asymptotic standard errors in parentheses. The estimates are obtained by the Berndt, Hall, Hall and Hausman (1974) algorithm using numerical derivatives.

The Ljung-Box test statistic for the standardized residuals,  $\hat{\epsilon}_t \hat{h}_{t|t-1}^{-1/2}$ , and the standardized squared residuals,  $\hat{\epsilon}_t^2 \hat{h}_{t|t-1}^{-1}$ , from the estimated GARCH(1,1)- $t$  model take the values  $Q(10) = 4.34$  and  $Q^2(10) = 8.60$ , respectively, cf. table 1, and thus do not indicate any further first or second order serial dependence. It is also interesting to note that the implied estimate of the conditional kurtosis,  $3(\hat{\nu} - 2)(\hat{\nu} - 4)^{-1} = 4.45$ , cf. section II, is in close accordance with the sample analogue for  $\hat{\epsilon}_t \hat{h}_{t|t-1}^{-2}$ ,  $\kappa = 4.63$ .

<sup>2</sup>The data were kindly provided by Dick Baillie.

TABLE 2.—GARCH(1,1)-*t* MAXIMUM LIKELIHOOD ESTIMATES AND TESTS

	$\mu$	$\omega$	$\alpha$	$\beta$	$1/\nu$	$LR_{1/\nu=0}$	$LR_{\alpha=\beta=0}$
<b>Exchange Rates</b>							
Britain	$-4.56 \cdot 10^{-4}$ ( $1.67 \cdot 10^{-4}$ )	$0.96 \cdot 10^{-6}$ ( $0.46 \cdot 10^{-6}$ )	.057 (.017)	.921 (.023)	.123 (.024)	41.26	65.21
Germany	$-5.86 \cdot 10^{-4}$ ( $1.78 \cdot 10^{-4}$ )	$0.13 \cdot 10^{-6}$ ( $0.59 \cdot 10^{-6}$ )	.095 (.021)	.881 (.027)	.072 (.022)	13.92	101.77
<b>Stock Price Indices</b>							
500 Composite	$5.63 \cdot 10^{-3}$ ( $1.46 \cdot 10^{-3}$ )	$0.17 \cdot 10^{-3}$ ( $0.13 \cdot 10^{-3}$ )	.074 (.045)	.768 (.148)	.139 (.055)	12.11	7.03
Industrial	$5.73 \cdot 10^{-3}$ ( $1.51 \cdot 10^{-3}$ )	$0.19 \cdot 10^{-3}$ ( $0.14 \cdot 10^{-3}$ )	.078 (.046)	.756 (.150)	.129 (.054)	10.46	6.87
Capital Goods	$5.78 \cdot 10^{-3}$ ( $1.73 \cdot 10^{-3}$ )	$0.27 \cdot 10^{-3}$ ( $0.23 \cdot 10^{-3}$ )	.065 (.048)	.751 (.183)	.140 (.052)	13.66	4.08
Consumer Goods	$5.53 \cdot 10^{-3}$ ( $1.48 \cdot 10^{-3}$ )	$0.14 \cdot 10^{-3}$ ( $0.07 \cdot 10^{-3}$ )	.103 (.041)	.782 (.081)	.161 (.052)	19.01	19.75
Public Utilities	$4.32 \cdot 10^{-3}$ ( $1.27 \cdot 10^{-3}$ )	$0.05 \cdot 10^{-3}$ ( $0.03 \cdot 10^{-3}$ )	.123 (.047)	.820 (.066)	.131 (.049)	11.78	37.82

Note: Asymptotic standard errors are in parentheses.

This estimate of the conditional kurtosis differs significantly from the normal value of three, as seen by the  $LR_{1/\nu=0}$  test for the GARCH(1,1) model with conditionally normal errors, equal to 41.26. Note, the Monte Carlo results mentioned in section II indicate that in this situation the LR test statistic is more concentrated towards the origin than a  $\chi_1^2$  distribution, and therefore evaluating  $LR_{1/\nu=0}$  in the  $\chi_1^2$  distribution leads to a conservative test for conditional normality, implying rejection at even higher levels.

On the other hand, the standardized *t*-distribution with constant conditional variance which has previously been suggested in the literature to characterize the distributional properties of foreign exchange rates, cf. Rogalski and Vinso (1978), fails to take account of the conditional dependence in the second moments. Indeed, the  $LR_{\alpha=\beta=0}$  test statistic equals 65.21, cf. table 2, which is highly significant at any level in the corresponding asymptotic  $\chi_2^2$  distribution (see Engle, Hendry and Trumble (1985) for a discussion of the small sample properties of ARCH tests and estimators).

The results for the Deutschmark are quite similar to the results for the British pound discussed above. Again, the GARCH(1,1)-*t* seems to provide a simple and parsimonious description of the time series properties. The Ljung-Box test statistics do not indicate any further first or second order dependence in the standardized residuals. The estimated value of the conditional kurtosis,  $3(\hat{\nu} - 2)(\hat{\nu} - 4)^{-1} = 3.61$ , is also in accordance with the sample analogue of  $\hat{\epsilon}_t^4 \hat{h}_{t-1}^{-2}$ ,  $\kappa = 3.76$ . The  $LR_{\alpha=\beta=0}$  test for absence of conditional heteroskedasticity is equal to 101.77 and highly significant at any level, as is the  $LR_{1/\nu=0}$  test for GARCH(1,1) with conditionally normal errors equal to 13.92.

Note, the estimated values for  $\alpha + \beta$  are close to one for both currencies. Thus, an Integrated GARCH(1,1) - *t* model imposing the restriction  $\alpha + \beta = 1$  might provide an even simpler characterization of the foreign exchange rates considered here. However, when  $\alpha + \beta = 1$  the unconditional variance does not exist, and the asymptotic properties of the maximum likelihood estimators are not clear. See Engle and Bollerslev (1986) for a precise definition of the IGARCH model, along with a discussion of the statistical problems that arise in that situation.

It would of course be interesting to see whether the results presented above carry over to other currencies and sampling intervals. The recent empirical findings in Milhøj (1987), Hsieh (1985) and McCurdy and Morgan (1985) are all suggestive. Even though the unconditional error distribution corresponding to the ARCH and GARCH models with conditionally normal errors are leptokurtic, they find that the models do not fully account for the leptokurtosis in the different foreign exchange rates analyzed.

We now turn to the second set of data which includes five different monthly stock price indices for the U.S. economy, namely Standard and Poor's 500 Composite, Industrial, Capital Goods, Consumer Goods and Public Utilities price indices. The indices are monthly averages of daily prices from 1947.1 to 1984.9, i.e., 453 observations.<sup>3</sup> It is well known that if the daily price changes are uncorrelated, averaging daily prices to obtain an average monthly price index,  $P_t$ , induces a spurious first order moving average correlation structure with moving average parameter close to 0.268 corresponding to a

<sup>3</sup>The data were taken from the *Citibank Economic Database*.

first order correlation coefficient equal to 0.250; cf. Working (1960). Thus, following Praetz (1982) we define the rate of return series for the stock price indices,  $y_t$ , to be

$$y_t = (1 + .268B)^{-1} \log_e(P_t/P_{t-1}), \quad (8)$$

where  $B$  denotes the usual backshift operator.<sup>4</sup>

From rows 3 through 7 in table 1 it is clear that even though the rates of return series tend to be uncorrelated over time, there is again a tendency for large and small residuals to cluster together as seen by the highly significant  $Q^2(10)$  statistics in column 2. The estimates of the GARCH(1,1)- $t$  model reported in the last five rows in table 2 also reflect this fact. Most of the coefficients are significant at traditional levels. Furthermore, none of the Ljung-Box portmanteau tests for the standardized residuals,  $\hat{\epsilon}_t \hat{h}_{t-1}^{-1/2}$ , reported in table 1 are significant at the usual 5% level, except for  $Q(10)$  for Public Utilities. It is interesting to note that all of the  $LR_{\alpha=\beta=0}$  tests for the standardized  $t$ -distribution, which have previously been suggested in the literature to characterize the stochastic behavior of stock price indices, cf. Praetz (1972), are significant at the 5% level or lower, except for Capital Goods. The  $LR_{1/\nu=0}$  tests for the GARCH(1,1) model with conditionally normal errors, are significant at the 1% level or lower using the conservation  $\chi^2_1$  distribution.

Summing up, the results presented in this section confirm the previous findings in the literature that speculative price changes and rates of return series are approximately uncorrelated over time but characterized by tranquil and volatile periods. The standardized  $t$ -distribution fails to take account of this temporal dependence, and the ARCH or GARCH models with conditionally normal errors do not seem to fully capture the leptokurtosis. Instead, the relatively simple GARCH(1,1)- $t$  model fits the data series considered here quite well. Of course, it remains an open question whether other conditional error distributions provide an even better description. Another interesting question is whether high order GARCH models might be called for when modelling other financial time series. The empirical relevance of the IGARCH model also deserves further investigation. We leave the answer to all of these questions for future research.

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<sup>4</sup>In practice, the expansion of  $(1 + .268B)^{-1}$  was truncated after its first four terms,  $1 - .2680B + .0718B^2 - .0192B^3 + .0052B^4$ .

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## THE LINK BETWEEN THE U.S. DOLLAR REAL EXCHANGE RATE, REAL PRIMARY COMMODITY PRICES, AND LDCS' TERMS OF TRADE

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*Abstract*—The correlation between a real appreciation of the U.S. dollar and a decline in U.S. relative primary commodity prices does not imply that such an appreciation leads to a deterioration in the terms of trade of commodity producing LDCs. In fact, empirical results reported in this note suggest that the dollar appreciation has had a beneficial impact on these countries' terms of trade.

### I. Introduction

The behaviour of non-oil primary commodity prices over recent years has induced observers to examine the role of "non-traditional" factors in the determination of price movements. In this context, a variable that has gained emphasis is the U.S. dollar effective exchange rate. Movement in the U.S. dollar exchange rate vis-à-vis other major currencies has an impact on commodity prices expressed in U.S. dollars since it influences the relative prices of substitutes or production factors.

In this note we examine the link between the U.S. dollar real effective exchange rate, industrial world real prices of non-oil primary commodities (prices of commodities relative to prices of manufactured goods exported by industrial countries), and LDCs' terms of trade. We argue that this link cannot be predicted a priori. Contrary to Dornbusch (1983), our empirical results suggest that the real appreciation of the U.S.

dollar vis-à-vis other industrial countries' currencies has led to an improvement in the LDCs' terms of trade.

The remainder of the paper is organized in the following way. In section II we examine the theory, while in section III we discuss our regression results. Conclusions are drawn in the last section.

### II. Theoretical Considerations

An analysis of the link between the U.S. dollar effective exchange rate and U.S. prices of primary commodities has been conducted in nominal terms by Sachs (1985, Appendix), and in real terms by Dornbusch (1985a, b, c). The argument is the following. Assume that the "law of one price" holds for primary commodities and that prices of manufactured goods (used as the deflator) are fixed in local currency terms both in the United States and abroad. Demand and supply of commodities in each area depend on the local relative price of commodities. A real appreciation of the U.S. dollar vis-à-vis currencies of major industrial countries immediately leads (with the dollar prices of commodities unchanged) to an increase in the local relative price of commodities in other industrial countries. Foreign demand for commodities is therefore reduced (foreign supply should increase as well), which should lead to a decrease in their equilibrium dollar price. The extent of the decline in dollar commodity prices depends on the weight of the United States as a producer and consumer of primary commodities. The more important is the United States, the less is the U.S. dollar price adjustment.

Therefore, one can expect that a real appreciation of the U.S. dollar vis-à-vis currencies of major industrial countries leads to a decrease in the real price of com-

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