The problem set is due by 2PM, Monday January 31. You may turn in your problem set to my secretary, Patricia Hamad in Baxter 112, or you may give it to me at the beginning of class, or you may email it to the TA, Heidi Kamp, heidi@caltech.edu. Lateness will be penalized.

Be sure to explain your answers and show your work.

For all problems, assume that each party is a selfish (non-altruistic), rational maximizer. That is, each party considers costs and benefits it directly incurs, and that each party chooses the action which maximizes benefits minus costs.

1) In class we modeled the tragedy of the commons as follows:

\[ S = \text{the total number of sheep} \]
\[ S_a \text{ is A’s herd} \]
\[ S_b \text{ is B’s herd} \]
\[ S_a + S_b = S \]

\[ B = \text{the total profits from grazing sheep} \]
\[ = 20S - S^2/2 \]

Each shepherd derives profits proportional to the size of its herd

\[ B_a = S_aB/S \]
\[ B_b = S_bB/S \]

In class, we showed that if shepherds are restricted to 5, 10 or 15 sheep, then the social optimum is that each grazes 10 sheep, and the Nash equilibrium is that each grazes 15 sheep. Suppose the shepherds can graze herds of any positive integer size. What is the social optimum? What is/are the Nash equilibrium/a? Is there more than one Nash equilibrium?
2) Consider the factory-shepherd problem we discussed in class, but now assume that there are 4 shepherds:

F = factory owner
S₁ = shepherd #1
S₂ = shepherd #2
S₃ = shepherd #3
S₄ = shepherd #4
p = percent pollution eliminated
B(p) = dollars F or S₁ or S₂ or S₃ or S₄ would pay to reduce pollution p%

B_F = B_S₁ = B_S₂ = B_S₃ = B_S₄ = 25p
C(p) = cost of reducing pollution p percent
C(p) = p²/2

A) What is the socially optimal amount of pollution reduction?

Suppose there are no laws against pollution, and the shepherds are negotiating with the factory owner to reduce pollution.

B) What is the factory owner’s threat value?

C) What are each of the shepherds’ threat values?

D) Is an agreement to reduce pollution possible?

E) Suppose one of the shepherds refuses to pay the factory owner anything to reduce pollution. Is it possible that the factory owner and the other three shepherds might still reach an agreement to reduce pollution?

F) Is an agreement more likely when there is one shepherd (as discussed in class) or four shepherds (as here)? What does this tell us more generally about transactions costs?
3) Theater owners can sell tickets in two ways. They can sell general admission tickets, allowing the ticket holder to sit anywhere, or they can sell tickets with assigned seats.

A) Using concepts discussed in class, discuss the advantages and disadvantages of each system.

B) Using your analysis in (A), explain why movie tickets are usually sold in the first way (general admission), while tickets to the symphony are usually sold in the second way (assigned seats).

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4) Congress is considering increasing the patent term from 20 to 25 years.

A) Discuss the advantages and disadvantages of this proposal.

B) Should Congress make the increase retroactive? That is, should the 25 year term apply only to inventions made after the legislation is enacted (e.g., after March 1, 2005) or should the 25 year term apply to all inventions currently under patent (e.g., including patents granted in 1986, 1987, etc.)

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5) Farmer contracted with Factory to supply 1 ton of grain for $200. Payment was to be made at delivery. Factory planned to use the grain to make bread. A ton of grain was sufficient to produce 1000 loaves of bread, which the farmer could sell at 10 cents profit per loaf. Farmer, however, did not deliver the grain. As a result of nondelivery of the grain, Factory produced no bread. Factory refused to pay for the nondelivered grain. In addition, Factory sued for damages.

A) How much should the court order Farmer to pay in damages?

B) Would you answer to (A) change if there existed an active spot market for grain, and, at the time of the scheduled delivery, the spot price of grain was $225 per ton. That is, at the time of delivery, there were other Farmers willing to sell grain for immediate delivery for $225 per ton?

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6) In general, the more equal a husband and wife’s income is, the greater the probability of divorce. Why?