MA140A, HOMEWORK 4
DUE FRIDAY, OCTOBER 30TH

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *The Law of Total Expectation.* Let $X \in \mathcal{L}^2$. Show that if $\mathcal{G}_2 \subseteq \mathcal{G}_1$ then $\mathbb{E}\left[\mathbb{E}\left[X|\mathcal{G}_1\right]|\mathcal{G}_2\right] = \mathbb{E}\left[X|\mathcal{G}_2\right]$.

(2) Prove that if $X, Y \in \mathcal{L}^1$ are independent, then $\mathbb{E}\left[X|Y\right]$ is the constant random variable $\mathbb{E}\left[X\right]$.

(3) Find a sequence of independent random variables $(X_1, X_2, \ldots)$ with $\mathbb{P}\left[X_n \in \{-n, n, 0\}\right] = 1$, $\mathbb{E}\left[X_n\right] = 0$, and such that the weak LLN holds but not the strong: for $Y_n = \frac{1}{n}\sum_{k \leq n} X_k$ it holds that $\mathbb{P}\left[|Y_n| \geq \varepsilon\right] \to 0$ but $\mathbb{P}\left[\lim_n Y_n = 0\right] \neq 1$.

(4) *Elchanan Mossel's die paradox.* Toss a six sided die until 6 comes up, and then stop. Conditioned on all tosses coming out even, what is the expected number of tosses?

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