PS/EC 172, SET 1
DUE FRIDAY, APRIL 14TH AT 1PM
RESUBMISSION DUE FRIDAY, APRIL 28ST AT 1PM

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *20 points*. What are the subgame perfect equilibria of the centipede game?

(2) Explain why the second player cannot force a victory in
  (a) *20 points*. Tic-tac-toe. Hint: assume the second player has a strategy that forces a victory. Explain how the first player can use this strategy to build a strategy that would force victory too, leading to a contradition. This can be done in a way that is independent of most of the details of the definition of the game.

  (b) *20 poinst*. The sweet fifteen game, as described in section 2.2 of the lecture notes. Hint: https://en.wikipedia.org/wiki/Magic_square.

(3) *Intransitive dice*. A die has six sides, each labeled with a number. Consider three dice that are labeled as follows
  (a) 2, 2, 4, 4, 9, 9.
  (b) 1, 1, 6, 6, 8, 8.
  (c) 3, 3, 5, 5, 7, 7.

  Players 1 and 2 play the following extensive form game with perfect information. First, player 1 picks one of these three dice. Then player 2 picks one of the two that are left over. The utility of a player is the probability, when the two picked dice are rolled, that their die shows the higher number.
  (a) *10 points*. Find a subgame perfect equilibrium of this game.

  (b) *9 points*. Who has the higher utility? Is there a subgame perfect equilibrium in which the other player has higher utility?

  (c) *1 point*. Read this: https://en.wikipedia.org/wiki/Intransitive_dice#Warren_Buffett.

(4) *20 points*. Find a subgame perfect equilibrium of the dollar auction extensive form game, as described in section 2.9 of the lecture notes.

(5) *Bonus question: countability via games*. Recall that a set S is *countable* if there exists a bijection (one-to-one correspondence) $f: S \to \mathbb{N}$ from S to the natural numbers. Equivalently, S is countable if it can be written as $S = \{s_1, s_2, \ldots\}$. Recall also that the interval $[0,1]$ is not countable (Cantor, 1874). We will prove this using a game. This proof is due to Grossman and Turett (1998).

  Consider the following game. Fix a subset $S \subseteq [0,1]$, and let $a_0 = 0$ and $b_0 = 1$. The players Al and Betty take alternating turns, starting with Al. In

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Al's n^{th} turn he has to choose some a_n which is strictly larger than a_{n-1}, but strictly smaller than b_{n-1}. At Betty's n^{th} turn she has to choose a b_n that is strictly smaller than b_{n-1} but strictly larger than a_n. Thus the sequence $\{a_n\}$ is strictly increasing and the sequence $\{b_n\}$ is strictly decreasing, and furthermore $a_n < b_m$ for every $n, m \in \mathbb{N}$.

Since a_n is a bounded increasing sequence, it has a limit $a = \lim_n a_n$. Al wins the game if $a \in S$, and Betty wins the game otherwise.

(a) *1 point*. Let $S = \{s_1, s_2, \ldots\}$ be countable. Prove that the following is a winning strategy for Betty: in her n^{th} turn she chooses $b_n = s_n$ if she can (i.e., if $a_n < s_n < b_{n-1}$). Otherwise she chooses any other allowed number.

(b) *1 point*. Explain why this implies that $[0,1]$ is uncountable.