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Bundling Customers: How to Exploit Trust Among Customers to Maximize Seller Profit

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Abstract

We consider an auction of identical digital goods to customers whose valuations are drawn independently from known distributions. Myerson's classic result identifies the truthful mechanism that maximizes the seller's expected profit.

Under the assumption that in small groups customers can learn each others' valuations, we show how Myerson's result can be improved to yield a higher payoff to the seller using a mechanism that offers groups of customers to buy bundles of items.

1 Introduction

1.1 Bundling items

Bundling is the practice of joining together a number of products into a "bundle", so that customers may not buy each product separately, but must choose to either buy the entire bundle or have non of the included items. Alternatively, customers may be allowed to purchase a single item, but at a higher cost; that is, the price of the bundle is set to below the sum of the prices of the individual items that comprise it. Examples range from McDonald's happy meals to enormous defense contracts [1] (see also the recent attention to bundling of scientific journal subscriptions [7]). Bundling has also received much attention from theorists (cf. [2, 10, 8] and many more).

However, consider a population of consumers who are potential customers for some mass produced product (i.e., the number of available items is unlimited). Assume also that customers generally have no need for more than one item. For example, the product might be an upgrade to an operating system, a cellphone data package or removal of tax offenses record. This class of products is sometimes referred to as *digital goods*.

Since each customer has no need for more than one item, bundling items does not seem to offer an advantage to the seller. Indeed, Myerson [11] shows that in a Bayesian setting the best strategy available to the seller is to offer a fixed per-item price[1]. Since customer valuations for a product may differ wildly, fixing a price often means forfeiting the customers who are willing to pay less, while undercharging the customers who are willing to pay more.

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[1] Different fixed prices may be offered to different customers in a practice called *price differentiation*.

1.2 Bundling customers

We consider a different kind of bundling, which, although also widespread, seems (to our knowledge) to have been largely overlooked by theorists. We propose that the seller may increase its profit beyond Myerson's bound by *bundling customers*: here customers are arbitrarily grouped into pairs and are offered to buy two items for a price that is lower than the sum of the prices of the individual items. The same can of course be done for larger groups of customers, so that a group of n customers are jointly offered to buy n items for a discount.

Our key assumption is what we call *group rationality*: namely that a bundle of customers will accept the group offer *if there is a way for them to share the cost so that all of them benefit.* For example, consider two customers who are each interested in buying a copy of a book whose (single item) price is set to 10 Gold Dinars. Let customer X be willing to pay at most 20 Dinars, and let customer Y be willing to pay at most 5 Dinars. Let the cost of a single book be set to 10 Dinars. Group rationality implies that if X and Y were offered to jointly buy two books for 11 Dinars then they would accept and find a way to split the cost, since both can benefit; customer X can contribute 8 Dinars and customer Y can contribute 3 Dinars, and then X has paid two Dinars less than she would have paid on her own, and Y was able to buy the book, which he wouldn't been able to do on his own. Note that assuming that the cost of printing a book is small, then the seller is also strictly better off.

Our *group rationality* assumption is novel in the context of Myerson auctions, and is in fact what allows us to increase the seller's profit past Myerson's bound on truthful auctions. We note that indeed there is no truthful mechanism for two customers to agree on a division of costs when a feasible one exists; this is nothing but the well known "splitting the dollar" game. In the example above, if customer Y manages to convince X that he is not willing to pay more than 2 Dinars, then X might settle for paying 9 herself, which still leaves her better off than buying a single book for 10 Dinars.

However, we argue that it is important to consider group rationality; it is in fact a phenomenon that, in other contexts, has been widely studied theoretically and experimentally, and falls under the general titles of cooperation and altruism (cf. [3, 12, 5, 13]).

Specifically, families and tribes are often group rational (for obvious evolutionary reasons, cf. [9]), as are other groups of people who expect to have to rely on each other in the future. A further argument to support group rationality in our setting is the observation that when the stakes (i.e., the savings) are high, one could expect that in any small group people would be sufficiently incentivized to find a way to compromise, trust and share, even if there is a danger of being short-changed; in reality, prisoners do sometimes choose to "cooperate" even when facing the risk of "defection" by cellmates, and the tragedy of the commons can be averted (cf. [6]).

1.3 Results

We consider a Bayesian setting with independent customer valuation distributions and *group rational* customers. Our main result is that under mild smoothness conditions of the customers' valuation distributions, the seller can expect a strictly higher profit when bundling customers into pairs, as compared to selling single items.

We also show that when valuations are uniformly bounded then, as the size of the bundle increases, the seller's expected profit from the customers approaches the sum of their expected valuations for the product, which is an upper bound on the seller's profit. This bound is achieved in single customer auctions only when the customer reveals its valuation to the seller.

Approaching this limit by bundling ever larger groups of customers would require ever more trust among them. Note that assuming group rationality for larger groups is a stronger assumption than group rationality for smaller groups. Indeed, as the size of the group grows, the believability of *group rationality* diminishes; all else being equal, it seems harder to expect honesty and trust among a hundred people than among a couple.

Our results can therefore be interpreted to show that the seller can exploit trust *among customers*[2] to increase its profit. And in fact, the more trusting the customers are (i.e., the larger the trusting group is), the higher the profit the seller can expect, up to the maximal profit possible.

2 Model

Let $[n] = \{1, \ldots, n\}$ be the set of customers. Each customer i has a private valuation V_i, which is the maximum price that it would be willing to pay for the product. These valuations are not known to the seller, who however has some knowledge of what they might be. We model the seller's uncertainty by assuming that each valuation V_i is picked independently[3] from some distribution with cumulative distribution function (CDF) F_i. This model is a special case of Myerson auctions [11].

We make a number of mild smoothness conditions on the distributions of valuations: We assume that F_i is non-atomic and differentiable with bounded density (PDF) f_i. We assume all valuations are in $[0, M]$ for some $M \in \mathbb{R}$, so that f_i is zero outside this interval for all i. We further assume that for some $\delta > 0$ it holds for all i that $\delta < f_i < 1/\delta$ in the interval $[0, M]$[4].

Let s be an auction mechanism or sales strategy. We assume that it can result in each of the customers either receiving or not receiving an item, and parting with some sum of money. In the context of s, we denote by R_i^s the event that customer i receives an item. We denote by P_i^s the price, or the amount of money customer i paid the seller for the item. We denote customer i's utility by C_i^s, where

$$C_i^s = \mathbf{1}_{R_i^s}(V_i - P_i^s). \tag{1}$$

and denote the customer's expected utility by $c_i^s = \mathbb{E}[C_i^s]$.

Let U_i^s denote the seller's utility from selling to customer i. We assume that the cost of an item is zero, and so define

$$U_i^s = P_i^s. \tag{2}$$

We denote the seller's expected utility by $u_i^s = \mathbb{E}[U_i^s]$. We denote the seller's total expected utility by $u^s = \sum_{i=1}^n u_i^s$.

We assume throughout that given a seller's strategy, the customer will pick a strategy that will maximize its expected utility. Given that, a seller will pick a strategy that will maximize its own total expected utility. We largely ignore the possibility of ties (i.e., two strategies that result in the same expected utility, for either the customer or the seller), since, as we assume the distribution of the valuations is non-atomic, it will be the case for the strategies that we consider that ties will occur with probability zero.

[2] Note that the customers are not required to trust the seller!

[3] Despite some recent progress [14], it seems that Myerson auctions are generally difficult to analyze when valuations are not independent. We conjecture that our results hold also for the case of correlated valuations.

[4] These assumptions can be significantly relaxed at the price of a significantly more technical and difficult to read paper.

2.1 Sales strategies

2.1.1 Single customer one time offer

We assume that the seller wishes to maximize the sum of the expected revenues it extracts from the customers. A possible strategy would be to give each customer i a one time offer to buy the product at price p_i. Myerson [11] shows that this sales strategy, of all the truthful strategies, maximizes the profit of the seller, for the appropriate choice of p_i.

The customer's utility in this case is $C_i = (V_i - p_i)\mathbf{1}_{R_i}$. Therefore, assuming the customer wishes to maximize its utility, it would buy iff $V_i \geq p_i$ (or equivalently $V_i > p_i$, since $\mathbb{P}[V_i = p_i] = 0$). Hence $R_i = \mathbf{1}_{V_i \geq p_i}$, the gain by the seller is $P_i = p_i \mathbf{1}_{R_i}$, and the seller's expected utility is

$$u_i(p_i) = \mathbb{E}[P_i] = p_i \cdot \mathbb{P}[V_i \geq p_i] + 0 \cdot \mathbb{P}[V_i < p_i] = p_i(1 - F_i(p_i)), \tag{3}$$

with

$$u_i'(p_i) = \frac{du_i(p_i)}{dp_i} = 1 - F_i(p_i) - p_i f_i(p_i). \tag{4}$$

If we assume that F_i is non-atomic, differentiable and only supported on $[0, M]$, then $u_i(p_i)$ is continuous and differentiable and must have a maximum in $[0, M]$. By solving $u_i'(p_i) = 0$ we can show that any p_i which maximizes u_i satisfies

$$p_i = \frac{1 - F_i(p_i)}{f_i(p_i)}. \tag{5}$$

Furthermore, under these assumptions $u_i(0) = u_i(M) = 0$, whereas clearly u_i is positive for some $0 < p_i < M$. Hence this maximum does not occur at 0 or M.

2.1.2 Bundling customers

We next consider the strategy of bundling the customers. Let $[n] = \{1, \ldots, n\}$ be a set of customers. The bundling strategy here is parametrized by a vector of single item prices $\bar{a} = (a_1, \ldots, a_n)$ and the bundle price b.

The n customers are given the option to buy a bundle of n items (i.e., each gets an item) for the total price of b. Additionally, each customer i may buy a single item for the price of a_i.

We assume *group rationality*, so that the customers choose to buy the bundle *if the cost can be shared in a way that is profitable for all.* That is, the customers buy the bundle if *there exist* $(P_1, \ldots, P_n)$ such that the following holds:

1. $\sum_i P_i = b$.

2. $P_i \leq V_i$ for all $i \in [n]$. That is, each customer's utility for buying the bundle is positive, or better than the utility for not buying.

3. $P_i \leq a_i$ for all $i \in [n]$. That is, each customer's utility for buying the bundle is better than the utility for buying individually.

Hence, we assume that if the cost of the bundle can be shared in a way that, for each customer, improves the utility over the other alternatives, then the customers will find a way to share the cost and will choose to buy the bundle. When this is not the case then each customer i, independently, decides to either buy or to buy, depending on whether $a_i \leq V_i$, as in the single customer case.

Formally, $R_i = 1$ iff the condition above holds or $a_i \leq V_i$. Note that "$\leq$" can be replaced by "$<$" throughout, since ties occur with probability zero.

Note that when the conditions above apply - i.e., accepting the bundle is group rational for some prices $\{P_i\}$ - then accepting the offer and paying P_i is a Nash Equilibrium: it is better for customer i to accept the offer for P_i rather than shop alone, since then it would have to pay more.

3 Results

3.1 Smoothness and boundedness conditions

We make the following assumptions on the distribution of customer valuations V_i. Recall that we denote by F_i and f_i the CDF and PDF of the distribution of V_i.

1. Customers valuations are independent and non-atomic.
2. There exists $M > 0$ such that, for all i, V_i is in $[0, M]$.
3. The distribution of V_i has a density (PDF) f_i.
4. There exists $0 < \delta < 1$ such that, for all i, $\delta < f_i(p) < 1/\delta$ for $p \in [0, M]$.

3.2 Theorem statements

In the statement of the following theorem we mark quantities related to the single customer strategy by s, and quantities related to the bundling strategy by b. E.g., U_i^s is the seller's utility from customer i using the single customer strategy, and u_i^b is the seller's expected utility from customer i using the pair bundling strategy.

Theorem 3.1. *Let $\{1, 2\}$ be a pair of customers with valuation distributions satisfying the smoothness and boundedness conditions in 3.1. Let*

$$u^s(p_1, p_2) = u_1^s(p_1) + u_2^s(p_2)$$

be the seller's total expected utility when using the single customer strategy with prices p_1 and p_2. Let

$$u^b(a_1, a_2, b) = u_1^b(a_1, a_2, b) + u_2^b(a_1, a_2, b)$$

be the seller's total expected utility when using the pair bundling strategy with prices a_1, a_2 and b. Then

$$\max_{a_1,a_2,b} u^b(a_1, a_2, b) > \max_{p_1,p_2} u^s(p_1, p_2). \tag{6}$$

That is, the best bundling strategy is strictly better than the best single customer strategy.

The next theorem shows that when valuations are bounded then, as the size of the bundle grows, the expected utility of the seller from the customers approaches the sum of their expected valuations.

Theorem 3.2. *Consider a set of n customers with valuation distributions satisfying the smoothness and boundedness conditions in 3.1.*

Let $\mu_i = \mathbb{E}[V_i]$ be customer i's expected valuation, and let $\mu = \sum_{i=1}^{n} \mu_i$ be the sum of the customers' expected valuations. Let

$$u^n(\bar{a}, b) = \sum_{i=1}^{n} u_i^b(\bar{a}, b)$$

be the seller's total expected utility when bundling all n customers with prices $\bar{a} = (a_1, \ldots, a_n)$ and b.

Then the seller's total expected utility satisfies

$$\max_{\bar{a},b} u^n(\bar{a}, b) \geq \left(1 - \frac{4}{\delta}\sqrt{\frac{\log n}{n}} - O\left(\frac{1}{n}\right)\right)\mu \tag{7}$$

Note that since a customer will never pay more than its valuation then

$$\max_{\bar{a},b} u^n(\bar{a}, b) \leq \mu.$$

4 Proofs

Proof of Theorem 3.1. Let (p_1, p_2) be the prices that maximize the seller's total expected utility for the single customer strategy. We will prove the theorem by showing that there exists $\epsilon > 0$ such that

$$u^b(p_1 + \epsilon, p_1, p_1 + p_2) > u^s(p_1, p_2). \tag{8}$$

Note that since p_1 is optimal for the single customer strategy, then

$$\frac{\partial u^s(p_1, p_1)}{\partial p_1} = \frac{du_1^s(p_1)}{dp_1} = 0.$$

Hence there exist a constant C_1 such that for all ϵ small enough it holds that

$$u^s(p_1 + \epsilon, p_2) > u^s(p_1, p_2) - C_1\epsilon^2. \tag{9}$$

Let B denote the event that the customers buy the bundle. Recall that in the bundling strategy with prices $(p_1 + \epsilon, p_2, p_1 + p_2)$ B occurs if and only if there exist P_1 and P_2 such that

$$\begin{aligned} P_1 + P_2 &= p_1 + p_2, \\ V_1 &\geq P_1, \\ V_2 &\geq P_2, \\ P_1 &\leq p_1 + \epsilon, \\ P_2 &\leq p_2. \end{aligned} \tag{10}$$

Using Eq. (10) we can substitute $P_2 = p_1 + p_2 - P_1$ and arrive at the following equivalent condition: B occurs if and only if there exists a P_1 such that

$$\begin{aligned} p_2 - V_2 \leq P_1 - p_1 &\leq V_1 - p_1, \\ 0 \leq P_1 - p_1 &\leq \epsilon. \end{aligned}$$

In this form it is apparent that B occurs if and only if

$$\begin{aligned} V_1 + V_2 &\geq p_1 + p_2, \\ V_1 &\geq p_1, \\ V_2 &\geq p_2 - \epsilon. \end{aligned}$$

We now partition our probability space into the disjoint events $\{A_1^\epsilon, A_2^\epsilon, A_3^\epsilon, A_4^\epsilon, A_5^\epsilon\}$ (see Fig. 1), where the ϵ in the superscripts denotes the fact that these events depend on ϵ. We compare $U^s = U^s(p_1, p_2)$ and $U^b = U^b(p_1 + \epsilon, p_2, p_1 + p_2)$ in each event.

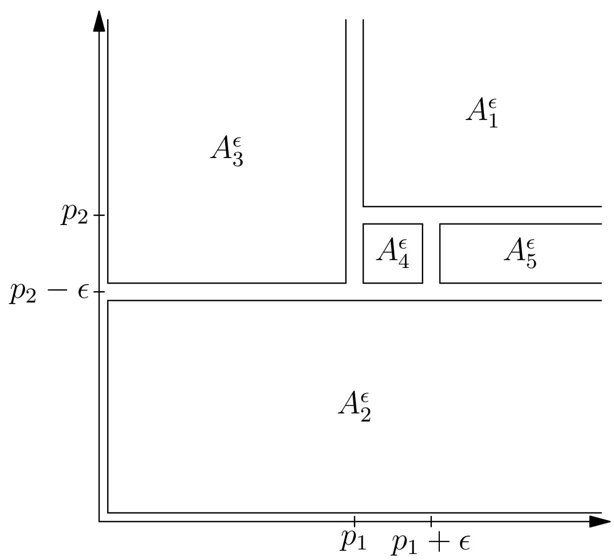

Figure 1: Disjoint union of $\mathbb{R}^{+2}$ into $\{A_i^\epsilon\}_{i=1}^5$.

1. Let A_1^ϵ be the event that $(V_1, V_2) \in [p_1, \infty) \times [p_2, \infty)$. Then in A_1, in the single customer strategy both customers buy an item for a total of $p_1 + p_2$, and in the bundling strategy the customers buy the bundle for $p_1 + p_2$. Hence

$$U^s \mathbf{1}_{A_1^\epsilon} = U^b \mathbf{1}_{A_1^\epsilon}$$

and

$$\mathbb{E}\left[U^b \mathbf{1}_{A_1^\epsilon}\right] = \mathbb{E}\left[U^s \mathbf{1}_{A_1^\epsilon}\right]. \tag{11}$$

2. Let A_2^ϵ be the event that $(V_1, V_2) \in [0, \infty) \times [0, p_2 - \epsilon)$. In this region the bundle is not bought, and neither does customer 2 buy an item on their own, in either strategies. Hence in this region customer 1 buys the item iff $V_1 \geq p_1 + \epsilon$ in the bundling strategy. Since V_1 and V_2 are independent then the (expected) utility for the seller in the bundling strategy is identical to what it would be when offering a single item to customer 1 for $p_1 + \epsilon$. Since, by Eq. (9), this expected utility is maximized when the price is p_1 (as is done in the single item strategy), then

$$\mathbb{E}\left[U^b \middle| A_2^\epsilon\right] \geq \mathbb{E}\left[U^s | A_2^\epsilon\right] - C_1 \epsilon^2.$$

and

$$\mathbb{E}\left[U^b \mathbf{1}_{A_2^\epsilon}\right] \geq \mathbb{E}\left[U^s \mathbf{1}_{A_2^\epsilon}\right] - C_1 \epsilon^2 \mathbb{P}\left[A_2^\epsilon\right]. \tag{12}$$

3. Let A_3^ϵ be the event that $(V_1, V_2) \in [0, p_1) \times [p_2 - \epsilon, \infty)$. In this region the bundle is not bought, and neither does customer 1 buy an item on their own, in either strategies. Customer 2, however, buys in both strategies. Therefore the seller's utility is identical in this region:
$$U^s \mathbf{1}_{A_3^\epsilon} = U^b \mathbf{1}_{A_3^\epsilon}$$
and
$$\mathbb{E}\left[U^b \mathbf{1}_{A_3^\epsilon}\right] = \mathbb{E}\left[U^s \mathbf{1}_{A_3^\epsilon}\right]. \tag{13}$$

4. Let A_4^ϵ be the event that $(V_1, V_2) \in [p_1, p_1 + \epsilon) \times [p_2 - \epsilon, p_2)$. In this case we note that
$$\mathbb{E}\left[U^b \mathbf{1}_{A_4^\epsilon}\right] = \mathbb{E}\left[U^b \middle| A_4^\epsilon\right] \mathbb{P}\left[A_4^\epsilon\right],$$
and
$$\mathbb{E}\left[U^s \mathbf{1}_{A_4^\epsilon}\right] = \mathbb{E}\left[U^s | A_4^\epsilon\right] \mathbb{P}\left[A_4^\epsilon\right].$$
Now, since we assumed that the distribution of (V_1, V_2) is non-atomic and since both U^b and U^s are bounded then there exists a constant C such that for ϵ small enough it holds that $\mathbb{P}\left[A_4^\epsilon\right] < C\epsilon^2$, and so there exists a constant C_2 such that
$$\mathbb{E}\left[U^b \mathbf{1}_{A_4^\epsilon}\right] \geq \mathbb{E}\left[U^s \mathbf{1}_{A_4^\epsilon}\right] - C_2\epsilon^2 \tag{14}$$
for ϵ small enough.

5. Finally, let A_5^ϵ be the event that $(V_1, V_2) \in [p_1 + \epsilon, \infty) \times [p_2 - \epsilon, p_2)$. Here in the single customer strategy customer 1 buys an item for p_1 and customer 2 does not buy. In the bundling strategy the customers purchase a bundle for $p_1 + p_2$. Hence
$$\mathbb{E}\left[U^s \mathbf{1}_{A_5^\epsilon}\right] = p_1 \mathbb{P}\left[A_5^\epsilon\right]$$
and
$$\mathbb{E}\left[U^b \mathbf{1}_{A_5^\epsilon}\right] = (p_1 + p_2)\mathbb{P}\left[A_5^\epsilon\right].$$
Since the distribution of (V_1, V_2) is supported on $[0, M]^2$, and since $p_2 > 0$ (see note at the end of Section 2.1.1), then there exists a constant C_3 such that for ϵ small enough $\mathbb{P}\left[A_5^\epsilon\right] > C_3\epsilon$. Hence
$$\mathbb{E}\left[U^b \mathbf{1}_{A_5^\epsilon}\right] \geq \mathbb{E}\left[U^s \mathbf{1}_{A_5^\epsilon}\right] + p_2 C_3 \epsilon. \tag{15}$$
for ϵ small enough.

Since the events $\{A_i^\epsilon\}$ are disjoint and since $\mathbb{P}\left[\cup_i A_i\right] = 1$ then
$$u^b = \mathbb{E}\left[U^b\right] = \sum_{i=1}^{5} \mathbb{E}\left[U^b \mathbf{1}_{A_i^\epsilon}\right],$$
with a similar expression for u^s. Therefore, as a conclusion of Eqs. (11), (12), (13), (14) and (15) we have that for ϵ small enough
$$u^b(p_1 + \epsilon, p_2, p_1 + p_2) \geq u^s(p_1, p_2) - (C_1 \mathbb{P}\left[A_2^\epsilon\right] + C_2)\epsilon^2 + p_2 C_3 \epsilon,$$
and therefore for ϵ small enough
$$u^b(p_1 + \epsilon, p_2, p_1 + p_2) > u^s(p_1, p_2).$$
□

Proof of Theorem 3.2. Consider the bundling strategy with individual prices $a_i = \infty$ for all $i \in [n]$ (i.e., no single item sales) and $b = \mu - 2M\sqrt{n \log n}$. In this case the customers will either buy the bundle if its cost is less than the sum of their valuations, and buy nothing at all otherwise. Denote the sum of their valuations by $V = \sum_{i=1}^{n} V_i$.

Since $V_i \in [0, M]$ then $\mathbb{E}\left[V_i^2\right] \le M^2$. Hence by a version of Bernstein's inequality [4][5] we have that,

$$\mathbb{P}\left[V < b\right] \le \exp\left(-\frac{4M^2 n \log n}{2M^2 n + 2M^2\sqrt{n \log n}/3}\right)$$

and hence

$$\mathbb{P}\left[V < b\right] \le \frac{1}{n}.$$

Since the customers buy the bundle when $V \ge b$ then the seller's expected utility equals $\mathbb{P}\left[V \ge b\right] b$ and it holds that

$$\mathbb{P}\left[V \ge b\right] b \ge \left(1 - \frac{1}{n}\right)\left(\mu - 2M\sqrt{n \log n}\right).$$

Since $f_i > \delta$ in the interval $[0, M]$ then $\mathbb{E}\left[V_i\right] > M\delta/2$ and $\mu > nM\delta/2$, and so it holds that

$$\begin{aligned} \mathbb{P}\left[V \ge b\right] b &\ge \left(1 - \frac{1}{n}\right)\left(1 - \frac{4}{\delta}\sqrt{\frac{\log n}{n}}\right)\mu \\ &\ge \left(1 - \frac{4}{\delta}\sqrt{\frac{\log n}{n}} - O\left(\frac{1}{n}\right)\right)\mu, \end{aligned}$$

where the second inequality follows from the fact that $\mu \ge n\epsilon$.

Since the optimal strategy yields at least as much utility to the seller as this one, then

$$\max_{\bar{a}, b} u(\bar{a}, b) \ge \left(1 - \frac{4}{\delta}\sqrt{\frac{\log n}{n}} - O\left(\frac{1}{n}\right)\right)\mu.$$

□

5 Conclusion

We showed how sellers may maximize profits by offering bundles of items to rational groups of customers. Our work suggest a number of future research directions we wish to mention.

[5] We use the following version of Bernstein's inequality: Let $X_1, \ldots, X_n$ be independent random variables such that $\mathbb{E}\left[X_i\right] = 0$ and $|X_i| < M$ for all i. Then for any $t > 0$ it holds that

$$\mathbb{P}\left[\sum_i X_i > t\right] \le \exp\left(-\frac{t^2/2}{\sum_i \mathbb{E}\left[X_i^2\right] + Mt/3}\right).$$

5.1 Optimal auctions and optimal profit

Our results show that it suffices to bundle pairs of customers to increase profits under mild conditions, and that if the customers are bundled in large groups it is possible to extract profit which approaches the theoretical bound, as the group size increase. For example - assume that there are N individuals - $N/2$ are paired into rational groups, another $N/3$ are partitioned into rational groups of size 3, and the rest $N/6$ are partitioned into rational groups of size 6. Assuming that all valuations are drawn i.i.d. from the same distribution F - what is the optimal auction and by how much is it better than the single item auction?

5.2 Overlapping Rational Groups and Social Networks

The problem presented in the previous subsection can be generalized further to a situation where an individual may belong to more than one rational group: for example an individual may belong to a family and to a small start-up company. The two groups are rational and are offered different bundles. Understanding the optimal auction in this setup and its relationship to the social network structure is, in our opinion, an interesting open problem.

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