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Warren Buffett, Black-Scholes and the Valuation of Long-dated Options

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Abstract

In his 2008 letter to Berkshire shareholders, Warren Buffett presented a critique of the Black-Scholes option pricing model as a tool for valuing long-dated options, including options that Berkshire had written. Given Mr. Buffett's track record, it worth investigating precisely why he thinks that the Black-Scholes model fails to provide a fair value for long-dated options. Unfortunately, the alleged deficiencies in the model are not transparent because Mr. Buffett's letter fails to develop his viewpoint in terms of option pricing theory. This short article fills the gap by interpreting Mr. Buffett's argument in the context of option pricing theory. It turns out that Mr. Buffett is really making a statement about political economics more than option pricing.

Introduction: Warren Buffett on long-dated options

Despite referring to derivatives as weapons of mass financial destruction, Warren Buffett has not been shy about investing in derivatives. In his 2008 letter to the shareholders of Berkshire Hathaway, Mr. Buffett explains that that Berkshire will enter into derivatives transactions if prices are sufficiently favorable. With respect to the investments already made, he states, "I believe each contract we own was mispriced at inception, sometimes dramatically so."

In light of their reported poor performance, some Berkshire derivative contracts have attracted particular attention. The source of the most controversy has been long dated put options that Berkshire wrote on major stock market indexes, primarily the S&P 500. Using the required mark-to-market accounting, in 2008 Berkshire reported losses exceeding $5 billion on these contracts. In light of the reported losses, Mr. Buffett went to some lengths to explain why he thought the long-dated options were good investments. His explanation was premised on the view that the Black-Scholes option pricing model was a poor tool for valuing long-date equity index put options. Referring to the Black-Scholes model, Mr. Buffett went so far as to say that "If the formula is applied to extended periods, however, it can produce absurd results." The purpose of this short paper is to explore exactly why Mr. Buffett believes the Black-Scholes model leads to absurd results by analyzing example calculations he presented in his shareholder letter.

II. Buffet's Argument and the Black-Scholes Option Pricing Model

Rather than directly analyzing the Black-Scholes model, Mr. Buffett frames his

critique of the model in terms of example calculations. In deference to Mr. Buffett, and to avoid mischaracterization, here is his entire explanation from the shareholder letter.

It's often useful in testing a theory to push it to extremes. So let's postulate that we sell a 100- year $1 billion put option on the S&P 500 at a strike price of 903 (the index's level on 12/31/08). Using the implied volatility assumption for long-dated contracts that we do, and combining that with appropriate interest and dividend assumptions, we would find the "proper" Black-Scholes premium for this contract to be $2.5 million.

To judge the rationality of that premium, we need to assess whether the S&P will be valued a century from now at less than today. Certainly the dollar will then be worth a small fraction of its present value (at only 2% inflation it will be worth roughly 14¢). So that will be a factor pushing the stated value of the index higher. Far more important, however, is that one hundred years of retained earnings will hugely increase the value of most of the companies in the index. In the 20th Century, the Dow-Jones Industrial Average increased by about 175-fold, mainly because of this retained-earnings factor.

Considering everything, I believe the probability of a decline in the index over a one-hundred-year period to be far less than 1%. But let's use that figure and also assume that the most likely decline – should one occur – is 50%. Under these assumptions, the mathematical expectation of loss on our contract would be $5 million ($1 billion x 1% x 50%).

But if we had received our theoretical premium of $2.5 million up front, we would have only had to invest it at 0.7% compounded annually to cover this

loss expectancy. Everything earned above that would have been profit. Would you like to borrow money for 100 years at a 0.7% rate?

Let's look at my example from a worst-case standpoint. Remember that 99% of the time we would pay nothing if my assumptions are correct. But even in the worst case among the remaining 1% of possibilities – that is, one assuming a total loss of $1 billion – our borrowing cost would come to only 6.2%. Clearly, either my assumptions are crazy or the formula is inappropriate.

When Mr. Buffett refers to a $1 billion put option in the example, he means that $1 billion is the maximum loss if the index were to go to zero. Given the hypothetical index level of 903, this implies that the put options are effectively written on 11,074,200 units of the index. To use the Black-Sholes model to price European put options on an index three inputs are required – the dividend yield, the risk-free rate and the volatility. At the time that Mr. Buffett published his letter reasonable approximations for each of these were 3%, 4.8% and 18%, respectively. The 3% dividend yield is both the approximate yield on the date of Mr. Buffet's letter and the approximate average payout rate over the prior fifty years. The 4.8% risk-free rate is somewhat greater than the yield on 30-year Treasury bonds because at the time of the Berkshire letter disruptions in the financial market led many analysts, including Mr. Buffett, to believe that Treasury yields were artificially low. The 18% is based on estimates of the long-run historical standard deviation of nominal returns for the S&P 500. These assumptions are evidently quite close to those used by Mr. Buffett because when they are substituted into the Black-Scholes model the estimated value of the put position is found to be $2.46 million, within rounding error of the $2.5 million figure that Mr. Buffett refers to in his letter.

Mr. Buffett clearly states that this value is much too high, but does not explicitly explain why the model fails to give the appropriate value. One thing he points to is inflation. Over the course of 100 years, depreciation in the purchasing power of the dollar will tend to increase nominal stock prices. Although Mr. Buffett is right to stress the impact of inflation, the Black-Scholes model takes account of this factor through the use of the nominal risk-free rate. As long as the inflation rate impounded in the risk-free rate is a reasonable estimate of long-run inflation, there will be no inflation bias in the model's estimate of value.

Although Mr. Buffett bases his example calculations on the future earnings Berkshire could achieve by investing the option premiums, those potential earnings are explicitly accounted for in the Black-Scholes model through the inclusion of the long-term risk-free rate in the model. Consequently, when Mr. Buffett speaks of the deficiencies of the model, he could not be referring to the failure to adequately account for the time value of money.

It should be noted, however, that a long-horizon makes estimated option values highly sensitive to the choice of the risk-free rate and the associated inflation rate that underlies it. For example, if the risk-free rate is dropped to 3.8% the estimated value of Mr. Buffett's hypothetical option position rises almost fivefold to $11.1 million, whereas if the rate is increased to 5.8% the estimated value falls by more than a factor of five to only $0.45 million. Nonetheless, Mr. Buffett does not appear to be arguing that the source of the mispricing is the failure to use a proper risk-free rate. The problem must lie elsewhere.

The estimated value of long dated puts is also highly sensitive to the choice of the dividend yield. Here, however, it is hard to quarrel with the choice of 3% because it represents both the approximate current yield on the S&P 500 and, serendipitously, the long-term average yield. Furthermore, Mr. Buffett does not claim in his letter that the failings of the model are somehow tied to use of an improper dividend yield.

The final factor to consider, other than the form of the model itself, is the long-term volatility. To review, the standard Black-Scholes model assumes that returns follow a lognormal diffusion. This implies that the time T distribution of the stock price, or index, is given by[1]

$$\text{Ln } S_T \sim \text{Normal } [\text{Ln } S_0 + (\mu - \sigma^2/2)*T, \sigma^2*T] \; . \qquad (1)$$

In equation (1), S_0 is the current stock price, S_T is the stock price at the date of expiration, T, μ is the drift which equals the expected return on the index minus the dividend yield, and σ^2 is the volatility.

To apply equation (1) an estimate of μ is required. Without reliving the extensive debate on the equity risk premium, the risk-free rate is selected as an estimate of for μ.[2] This implies that the equity premium equals the dividend yield. Given this assumption, in addition to those previously discussed, equation (1) implies that the probability that S_T will be less than S_0 100 years hence comes to about 4%, a number greater than the 1% probability that Mr. Buffet claims is much too large. This means that Buffett has two

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[1] See, for example, Hull (2009)

[2] See, for example, Cornell (1999).

possible beefs. First, the equity premium, and therefore, the drift should be larger. Second, something is wrong with the volatility.

The culprit is unlikely to be the drift. Although variation in the drift does impact the probability that the index will be below its initial level at expiration, it does not affect the estimate of value produced by the Black-Scholes model. Because Mr. Buffett's ultimate view is that the model overvalues the puts, his main criticism cannot be based to any meaningful extent on the drift term.

The final candidate, other than the arbitrage argument on which the model is based, is the volatility. If Mr. Buffett is criticizing the use of the lognormal diffusion assumption when pricing long-term options, he is not alone. Recall that the lognormal assumption implies that volatility increases linearly with respect to the horizon over which it is measured as shown in equation (1). There is empirical evidence which indicates that the linearity assumption fails to hold at long horizons. For example, Siegel (2008) reports that the variance of real returns on the S&P 500 historically have failed to rise linearly with the horizon. If the long-run volatility is lower, the value of long-term put options will be less. For instance, a volatility of 15%, instead of 18%, reduces the estimated value of Mr. Buffett's hypothetical put position to $1.5 million. It also reduces the probability that the index will be lower at expiration than at initiation.

The assumption that volatility fails to rise linearly with the horizon of which it is measured is not without controversy. First, the reduction that Siegel points to is less true of *nominal* returns than *real* returns. Whereas Siegel (2008) reports that historical volatility at 20-year horizons is about 25% below the prediction of the lognormal diffusion model for real returns, the number using nominal returns is less than 12%. In

addition, the conclusion is sample dependent. At horizons of 10 years the lognormal model essentially matches the historical data. Furthermore, some scholars, including Pastor and Stambaugh (2009), argue that return volatility as a function of the horizon may actually increase more than the lognormal assumption predicts.

In any event, the evidence regarding the volatility of nominal returns over the long run is not strong enough to support Mr. Buffett's contention that the Black-Scholes model produces absurd results. He must have had something else in mind. It is unlikely that Mr. Buffett put billions of dollars of Berkshire's capital at risk on the basis of an academic debate regarding the historical estimation of stock price volatility.

More fundamentally, the debate over historical estimation of volatility is ultimately irrelevant because the model requires an estimate of future volatility. The Black-Scholes model will produce "absurd results" if it is based historical estimates of volatility that have become outdated. It is here, I believe, that Mr. Buffett parts ways with the application of the Black-Scholes model on which Berkshire's mark-to-market accounting is based.

As Mr. Buffett notes, the value of long-dated put options is derived from the extreme right hand tail of the distribution of outcomes for which the ending index value is less than the strike price. Historically, this has been the result of a combination of two factors – negative real stock returns and falling prices. For example, the worst period in the history of the American stock market was from 1929 through 1932 when real returns fell and there was significant deflation making nominal returns even more negative. The value of long-dated puts is based on the likelihood that such events will occur again with a likelihood that is determined largely by the volatility parameter. The key question,

therefore, is whether the assumption of lognormal diffusion assumption accurately approximates the likelihood of such recurrences. Clues elsewhere in his letter suggest that Mr. Buffett believes that it does not.

Early on in his letter, Mr. Buffett observes, with respect to the current economic crisis, that

> *This debilitating spiral has spurred our government to take massive action. In poker terms, the Treasury and the Fed have gone "all in." Economic medicine that was previously meted out by the cupful has recently been dispensed by the barrel. These once-unthinkable dosages will almost certainly bring on unwelcome aftereffects. Their precise nature is anyone's guess, though one likely consequence is an onslaught of inflation.*

Mr. Buffett's observations regarding response of the government to the current crisis suggest that he believes a repeat of the 1930s is not possible. The reason being that governments will respond strongly and aggressively to prevent downturns, even deep downturns, from turning into deflationary spirals by providing whatever liquidity is needed to halt deflation. If this is so, then the assumption that nominal stock returns follow a lognormal diffusion over the long term with a volatility that can be estimated from historical data is not realistic. Instead, the lower tail of the probability distribution will be impacted by something akin to a soft reflecting barrier induced by the inflationary reactions of governments and central banks to economic crises. Cornell, Cvitanic and Goukasian (2009) show that the existence of such limitations on possible stock price paths, compared with the assumption of a lognormal diffusion, has a significant impact on option prices. In this case, the impact of inflationary government responses would be

to markedly reduce the estimated value of long-dated put options. Depending on the aggressiveness of the governmental response to potential deflation, the values for long-dated puts given by the standard Black-Scholes model using historical volatility can turn out to be "absurdly high" as Mr. Buffett asserts.

There is one final clue that supports the foregoing interpretation of Mr. Buffett's investment in long-dated index puts. Remember that Mr. Buffett states that "if the formula is applied to *extended time periods*, however, it can produce absurd results." This implies that over shorter time intervals Mr. Buffett believes that the formula yields more reasonable results. Such reasoning is consistent with the political economics interpretation of Mr. Buffett's critique presented previously. During normal times, when aggressive injection of liquidity is not required, the lognormal approximation is reasonable. Where it breaks down is near the left-hand tail when extensive government intervention is required to prevent deflation. But it is precisely that tail from which long-dated puts derive their value. In the short-run, the probability of reaching the tail is too small to be meaningful. But if volatility grows linearly with time, eventually the left-hand tail becomes important. Government policies which cut off that tail, therefore, have a dramatic impact on the value of long-dated puts but have little impact on short-dated contracts except possibly during times of crisis.

In conclusion, although Mr. Buffett's critique of the Black-Scholes model might appear to be another attack on "geeks bearing formulas," it is better seen as a statement about the changing nature of America's political economy. If threat of severe recession and deflation cause governments to respond with massive injections of liquidity, then naïve application of the Black-Scholes model will produce absurd results for long-dated

options if it is based on standard assumptions and historical data. The problem, however, is not the theory on which the model is based, but that government intervention implies that the long-run distribution of returns will not be lognormal. In particular, the left-hand tail will be truncated, sharply reducing the value of long-dated put options. If this is so, Mr. Buffett has made another wise investment even if his rationalization of it somewhat misses the mark.

II. Conclusion

In his 2008 letter to Berkshire shareholders, Warren Buffett criticizes the Black-Scholes option pricing model arguing that it can produce "absurd" values for long-dated put options. Though Mr. Buffett does not explicitly say so, a careful analysis of his viewpoint reveals that his criticism boils down to the belief that future nominal stock prices are not well approximated by a lognormal distribution with a volatility estimated from historical data. Instead, Mr. Buffett apparently believes that inflationary policies of governments and central banks will limit future declines in nominal stock prices compared with those predicted by an historically estimated lognormal distribution. If Mr. Buffett is right on this point, then the Black-Scholes model will indeed significantly overvalue long-dated put options.

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